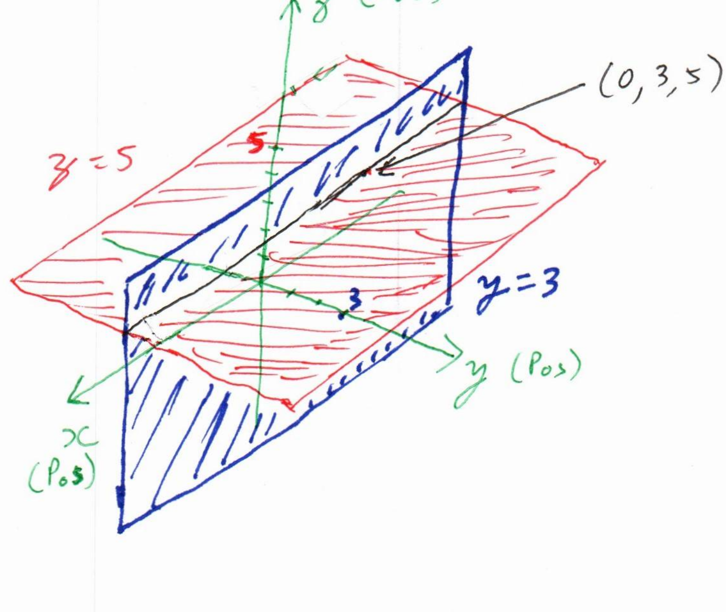
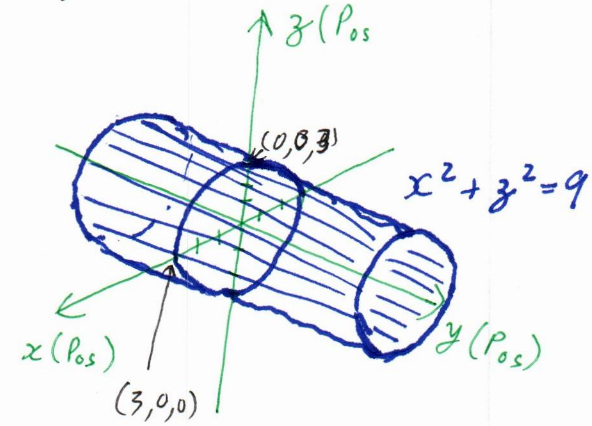
6) In $\mathbb{R}^3$, the equation $y=3$ represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation $z=5$ represents a horizontal plane parallel to the xy-plane and 5 units above it.

The pair of equations $y=3$, $z=5$ represents the set of points that are simultaneously on both planes, or in other words, the line of intersection of the planes $y=3$, $z=5$. This line can also be described as the set $\{(x,3,5) \mid x \in \mathbb{R}\}$, which is the set of all points in $\mathbb{R}^3$ whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point $(0,3,5)$.



8) $x^2 + z^2 = 9 \rightarrow x^2 + z^2 = (3)^2$

The equation $x^2 + z^2 = 9$ has no restriction on y, and the x- and z-coordinates satisfy the equation of a circle of radius 3 with center on the origin. Thus the surface $x^2 + z^2 = 9$ in $\mathbb{R}^3$ consists of all possible vertical circles (parallel to the xz plane) $x^2 + y^2 = 9$, $y = k$, and is therefore a circular cylinder with radius 3 whose axis is the y-axis.



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18) sphere passes origin $(0,0,0)$ center $(1,2,3)$

equation of sphere: $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

$r^2 = ((0)-(1))^2 + ((0)-(2))^2 + ((0)-(3))^2$

$r^2 = (-1)^2 + (-2)^2 + (-3)^2$

$r^2 = (1) + (4) + (9)$

$r^2 = 14$

$(x-(1))^2 + (y-(2))^2 + (z-(3))^2 = (14)$

$(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$

12.1/3

20) $x^2+y^2+z^2=6x-4y-10z$

$\frac{b_x}{2}=\frac{-6}{2}=-3$ $\quad \left(\frac{b_x}{2}\right)^2=(-3)^2=9$

$\frac{b_y}{2}=\frac{4}{2}=2$ $\quad \left(\frac{b_y}{2}\right)^2=(2)^2=4$

$\frac{b_z}{2}=\frac{10}{2}=5$ $\quad \left(\frac{b_z}{2}\right)^2=(5)^2=25$

$$x^2-6x+y^2+4y+z^2+10z=0$$

$$(x^2-6x+9)+(y^2+4y+4)+(z^2+10z+25)=0+(9)+(4)+(25)$$

$$(x-3)^2+(y+2)^2+(z+5)^2=38$$

$$(x-(3))^2+(y-(-2))^2+(z-(-5))^2=(\sqrt{38})^2$$

center: $(3,-2,-5)$

radius: $r=\sqrt{38}$

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22) $4x^2+4y^2+4z^2=16x-6y-12$

