Some Math 390 sample questions

Q 1 Let $\vec{r}(t) = (1+t^2)\,\hat{\imath} + (7-t^2)\hat{\jmath}$ be a curve in the plane. Find the arclength function $s(t)$ along the curve from the point $(1,7)$ and reparameterize the curve with respect to arclength.

Q 2 Show that the curve $\vec{r}(t) = t\cos(t)\hat{\imath} - t\hat{\jmath} + t\sin(t)\hat{k}$ lies on the cone $y^2 = x^2 + z^2$ and use that fact to sketch the curve. Parameterize the tangent line to the curve when $t = 2\pi$. Include in your sketch the unit tangent vector T at time $t = 2\pi$.

Q 3 Evaluate the line integral $\int_C (8x + \sqrt{y})\,ds$, where c the the path along the parabola $y = x^2$ from the origin to $(3,9)$.

Q 4 Let $\vec{F}(x,y) = 2y\hat{\imath} - 2x\hat{\jmath}$. Find the work done by the vector field $\vec{F}(x,y)$ along the arc of a semicircle centered at the origin from $(0,-2)$ to $(0,2)$ travelled in a clockwise direction. Does Green's Theorem apply here, and if so, state which version.

Q 5 Give the definition of: a vector field being conservative on a region R, a vector field being path-independent on a region R, a region R being connected, a region R being simply connected, a region R being open, a region R being closed, the boundary of a region R.

Q 6 Let $\vec{F}(x,y) = (2x-y)\hat{\imath} + (3x-5y)\hat{\jmath}$. Find the flux of the vector field $\vec{F}(x,y)$ outward across the circle of radius 3 centered at the origin. Does Green's Theorem apply here, and if so, state which version.

Q 7 Find the surface area of S, where S is the part of the paraboloid $z = x^2 + y^2$ inside $x^2 + y^2 = 16$.

Q 8 State Green's Theorem in the plane in the component (or vector) form for work (or flux) and/or the multi-curve version.

Q 9 Parameterize a curve from $(0,0,-5)$ to $(0,0,5)$ which passes through $(3,4,0)$ and remains a constant distance from the origin. Parameterize the tangent line to your curve at $(3,4,0)$ and sketch.

Q 10 Use Green's Theorem to find the work done by the vector field $\vec{F} = (4x\sin(2x) - 3y)\hat{\imath} + (5x + y^3)\hat{\jmath}$ along the circle of radius 4 centered the origin traversed clockwise.

Q 11 Prove that if the length of a vector function $\vec{r}(t)$ is constant, then $\vec{r}(t)$ and its derivative $\vec{r}'(t)$ are perpendicular.

Q 12 Find the surface area of S, where S is the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 9$.

Q 13 Find the surface area of S,where S is the part of the paraboloid $z = 9 - x^2 - y^2$ above the xy plane.

Q 14 State the definition of a region in space being connected.

Q 15 Find the surface area of S, where S is the part of the paraboloid $z = 9 - x^2 - y^2$ in the first octant.

Q 16 Prove the following: If $\vec{F}(x,y,z) = P(x,y,z)\hat{\imath} + Q(x,y,z)\hat{\jmath} + R(x,y,z)\hat{\mathrm{k}}$ is a vector field with continuous second partial derivatives, then the divergence of the curl of $\vec{F}$ is zero.

Q 17 The planar sets $1 \leq \Big\{(x,y)|(x^2+y^2) < 4\Big\}$, $\Big\{(x,y)|(x^2+y^2) < 4\Big\}$, $\Big\{(x,y)|(x^2+y^2) \leq 4\Big\}$, $\Big\{(x,y)|(x^2+y^2) \leq 9\Big\}$, are each: **Open** **Closed** **Neither** **Both**

Q 18 Prove the following: If f is a function with continuous second partial derivatives, then the curl of the gradient of f is zero.

Q 19 Evaluate the surface integral: $\int\int_K z^3\, dS$, where K is the part of the sphere $x^2 + y^2 + z^2 = 4$ which lies below the xy plane with $x \geq 0$ and $y \leq 0$.

Q 20 State the definition for a region in space to be simply connected and give an example of a region which is not simply connected.

Q 21 Evaluate the surface integral: $\int\int_K 2z^5\, dS$, where K is the part of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant.

