

$$14) 4x^2 + 9y^2 + 9z^2 = 36$$

$$\frac{4x^2}{36} + \frac{9y^2}{36} + \frac{9z^2}{36} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{4} = 1$$

$$\frac{x^2}{(3)^2} + \frac{y^2}{(2)^2} + \frac{z^2}{(2)^2} = 1$$

if $x=0$ then $\frac{y^2}{(2)^2} + \frac{z^2}{(2)^2} = 1$
circle of radius $r=2$

if $y=0$ then

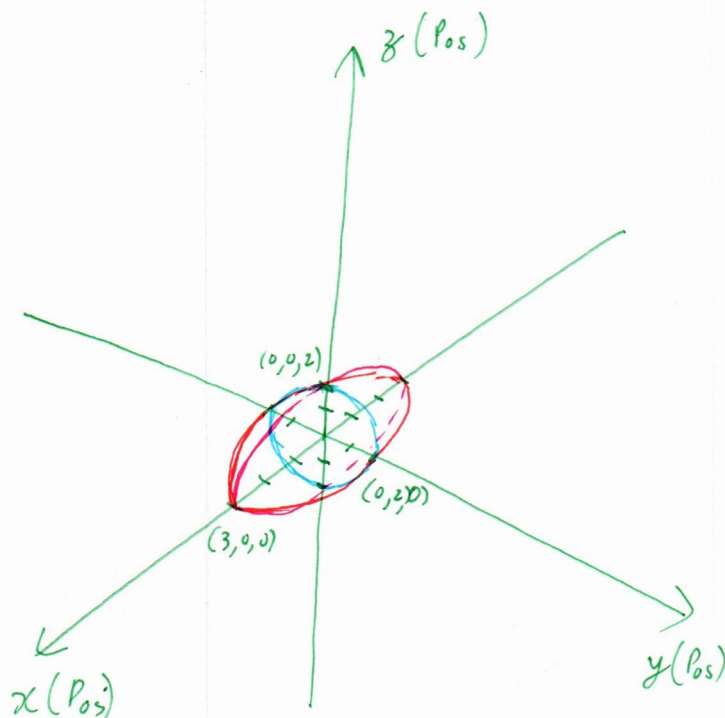
$$\frac{x^2}{(3)^2} + \frac{z^2}{(2)^2} = 1$$

ellipse with longer side on x

if $z=0$ then

$$\frac{x^2}{(3)^2} + \frac{y^2}{(2)^2} = 1$$

ellipse with longer side on x



ellipsoid with centered at origin
with intercepts: $x=\pm 3, y=\pm 2, z=\pm 2$

$$16) z^2 - 4x^2 - y^2 = 4$$

if $x=k$ then

$$z^2 - 4k^2 - y^2 = 4$$

$$z^2 - y^2 = 4k^2 + 4$$

hyperbola opening on z

if $y=k$ then

$$z^2 - 4x^2 - k^2 = 4$$

$$z^2 - 4x^2 = k^2 + 4$$

hyperbola opening on z

16) continued...

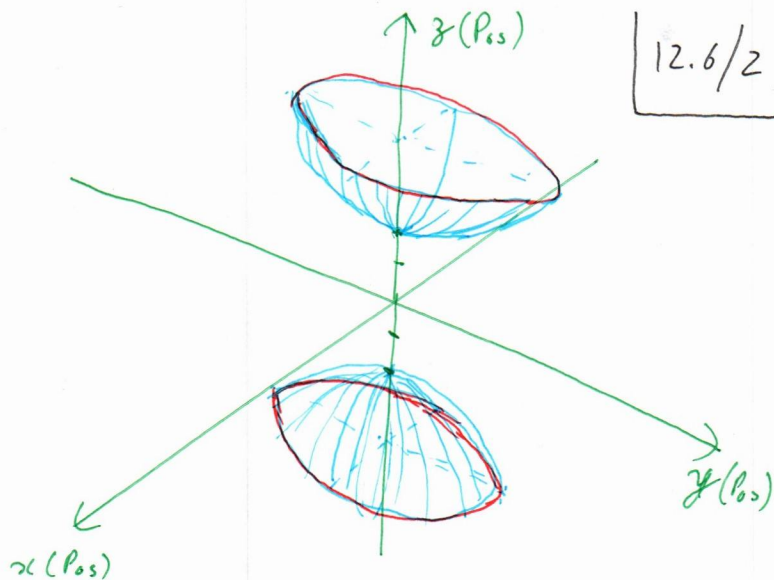
if $z = k$ then

$$k^2 - 4x^2 - y^2 = 4$$

$$k^2 - 4 = 4x^2 + y^2$$

$$1 = \frac{x^2}{\frac{k^2-4}{4}} + \frac{y^2}{k^2-4}$$

ellipse with longer side on y



12.6/2

hyperboloid of 2 sheets (with z -axis)