$\frac{b_x}{2}=\frac{-4}{2}=-2$ $\quad \left(\frac{b_x}{2}\right)^2=(-2)^2=4$

$\frac{b_y}{2}=\frac{-\frac{3}{2}}{2}=\frac{-3}{4}$ $\quad \left(\frac{b_y}{2}\right)^2=\left(\frac{-3}{4}\right)^2=\frac{9}{16}$

$$4x^2-16x+4y^2-6y+4z^2=-12$$

$$4(x^2-4x)+4(y^2-\tfrac{3}{2}y)+4z^2=-12$$

$$4(x^2-4x+4)+4(y^2-\tfrac{3}{2}y+\tfrac{9}{16})+4z^2=-12+4(4)+4(\tfrac{9}{16})$$

$$4(x-2)^2+4(y-\tfrac{3}{4})^2+4(z-(0))^2=4+\frac{9}{4}=\frac{16}{4}+\frac{9}{4}=\frac{25}{4}$$

$$4\{(x-2)^2+(y-\tfrac{3}{4})^2+(z-0)^2\}=\frac{25}{4}$$

$$(x-(2))^2+(y-(\tfrac{3}{4}))^2+(z-(0))^2=\frac{25}{16}=\left(\frac{5}{4}\right)^2$$

center: $(2,\frac{3}{4},0)$

radius: $r=\frac{5}{4}$

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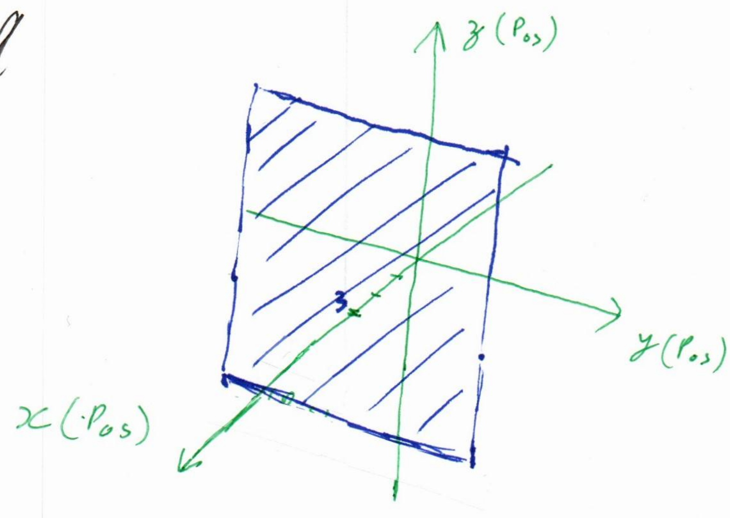
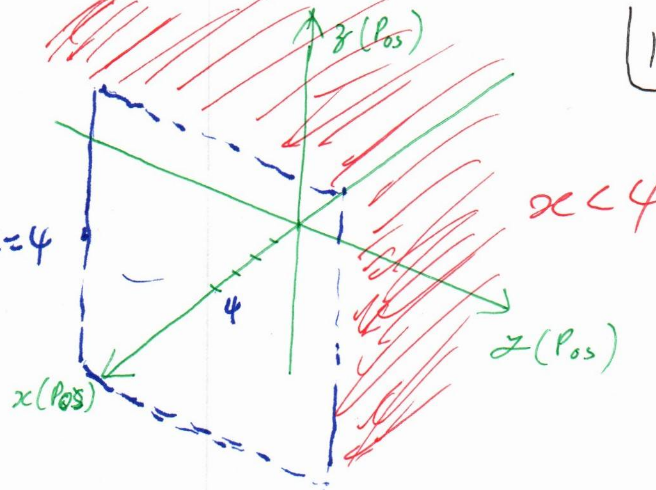
28) $x=3$ plane parallel to the yz-plane and 3 units front of it.