Q 22 Parameterize the torus obtained by rotating the circle of radius 3 centered at (5,0) in the xz plane around the z axis. Use your parameterization to find an equation of a tangent plane to a point on the torus in the yz plane.

Q 23 Evaluate the surface integral: $\int\int_K z^4\, dS$, where K is the part of the sphere $x^2 + y^2 + z^2 = 9$ which lies above the xy plane with $x \leq 0$ and $y \geq 0$.

Q 24 Let C be the part of the round cylinder of radius 3 around the z-axis between the planes $z = 2$ and $z = 4$, with $x \geq 0$. Compute the surface integral via an efficient method: $\int\int_C (2x + 3y^3 + y^2 + x^2 + xyz + xy^3z + 3z^4 + x^2y^3z)\, dS$

Q 25 Evaluate the surface integral: $\int\int_K 4z^3\, dS$, where K is the part of the sphere $x^2 + y^2 + z^2 = 4$ which lies below the xy plane with $x \geq 0$ and $y \geq 0$.

Q 26 Let S a thin metal plate given by the surface $z = 3x^3 + 2xy + 2y^4$ with $1 \leq x \leq 5$ and $2 \leq y \leq 3$, with variable density given by $\delta(x,y,z) = 2 + x^2 + xy + 3z$. Set up an integral to find the mass of the surface.

Q 27 Let $\vec{F}(x,y) = (2+3z)\hat{\imath} + (3x^2 - 5y^2)\hat{\jmath} + (3x+2z)\hat{k}$. Find the flux of the vector field $\vec{F}(x,y)$ upward across the part of the sphere of radius 5 centered at the origin which lies above the plane $z = 3$.

Q 28 Find the flux of the vector field $\vec{F} = 2x\hat{\imath} + 2z\hat{\jmath} + 3y\hat{k}$ upward across the part of the plane $x + y + z = 4$ in the first octant.

Q 29 Let S be the sphere centered at the origin of radius 2. Compute the surface integral $\int\int_S (2x + 3y^3 + xy^2 + 2x^3z^2 + xyz + xy^2z + 3z^4 + x^2y^2z)\, dS$ via whatever method seems most convenient.

Q 30 The parametric surface S is given by the vector function
$\vec{r}(u,v) = u\hat{\imath} + u^2\cos(v)\hat{\jmath} + u^2\sin(v)\hat{k}$ with the domain $0 \le u \le 4, 0 \le v \le \pi$.
a) Give an equation of the tangent plane to the surface at the point where $u = 1, v = \pi$
b) Set up an integral to find the area of the surface.

Q 31 Let S be the surface of revolution obtained by rotating the curve in the xz-plane given by $z = 1 + x^2$ between $x = 1$ and $x = 2$ around the z-axis. Set up a definite integral to find the surface area of S.

Q 32 Use Green's Theorem to find the area inside the ellipse $x^2 + 4y^2 = 16$.

Q 33 Let S a surface parameterized as $\vec{r}(u,v) =< u - v, 3v, u + v >$ with $2 \le u \le 4$ and $0 \le v \le 3$. Find the surface area of S.

Q 34 For each of the following regions in space, state whether or not they are open, connected, simply connected: $\{x^2+y^2+z^2 < 4\} \cap \{x^2+z^2 > 2\}$, $\{1 \le x^2+y^2+z^2 \le 4\}$, the domain of $f(x,y,z) = \frac{1}{xyz}$, $\{x^2+y^2+z^2 > 4\}$, $\{1 < x^2+y^2 < 4, 0 < z < 5\} \cup \{-2 < x < 2, -2 < y < 2, z = 2\}$

Q 35 Let $\vec{F}(x,y) = (2x - y)\hat{\imath} + (3x - 5k)\hat{\jmath} + (3x + 2z)\hat{k}$. Find the flux of the vector field $\vec{F}(x,y)$ outward across the surface of the tetrahedron with corners at the origin, (2,0,0), (2,2,0), and (2,2,4).

Q 36 Let S be the surface of revolution obtained by rotating the curve in the first quadrant of the xz-plane given by $x = f(t), z = g(t), a < t < b$ around the z-axis. Set up a definite integral to find the surface area of S.

Q 37 Either prove the following statement or give an example to prove that it is false: Every smooth vector field which is defined on all space is the gradient of a scalar function.