18) $3x^2 + y + 3z^2 = 0$

$$y = -3x^2 - 3z^2$$

if $x = 0$, then

$$y = -3z^2$$

parabola opening toward $-y$

if $z = 0$, then

$$y = -3x^2$$

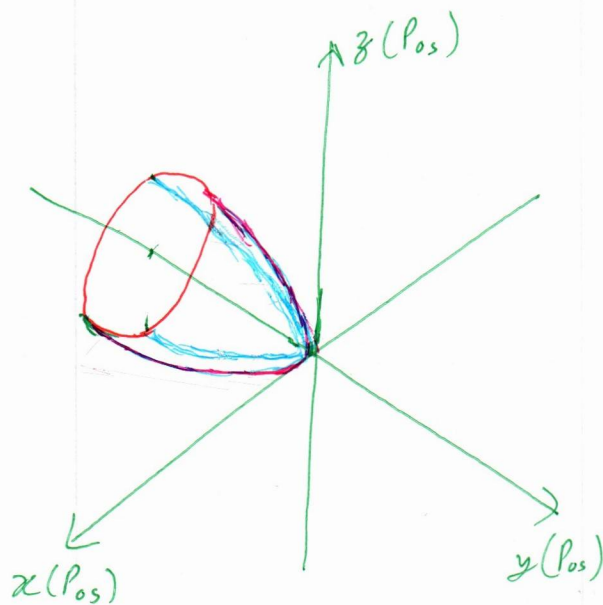
parabola opening toward $-y$

if $y = k$, then

$$k = -3x^2 - 3y^2 \text{ or } -k = 3x^2 + 3y^2$$

so if $k < 0$, then it is

a circle



a circular paraboloid with the y -axis, opening in the negative y -direction, and vertex the origin.