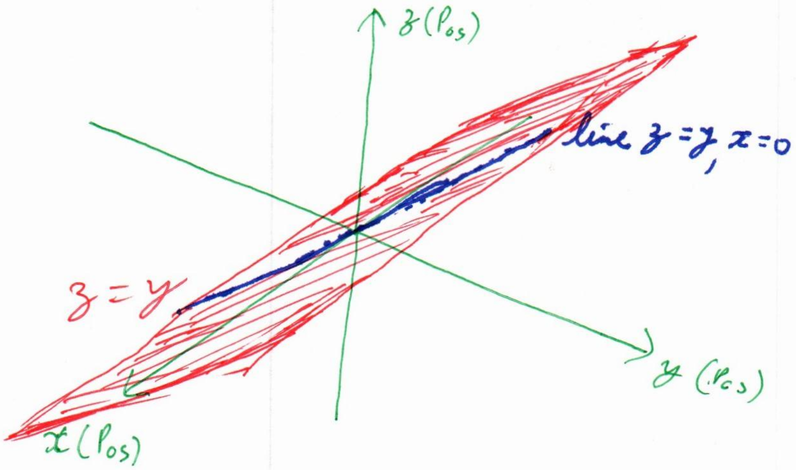
30) $x < 4$

represents half-space consisting of all the points behind the plane $x = 4$



32) $z = y$

represents a plane perpendicular to the yz-plane and intersecting the yz-plane in the line $z = y$, $x = 0$

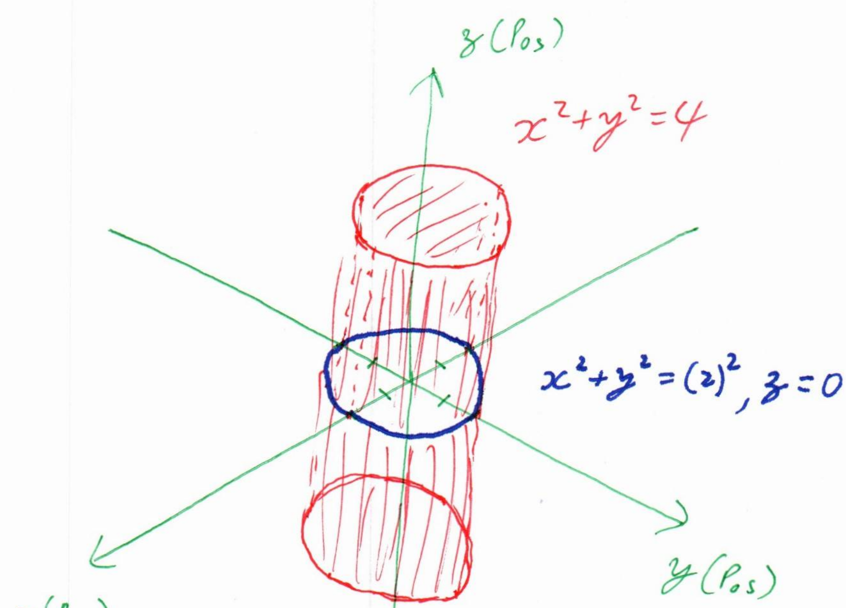


34) $x^2 + y^2 = 4$

no restriction on z

circular cylinder consisting of all possible circles

$x^2 + y^2 = (2)^2, z = k$

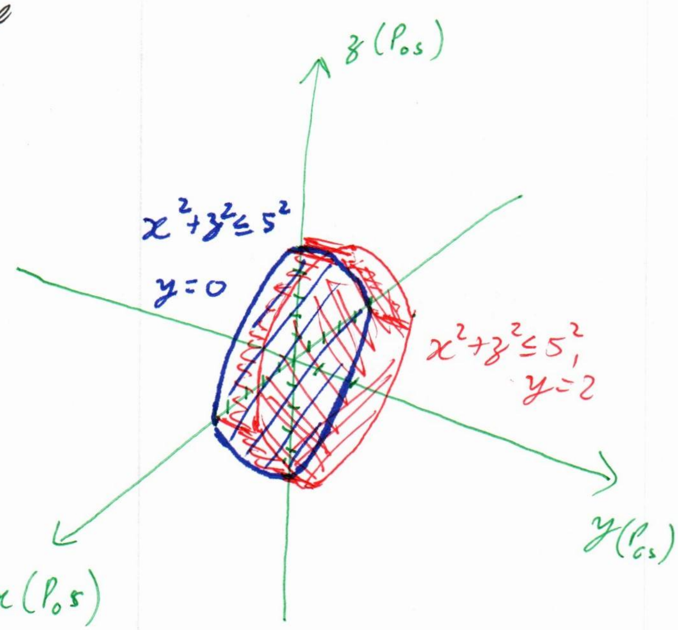


36) $x^2+z^2 \leq 25$ $\qquad 0 \leq y \leq 2$

$x^2+z^2 \leq 25 \rightarrow \sqrt{x^2+z^2} \leq 5$ describes the set of all points in $\mathbb{R}^3$ whose distance from the y-axis is at most 5.

$0 \leq y \leq 2$ consist of the points between the planes $y=0$ to $y=2$.

Thus these inequalities represent the region consisting all points inside a circular cylinder of radius 5 $\{x^2+z^2 \leq (5)^2\}$ with the y-axis from $y=0$ to $y=2$.