Q 38 Let $\vec{F}(x,y) = (2x - y)\hat{\imath} + (3x - 5k)\hat{\jmath} + (3x + 2z)\hat{k}$. Find the flux of the vector field $\vec{F}(x,y)$ outward across the surface of the round cylinder of radius 3 around the z-axis between the planes $z = 2$ and $z = 4$, together with the top and bottom end caps.

Q 39 Let K be the surface defined by $\left\{x^2 + y^2 + z^2 = 4, x > 0, y < 0, z < 0\right\}$ and let S be the entire sphere of radius 2 defined by $\{x^2 + y^2 + z^2 = 4\}$.
a) Compute the following surface integral: $\iint_K \vec{F} \cdot \mathrm{n}\, dS$, for $\vec{F} = 3x\hat{\imath} + 2\hat{\mathrm{k}}$.
b) Compute the following surface integral: $\iint_S \vec{F} \cdot \mathrm{n}\, dS$, for $\vec{F} = (2x^2 + x^3)\hat{\imath} + x^2\hat{\jmath}$

Q 40 Let $\vec{F}(x, y) = (2x + y)\hat{\imath} + (3x - 5k)\hat{\jmath} + (3x + 2z)\hat{\mathrm{k}}$. Find the flux of the vector field $\vec{F}(x, y)$ inward across the surface of the sphere of radius 3 centered at the origin.

Q 41 Let R be the region $\{x^2 + z^2 \geq 25\}$. If $\vec{F}$ is defined on R and $\nabla \times \vec{F} = 0$ on R, is there necessarily any closed curve C in R so that $\oint_C \vec{F} \cdot \mathrm{T}\, ds > 0$? Explain your answer carefully.

Q 42 Let S be the sphere centered at the origin of radius 3. Compute $\iint_S y^2\, dS$ via whatever method seems most convenient.

Q 43 Find the area of the region between the line y = x and the curve given by $x = t^2 + t^3, y = t^6 + t^8, t = [0, 1]$. Hint: it may help to use Green's Theorem.

Q 44 Let $\vec{F}(x, y) = (2x - y)\hat{\imath} + (3x - 5k)\hat{\jmath} + (3x + 2z)\hat{\mathrm{k}}$. Find the flux of the vector field $\vec{F}(x, y)$ outward across the surface of the cube with corners at the origin and $(2, 2, 2)$.

Q 45 Use Stokes' Theorem to find the work done by the vector field
$\vec{F} = (3xy^2 + 5yz + 3y)\hat{\imath} + (4xyz + 4x - 2y + 6z)\hat{\jmath} + (3xy^2 + 6x + 2z)\hat{\mathrm{k}}$ along the circle of radius 4 centered in the xz plane beginning at the point $(4, 0, 0)$ and traveling initially in the direction toward $(0, 0, 4)$.

Q 46 State Stokes' Theorem, Divergence Theorem, etc.

Q 47 Evaluate the surface integral $\iint_T (x + 2y + 2z)\, dS$, where T the part of the plane $x + y + z = 6$ that lies inside the cylinder $x^2 + y^2 = 4$.

Q 48 Let S be the sphere given by: $x^2 + y^2 + z^2 = 16$. Find the flux of the vector field $(6x + 7yz^2)\hat{\imath} + (4x + 3y - 8z)\hat{\jmath} + (2 - z)\hat{\mathrm{k}}$ outward through S. Explain the method you use to compute the flux.

Q 49 Let S be the hemisphere given by: $x^2 + y^2 + z^2 = 16, z \leq 0$. Find the flux of the vector field $(6x + 7yz^4)\hat{\imath} + (4x + 3y - 8z)\hat{\jmath} + (2 + xz + y^3z - z)\hat{\mathrm{k}}$ downward through S. Explain the method you use to compute the flux.

Q 50 Use Green's Theorem to find the work done by the vector field
$\vec{F} = (3xe^x + 9y)\hat{\imath} + (2y\cos(2y) - 4x + 3)\hat{\jmath}$ along the path of four segments going from $(0, 0)$ to $(6, 0)$ to $(6, 3)$ to $(0, 3)$ back to $(0, 0)$. If F represents the wind, is the path overall more against the wind or with the wind?

Q 51 Prove the following:
If $\oint_C \vec{F} \cdot \mathrm{T}\, ds = 0$ for any closed curve in a region R, then $\vec{F}$ is path-independent in R

Q 52 Evaluate the line integral $\int_C (2x+3\sqrt{y})\, ds$, where c the the path along the parabola $y = x^2$ from the origin to $(3, 9)$.