$$20) 3x^2 - y^2 + 3z^2 = 0$$

if $x = k$, then

$$3k^2 - y^2 + 3z^2 = 0$$

$$3k^2 = y^2 - 3z^2$$

hyperbolas opening on y

if $k \neq 0$

and if $k = 0$, a line

$$y = \sqrt{3}z$$

if $y = k$, then

$$3x^2 - k^2 + 3z^2 = 0$$

$$3x^2 + 3z^2 = k^2$$

circle if $k \neq 0$

and a point if $k = 0$

if $z = k$, then

$$3x^2 - y^2 + 3k^2 = 0$$

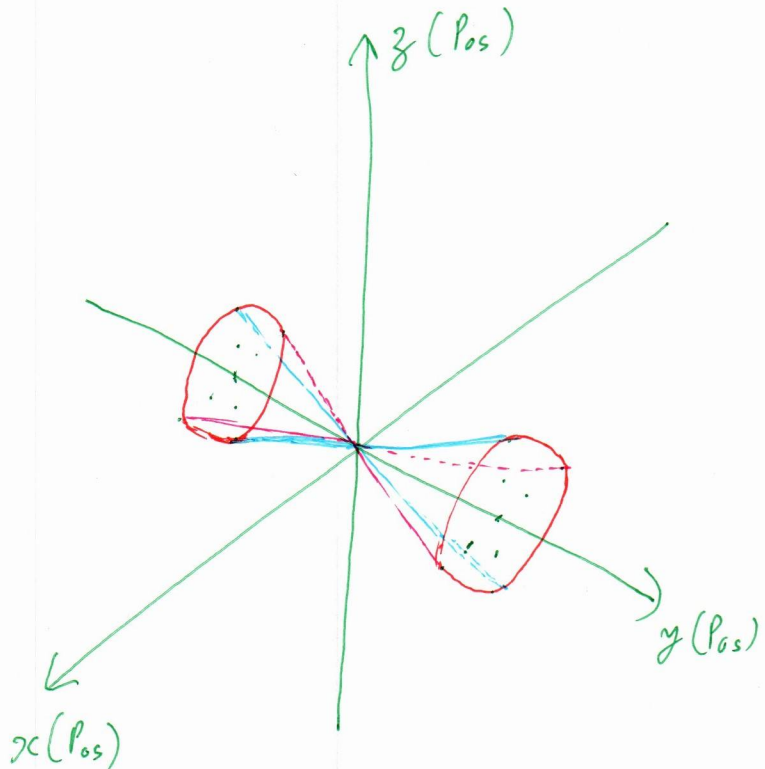
$$3k^2 = y^2 - 3x^2$$

hyperbolas opening on y

if $k \neq 0$

and if $k = 0$, a line

$$y = \sqrt{3}x$$



a circular cone with the y -axis
and vertex the origin.

