Math 360 Week of April 28 Sheet

1. Find the matrix of the Weingarten map $\mathcal{W}_{p,S}$ of surface of revolution $\sigma(f(u)\cos v, f(u)\sin v, g(u))$ with respect to the basis $\{\sigma_u, \sigma_v\}$ of T_pS, when the generating curve is parametrized by arc length, i.e. $f'(u)^2 + g'(u)^2 = 1$.

2. Use the previous problem to find matrices of the Weingarten maps for the following surfaces S (when a is a positive constant):
   (a) the sphere $x^2 + y^2 + z^2 = a^2$.
   (b) the cylinder $x^2 + y^2 = a^2$.

3. Using the parametrization $\sigma(u, v) = a(\cos u \cos v, \cos u \sin v, \sin u)$ of the unit sphere $x^2 + y^2 + z^2 = 1$, find the geodesic curvature of each of the following curves on the unit sphere.
   (a) The equator $\gamma_1(t) = \sigma(0, t)$.
   (b) The curve $\gamma_2(t) = \sigma(\frac{\pi}{4}, t)$.

4. Forgetting the pseudosphere of revolution we will study the upper-half (v, w) plane $w > 0$ with metric

$$\frac{dv^2 + du^2}{w^2}.$$

   This will be our model of hyperbolic geometry.
   (a) Show that the geodesic ODEs for for hyperbolic geometry reduce to

$$\begin{aligned} v'' - \frac{2v'w'}{w} &= 0 \\ w'' + \frac{v'^2 - w'^2}{w} &= 0. \end{aligned}$$

   (b) Show that the lines vertical half-lines $v = v_0, w > 0$ are geodesics in hyperbolic geometry.
   (c) Show that the semi-circles

$$(v(t), w(t)) = (v_0 + r\cos\theta(t), r\sin\theta(t)), 0 \leq \theta(t)\pi, r > 0$$

   centered on the v-axis are geodesics of the hyperbolic (v, w) plane when parametrized by arc length. (Hint: first show that $\theta' = k^2 \sin^2\theta$ and $\theta'' = k^2 \sin\theta\cos\theta$ for some constant k because (v, w) is parametrized by arc length).

5. (Pressley): $7.3.1, 7.3.2, 7.3.3, 9.1.2, 9.2.1 - 9.2.5$.