Q 53 Let $F = (y^2 - 2x)\hat{\imath} + (3y^2 + 2xy)\hat{\jmath}$. Find the work done along the path $\vec{r}(t) = (4+t^2+t^3)\hat{\imath}+(2-t^2+2t^7)\hat{\jmath}$, with t in $[0, 1]$. It may help to notice that $\vec{F}$ is conservative.

Q 54 Gracie the Goose is currently at the north pole, at (0,0,5). She would like to fly along the surface of the spherical earth to a more tropical location and decides to fly to visit a friend on the equator in Gabon at (0,5,0). The vector field representing winds on the surface of the earth is given by $\vec{F} =< 3yz, 3xy + z, xyz - y >$. If she flies directly there, how much work does she do against the wind? In particular, does she on average have a tailwind or a headwind?

Q 55 Prove the following:
If $\vec{F}$ is a vector field with continuous first and second derivatives on all of space , then $\int\int_S \nabla \times \vec{F} \cdot \mathbf{\hat{n}}\, dS = 0$ for any sphere S.

Q 56 Let H be the surface $x^2 + z^2 = \frac{1}{y^2}$ with $1 \le y \le 2$, which can be obtained by rotating the graph of $z = \frac{1}{y}$ in the $y - z$ plane around the y-axis. Sketch the surface, parameterize the surface explicitly as functions of u and v and set up an integral to find its area using the parametric method.

Q 57
Evaluate the surface integral

$$\int\int_S 3\sqrt{x}\, dS$$

where S is the part of an elliptic paraboloid parameterized as $\vec{r}(u, v) =< u^2, u\sin(v), u\cos(v) >$, with $0 \le u \le 2$ and $0 \le v \le \pi$.

Q 58 Evaluate the surface integral $\int\int_T (4-x-y-z)\, dS$, where H the part of the plane $x + y + z = 1$ that lies in the first octant.

Q 59 Compute $\int\int_B \nabla \times \vec{F} \cdot \hat{n}\, dS$ where F is the vector field
$\vec{F} = x\hat{\imath} + (y + z + 4)\hat{\jmath} + (2xyz^8 + 6xz)\hat{\mathrm{k}}$, B is the part of the sphere $x^2 + y^2 + z^2 = 25$ which lies below the plane $z = -4$, and $\hat{n}$ is the outward unit normal to the sphere.

Q 60 Compute the flux of the vectorfield $\vec{F} = (2x+7y^2)\hat{\imath}+(3x-5y+xz)\hat{\jmath}+(3x^2y+6z)\hat{\mathrm{k}}$ outward across the cube with opposite corners at $(1, 1, 1)$ and $(-1, -1, -1)$.

Q 61
Prove the following: If $\vec{F}(x,y,z) = P(x,y,z)\hat{\imath} + Q(x,y,z)\hat{\jmath} + R(x,y,z)\hat{\mathrm{k}}$ is a vector field with continuous second partial derivatives defined on all of space, then the divergence of the curl of $\vec{F}$ is zero.

Q 62 Prove the following: If f is a function with continuous second partial derivatives, then the curl of the gradient of f is zero.

Q 63 Evaluate the surface integral $\displaystyle\int\int_K z\,dS$, where K the part of the sphere $x^2+y^2+z^2 = 9$ with $x \le 0, y \ge 0, z \le 0$.

Q 64 Compute $\displaystyle\int\int_P \nabla \times \vec{F} \cdot \hat{n}\,dS$ where F is the vector field
$\vec{F} = xz\hat{\imath} - yz\hat{\jmath} + (2xy^2z^2 + 6xz)\hat{\mathrm{k}}$, P is the part of the paraboloid $z = 25 - x^2 - y^2$ which lies above the plane $z = 9$, and $\hat{n}$ is the upward unit normal to the paraboloid.

Q 65 Compute the flux of the vectorfield $\vec{F} = (2x + 7y)\hat{\imath} + (3x - 5y + xz)\hat{\jmath} + (6z - 2)\hat{\mathrm{k}}$ upward across the hemisphere $x^2 + y^2 + z^2 = 4$ with $z \ge 0$.