$$22) x = y^2 - z^2$$

if $x = k$, then

$$k = y^2 - z^2$$

if $k = 0$, then 2 lines $y = \pm z$

if $k > 0$, then

hyperbola opening on y

if $k < 0$, then

hyperbola opening on z

if $y = k$, then

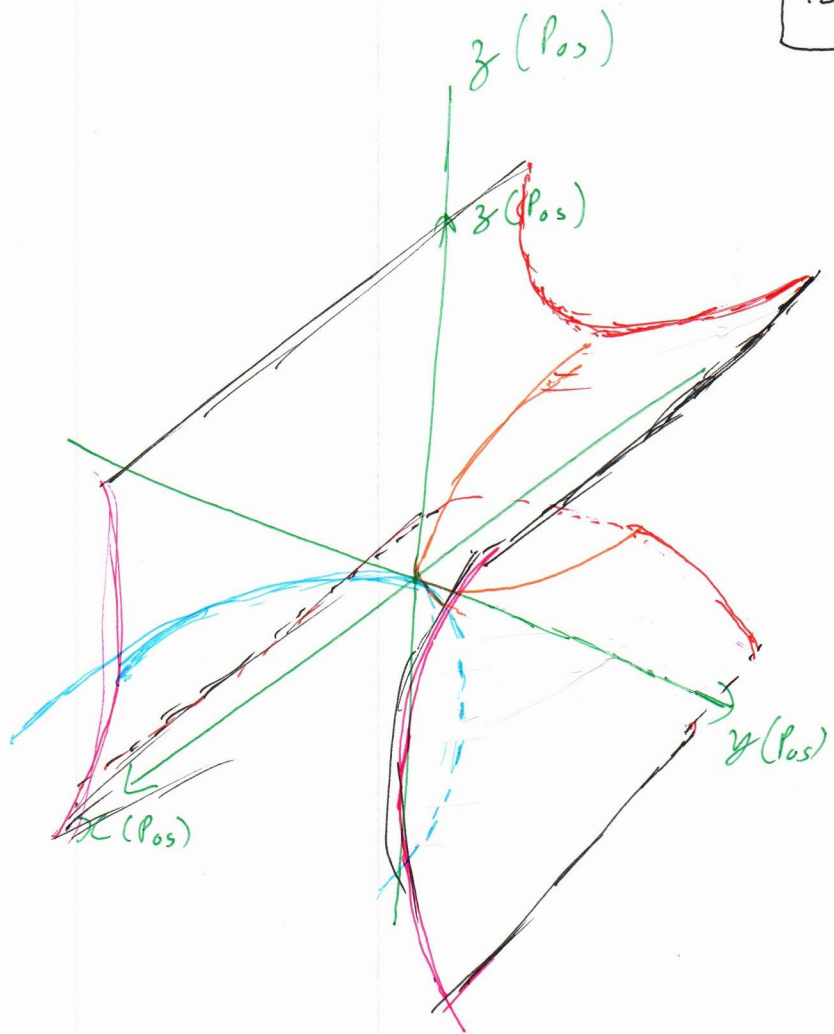
$$x = k^2 - z^2$$

parabola opening on $-x$ direction

if $z = k$, then

$$x = y^2 - k^2$$

parabola opening on $+x$ direction



hyperbolic paraboloid

centered at $(0,0,0)$

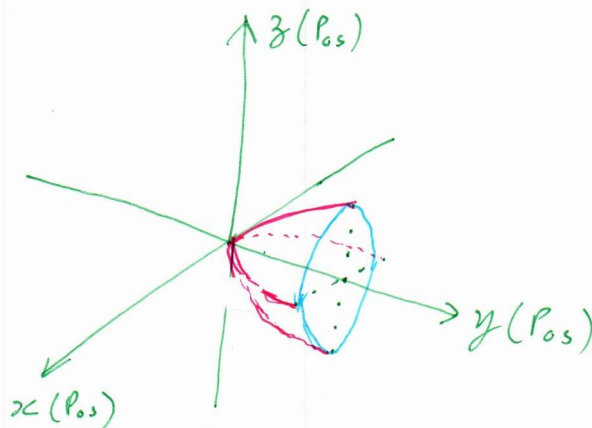
also known as saddle surface.

$$34) 4x^2 - y + 2z^2 = 0$$

$$4x^2 + 2z^2 = y$$

$$\frac{x^2}{\frac{1}{4}} + \frac{z^2}{\frac{1}{2}} = y$$

$$\frac{x^2}{(\frac{1}{2})^2} + \frac{z^2}{(\frac{1}{\sqrt{2}})^2} = y$$



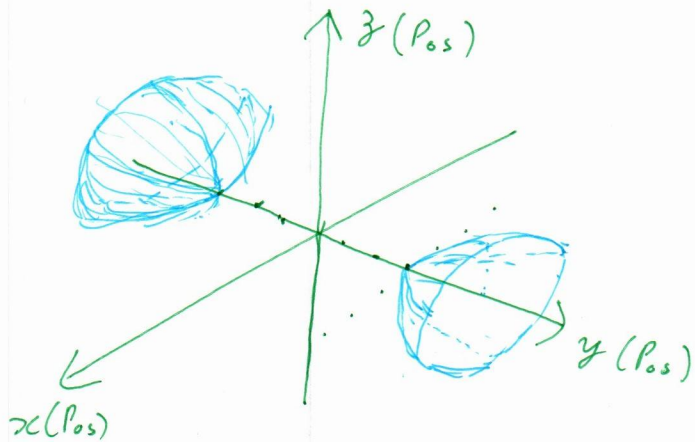
elliptic paraboloid with vertex $(0,0,0)$ and the y -axis

$$36) y^2 = x^2 + 4z^2 + 4$$

$$-x^2 + y^2 - 4z^2 = 4$$

$$-\frac{x^2}{4} + \frac{y^2}{4} - z^2 = 1$$

$$\frac{-x^2}{(2)^2} + \frac{y^2}{(2)^2} - \frac{z^2}{(1)^2} = 1$$



hyperboloid of 2 sheets with the y-axis

$$38) x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$$

$$x^2 - 4x - y^2 - z^2 - 2z = -3$$

$$\frac{b_x}{2} = \frac{-4}{2} = -2$$

$$\left(\frac{b_x}{2}\right)^2 = (-2)^2 = 4$$

$$\frac{b_z}{2} = \frac{2}{2} = 1$$

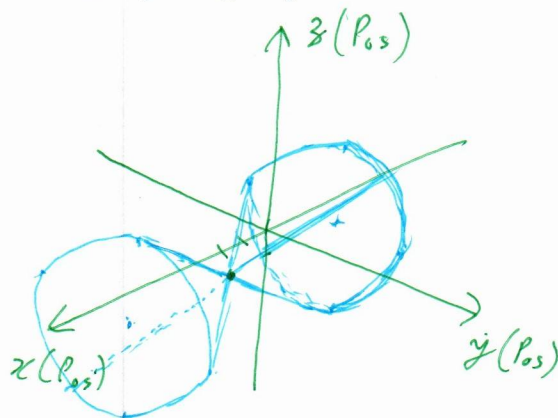
$$\left(\frac{b_z}{2}\right)^2 = (1)^2 = 1$$

$$(x^2 - 4x + 4) - y^2 - (z^2 + 2z + 1) = -3 + (4) - (1)$$

$$(x-2)^2 - (y-0)^2 - (z+1)^2 = 0$$

$$(x-2)^2 - (y-0)^2 - (z-(-1))^2 = 0$$

$$(x-2)^2 = (y-0)^2 + (z-(-1))^2$$



a circular cone with vertex
(2, 0, -1) and axis the
horizontal line $y=0, z=-1$.

