

The City College Department of Mathematics

Fall 2012 Math 20200 Final Exam

PART I: Answer ALL questions in this part. (70 points)

PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 7): Answer all questions (70 points)

1) Find the derivative for each of the following (6 points each).

a. $y = \ln\left(\frac{x^3}{\sqrt{x+1}}\right)$ b. $y = \arctan(x^2)$ c. $y = x^x + \sin(x)$

2) Compute each of the following integrals (6 points each).

a. $\int \frac{x^3 + 1}{x^3 + x} dx$ c. $\int \frac{1}{x^4 \sqrt{x^2 - 4}} dx$
b. $\int_0^{\pi/4} \tan^3(x) \sec(x) dx$ d. $\int_1^2 x \ln(x) dx$

3) Evaluate the limits (4 points each).

a. $\lim_{x \rightarrow \infty} \frac{(\ln(x))^2}{x}$ b. $\lim_{x \rightarrow \pi} \frac{\cos(x)}{x^2 + 1}$

4) (12 points) The region R in first quadrant of the xy plane is bounded by the curves $y = \ln(x)$, $y = 2$ and $x = 1$. Set up two integrals (method of washers and method of shells) for the volume of the solid obtained by rotating R around the x -axis. Use one of these to compute the volume.

5) (8 points) Sketch the curve given by the equation $r = 1 + \cos(\theta)$ in polar coordinates, labeling the x and y intercepts, and compute the area it encloses.

END OF PART I

Part II: Answer 3 complete questions (10 points each)

6) A conical tank, supported with point down, has diameter 8 feet and height 10 feet. Assuming it contains water to a height of 7 feet of water (which has density 62.5 pounds per cubic foot), compute the work done pumping all the water out over the top.

7)

- a. For the curve given parametrically by $x = t^2, y = t^3 - 3t$, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- b. For the graph of the function $y = \tan(x)$, set up an integral to compute the arc-length from $x = 0$ to $x = \pi/4$. Do not attempt to compute the integral.

8)

- a. Evaluate the integral or show it is divergent: $\int_0^\infty te^{-t} dt$.
- b. Write out the form of the partial fraction decomposition of the following function. Do not attempt to determine the numerical values of the coefficients.

$$f(x) = \frac{x^2 + 1}{x(x-1)^3(x^2 + 9)}$$

9)

- a. Draw a sketch of the conic whose equation is $y^2 + 2y = 9x^2 + 35$. Identify which sort of conic it is. On your sketch, show and label whichever of the following are present: vertices, asymptotes, and foci.
- b. Use the definitions of the hyperbolic trig functions to derive the identity: $\cosh^2(t) - \sinh^2(t) = 1$.

10)

- a.** A radioactive substance has a half-life of 15 years. If you begin with 400 pounds of the substance, how much will be left after 60 years?
- b.** Sketch the curves $y = x^3$ and $y = 4x$. Compute the area of the entire bounded region which has these two curves as boundaries.