

The City College Department of Mathematics

Fall 2014 Math 20200 Final Exam

PART I: Answer ALL questions in this part. (70 points)

PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 7): Answer all questions (70 points)

1) Find the derivative for each of the following (6 points each).

a. $y = \ln(x^2 + 2^x + \ln 2)$ b. $y = e^{\arctan^2 x}$ c. $y = x\sqrt{x}(x^{\ln x})$

2) Compute each of the following integrals (6 points each).

a. $\int \frac{x^2 + 11x}{(x-1)(x+1)^2} dx$ c. $\int_{\frac{1}{2}}^1 \frac{1}{x^2\sqrt{x^2+4}} dx$

b. $\int \tan^5 x \sec^3 x dx$ d. $\int_1^2 (\ln x)^2 dx$

3) Evaluate the limits (4 points each).

a. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x \sin x}$ b. $\lim_{x \rightarrow 0} \frac{e^x + x^2}{\cos^2 x}$

4) (8 points) Find the volumes of the solids obtained by rotating the region bounded by the curves $y = x^2$ and $y = x$ about the following lines.

a. The x -axis b. The y -axis c. $y = 2$

- 5) (4 points) Evaluate the integral or show that it is divergent: $\int_0^1 \frac{\ln x}{x^2} dx$.
- 6) (8 points) Sketch the curve given by the equation $r = \cos(3\theta)$ in polar coordinates, labeling the x and y intercepts, and compute the area it encloses.
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END OF PART I

Part II: Answer 3 complete questions (10 points each)

7) A leaky 4 lb bucket and a rope of negligible weight are used to draw water from a well that is 100 ft deep. The bucket is filled with 50 lb of water and pulled up at a rate of 2 ft/s. The water leaks at the constant rate and when the bucket reaches the top, the bucket has 30 lb of water left. How much work is done in pulling this bucket to the top of the well?

8)

- a. For the curve given parametrically by $x = 3t^3 - 5t$, $y = t^4 + 1$, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- b. For the graph of the function $y = \ln(\cos x)$, set up an integral to compute the arc-length from $x = 0$ to $x = \frac{\pi}{3}$. Do not attempt to compute the integral.

9)

- a. Without attempting to evaluate it, determine whether the following integral converges or diverges. Justify your answer.

$$\int_1^{\infty} \frac{1}{\sqrt{x^5 + 2}} dx$$

- b. Write out the form of the partial fraction decomposition of the following function. Do not attempt to determine the numerical values of the coefficients.

$$f(x) = \frac{3x^3 - 5x + 4}{x(2-x)^2(5x^2+1)^3}$$

10)

- a. Draw a sketch of the conic whose equation is $4y^2 - x^2 - 2x - 24y + 31 = 0$. Identify which sort of conic it is. On your sketch, show and label whichever of the following are present: vertices, asymptotes, and foci.
- b. Compute $\cosh x$ and $\tanh x$, assuming that $\sinh x = 0.8$.

11)

- a. An insect population triples in size after 5 months. Assuming exponential growth, when will it quadruple in size?
- b. Sketch the curves $y = 2 - x^2$ and $y = x$. Compute the area of the entire region bounded by these two curves.