Math 323 — **Quiz 1** — March 8, 2024

The questions are to be answered directly on this paper as indicated. Total: 50 pts.

Name:_____

1. Give **definitions** of each of the following: (20 pts.)

   (a) The sequence of reals (s_n) has limit (i.e. diverges to) $-\infty$.

   (b) $\limsup s_n$ of a sequence (s_n) of reals.

   (c) The sequence (s_n) of reals is **Cauchy**.

   (d) $\sup(S) = M \in \mathbb{R}$, using inequalitites equivalent to the words in definition 4.3

2. **Define** what it means for a sequence (s_n) of reals to converge to $s \in \mathbb{R}$, and then **use** (10 pts.) **the definition to prove** that $\lim \frac{n}{n+1} = 1$:

3. **True or false?** Indicate your answer for each statement by placing **T** or **F** in the (10 pts.)
blank at the start. **Extra credit:** give a counterexample below each false statement.

_____ Every monotonic sequence of reals has a limit.

_____ Every bounded sequence of reals is convergent.

_____ If $\limsup s_n = \liminf s_n$ then the sequence (s_n) converges.

_____ Every convergent sequence of reals is Cauchy.

_____ If $\limsup s_n = 1$ then the sequence (s_n) is bounded.

4. **Prove** from basic principles and definitions that a convergent sequence is bounded: (10 pts.)