Q 66 The parametric surface S is given by the vector function
$\vec{r}(u,v) = uv\hat{\imath} + (u + v)\hat{\jmath} + (u - v)\hat{\mathrm{k}}$ with the domain $1 \le u^2 + v^2 \le 16$. Give an equation of the tangent plane to the surface at the point where $u = -2, v = 1$ and find the area of the surface.

Q 67 Let V be the part of the solid ball $x^2 + y^2 + z^2 \le 4$ which lies below the plane $z = 1$. Let $\vec{F}$ be the vector field $2x\hat{\imath} + 2y\hat{\jmath} + 2z\hat{\mathrm{k}}$. Let $\vec{n}$ be the outward unit normal on the surface S which encloses V. Calculate the flux outward of $\vec{F}$ across S in two ways:
(a) directly, as a surface integral AND
(b) as a triple integral, using the Divergence Theorem.

Q 68
Find the work done around the circle of radius 3 centered at the origin in the xy plane by the vector field $6y\hat{\imath} + 4xy\hat{\jmath}$ if the circle is traversed clockwise.

Q 69 Let C be the intersection of the elliptic paraboloid $y = x^2 + z^2$ and the plane $y = 4$. Calculate the work $\displaystyle\int_C \vec{F} \cdot \mathrm{T}\,ds$ done around C travelled in a counter-clockwise direction when viewed from direction of the positive y axis, where $\vec{F} = -y\hat{\imath} + x^2\hat{\jmath} + z^3\hat{\mathrm{k}}$ in two ways:
(a) directly as a line integral AND
(b) as a double integral, by using Stokes' Theorem.

Q 70 Let $\vec{F} =< 3y\cos(z) + x, x^2z - y, x\sin(y) >$.
a) Find $\vec{G} = curl(\vec{F})$.
b) Show that the flux of $\vec{F}$ is zero over any closed surface R in space which encloses a solid region Q.

Q 71 Evaluate the surface integral $\int\int_T (x+2y+2z)\,dS$, where T the part of the plane $x+y+z=6$ that lies inside the cylinder $x^2+y^2=4$.

Q 72 Let S be the sphere given by: $x^2+y^2+z^2=16$. Find the total flux $\int\int_S \vec{F}\cdot\hat{n}\,dS = \int\int_S \vec{F}\cdot d\vec{\mathbf{S}}$ of the vector field $(6x+7yz^2)\hat{\imath}+(4x+3y-8z)\hat{\jmath}+(2-z)\hat{\mathrm{k}}$ outward through S. Explain the method you use to compute the flux.

Q 73
Find the flux of the vector field $\vec{F} = 5x^3\hat{\imath} + 5y^3\hat{\jmath} + (5z^3+4)\hat{\mathrm{k}}$ downward across the hemisphere of radius 2 centered at the origin below the xy plane.

Q 74 Calculate the surface integral to find the flux

$$\int\int_S \vec{F}\cdot d\vec{S} = \int\int_S \vec{F}\cdot \mathrm{n}\,dS,$$

where S is the part of the sphere $x^2+y^2+z^2=9$ with $z\le 0, y\ge 0$ and $\vec{F} = x\hat{\imath}+y\hat{\jmath}+(z+2)\hat{\mathrm{k}}$, and $\vec{n}$ is the unit normal vector field along S directed away from the origin.

Q 75 Evaluate the surface integral

$$\int\int_S 4yz\,dS$$

where S is part of an elliptic paraboloid parameterized as $\vec{r}(u,v) =< u^2, u\sin(v), u\cos(v) >$, with $0\le u\le 2$ and $0\le v\le \pi$.

Q 76 Find the work done by the vector field

$$\vec{F} = (e^x + x^2y)\hat{\imath} + (e^y - xy^2)\hat{\jmath}$$

around the circle of radius 3 centered at the origin travelled clockwise.

Q 77 Let C be the intersection curve of the surfaces $z=3x-7$ and $x^2+y^2=1$, oriented clockwise as seen from above. Let $\vec{F} = (4z-1)\hat{\imath}+2x\hat{\jmath}+(5y+1)\hat{\mathrm{k}}$. Compute the work integral $\int_c \vec{F}\cdot d\vec{r} = \int_c \vec{F}\cdot\vec{T}\,ds$ two ways:
(a) directly as a line integral
(b) as a double integral, using Stokes' Theorem.