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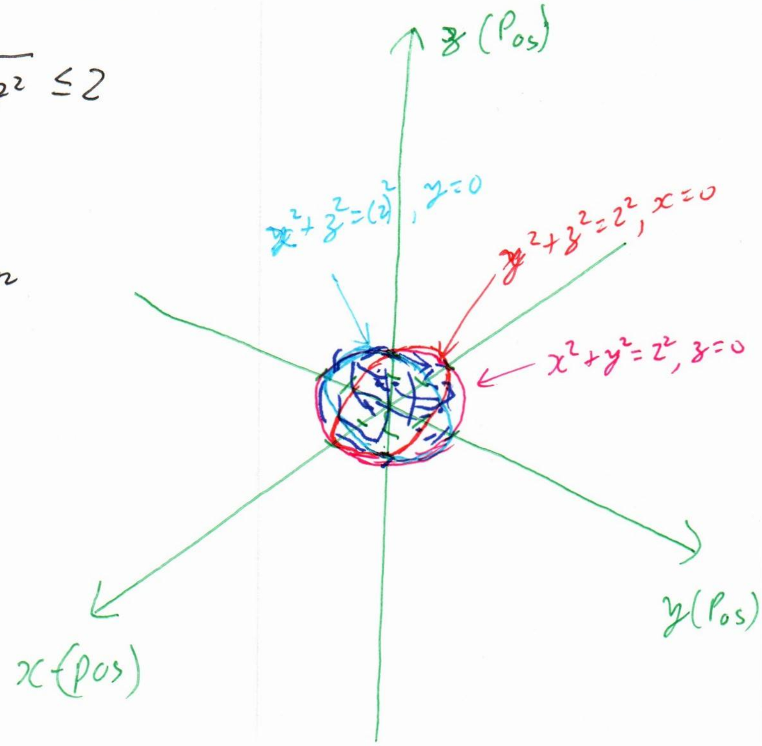
38) $x^2+y^2+z^2 \leq 4 \rightarrow \sqrt{x^2+y^2+z^2} \leq 2$

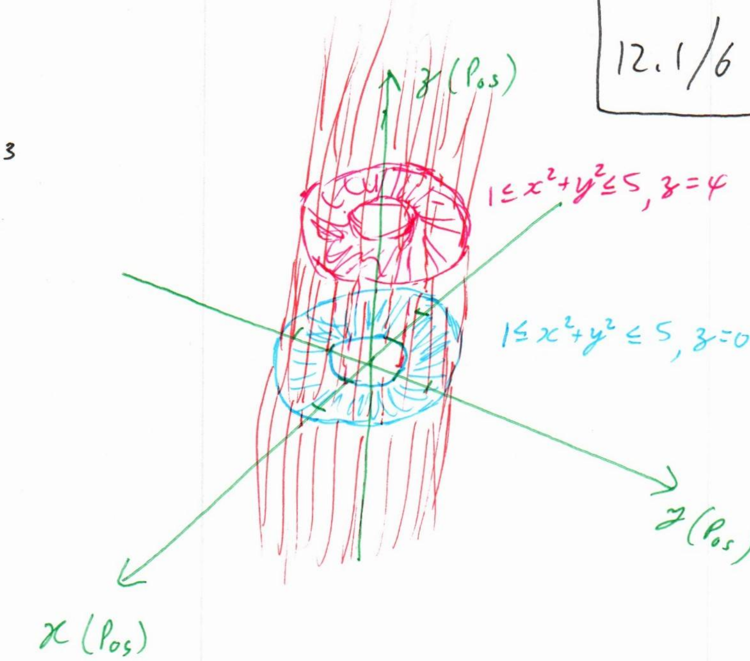
$(x-0)^2+(y-0)^2+(z-0)^2 \leq (2)^2$

This is the set of all points on or inside a sphere with radius 2 and center $(0,0,0)$.



40) $1 \le x^2 + y^2 \le 5 \rightarrow 1 \le \sqrt{x^2+y^2} \le \sqrt{5}$

represents the set of all points in $\mathbb{R}^3$ whose distance is at least 1 and at most $\sqrt{5}$ from the z-axis. The region consists of all points on or between a circular cylinder of radius 1 and a circular cylinder of radius $\sqrt{5}$ with the z-axis.



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42) $x^2 + y^2 + z^2 > 2z$

$$x^2 + y^2 + z^2 - 2z > 0$$

$$x^2 + y^2 + z^2 - 2z + 1 > 1$$

$$(x-0)^2 + (y-0)^2 + (z-1)^2 > (1)^2$$

or equivalent to

$$\sqrt{x^2 + y^2 + (z-1)^2} > 1$$

the region consists of those points whose distance from the point $(0,0,1)$ is greater than 1. The set of all points outside the sphere with radius 1 and center $(0,0,1)$