$$40) 4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0$$

$$\frac{dx}{dz} = -\frac{6}{2} = -3 \quad \frac{dy}{dz} = -\frac{8}{2} = -4$$

$$12.6/6$$

$$4x^2 - 24x + y^2 - 8y + z^2 + 4z = -55$$

$$\left(\frac{dx}{dz}\right)^2 = (-3)^2 = 9$$

$$\left(\frac{dy}{dz}\right)^2 = (-4)^2 = 16$$

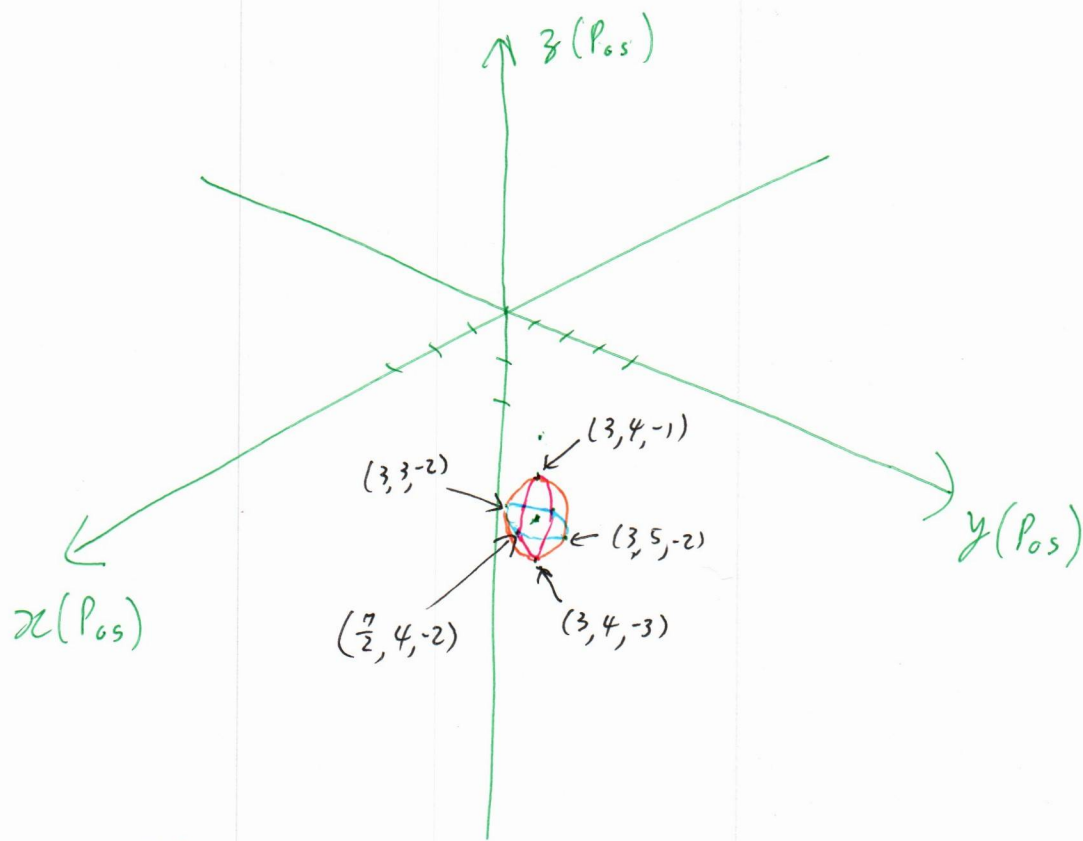
$$\frac{dx}{dz} = \frac{4}{2} = 2$$

$$\left(\frac{dx}{dz}\right)^2 = (2)^2 = 4$$

$$4(x^2 - 6x + 9) + (y^2 - 8y + 16) + (z^2 + 4z + 4) = -55 + 4(9) + (16) + (4)$$

$$4(x - 3)^2 + (y - 4)^2 + (z + 2)^2 = 1$$

$$\frac{(x - 3)^2}{\left(\frac{1}{2}\right)^2} + \frac{(y - 4)^2}{(1)^2} + \frac{(z + 2)^2}{(1)^2} = 1$$



an ellipsoid with center $(3, 4, -2)$