Q 78 Let T be the part of the surface $z=9-x^2-y^2$ which lies above the xy plane, with upward unit normal. Let B be the disk of radius 3 in the xy plane centered at the origin with downward unit normal. Find the total combined flux of the vector field $\vec{F} = (2x+9y-2yz)\hat{\imath}+(3x+y-5z)\hat{\jmath}+(x^2+y^2+2z^2)\hat{\mathrm{k}}$ across T and B in the directions of their given normals.

Q 79 Let C be the path from $P = (2,0,0)$ to $Q = (0,2,0)$ to $R = (0,0,2)$ and then back to P, each time along a straight segment. Calculate the work done around C in the given direction, where $\vec{F} = (2x+4y)\hat{\imath} + 2yz\hat{\jmath} + y^2\hat{\mathrm{k}}$, in two ways:
(a) directly, via line integrals AND
(b) as a double integral, by using Stokes' Theorem.

Q 80 Suppose $\vec{F}$ and $\vec{G}$ are vector fields such that for some non-zero real number r, $\nabla \times (\vec{F} - r\vec{G}) = 0$ for all of space. Suppose that for some simple closed curve C in space, $\oint_C \vec{F} \cdot \mathrm{T}\, ds = a$. Prove that $\oint_C \vec{G} \cdot \mathrm{T}\, ds = \frac{a}{r}$.

Q 81 Let $\vec{F}$ be a C^2 vector field defined on all of space except the z-axis. Let S be the unit sphere centered at the origin. Think of the unit sphere as the surface of a globe; that is, the north pole is the point (0,0,1) and the south pole is the point (0,0,-1). Say $\nabla \times \vec{F} = 0$ at all points where $\vec{F}$ is defined on the surface of the sphere, but $\nabla \times \vec{F} \neq 0$ at some points not on the surface of the sphere. Say that $\displaystyle\oint_E \vec{F} \cdot \mathrm{T}\, ds = \sqrt{2}$ for E the curve going around the equator once, parameterized in a clockwise direction when viewed from above.
a) Evaluate: $\displaystyle\oint_C \vec{F} \cdot \mathrm{T}\, ds$ for C the curve which goes around the arctic circle, always going east. If you apply a theorem to evaluate the integral, refer to the name of the theorem and describe accurately the regions involved in the application and their orientations. (The arctic circle is the line of latitude 23° away from the North Pole.)
b) Evaluate: $\displaystyle\oint_D \vec{F} \cdot \mathrm{T}\, ds$ for D the a curve which passes through the following cities in the order given, never crossing the arctic circle or the antarctic circle and always taking the most direct route:

- New York NY, USA
- Seattle WA, USA
- Honolulu HI, USA
- Tokyo, Japan
- Calcutta, India
- Istanbul, Turkey
- Paris, France
- New York NY, USA
- Seattle WA, USA
- Seoul, Korea
- Melbourne, Australia
- Cairo, Egypt
- Sao Paulo, Brazil
- Mexico City, Mexico

- Chicago IL, USA
- New York NY, USA

Q 82 Let S a surface parameterized as $\vec{r}(u,v) =< u-v, 3v, u+v >$ with $2 \leq u \leq 4$ and $0 \leq v \leq 3$. Find the surface area of S.

Q 83 Let $\vec{F}$ be a vector field defined on all of space except for the region $x^2+y^2+z^2 < 1$, with $\nabla \cdot \vec{F} = 0$ everywhere where $\vec{F}$ is defined. If the net flux of $\vec{F}$ outward across the sphere of radius 2 centered at the origin is 7π, find the net flux of $\vec{F}$ outward across the these two surfaces. Make sure to explain your analysis.
a) $x^2+y^2+z^2 = 16$ b) $(x-3)^2+(y+3)^2+(z-5)^2 = 9$

Q 84
Find the surface area of S, where S is the part of the paraboloid $z = x^2+y^2$ below the plane $z = 9$.
Find the surface area of S, where S is the part of the paraboloid $z = x^2+y^2$ inside $x^2+y^2 = 9$.
Find the surface area of S,where S is the part of the paraboloid $z = 9-x^2-y^2$ above the xy plane.
Find the surface area of S, where S is the part of the paraboloid $z = 9-x^2-y^2$ in the first octant.

Q 85 Find the flux of the vector field $\vec{F} = 2x\hat{\imath} + 2z\hat{\jmath} + 3y\hat{\mathrm{k}}$ upward across the part of the plane $x+y+z = 4$ in the first octant directly and also using one of the integration theorems.