

Please PRINT your name below. SHOW ALL WORK and SIMPLIFY answers. Write clearly and cross-out work not to be graded. The questions are to be answered directly on this paper as indicated. NO electronic devices allowed. Total: 100 pts.

Name: _____

1. FOR THIS ENTIRE PROBLEM, let $f(x, y) = x^2 + y^2$.

(a) Find the gradient of f , $\nabla f(x, y)$: (10 pts.)

(b) Find $D_{\mathbf{u}}f(1, -1)$, the directional derivative of f at $(1, -1)$ in the direction $\mathbf{u} = \langle 1/2, \sqrt{3}/2 \rangle$: (20 pts.)

(c) Find the direction of maximum increase in the function $f(x, y)$ at the point $(1, -1)$: (10 pts.)

(d) Find the differential dz when $z = f(x, y)$: (10 pts.)

- (e) Find the surface area of the portion of the graph $z = f(x, y) = x^2 + y^2$ (25 pts.) that lies *below* the plane $z = 4$:

2. Use spherical coordinates to compute the volume of the region below (25 pts.) $x^2 + y^2 + z^2 = 4$, above $z = \sqrt{x^2 + y^2}$, between $y = x$ and $x = 0$ with $y \geq 0$ (your limits of integration must correspond to the indicated region):