2) $r = e^{\theta}$, $\frac{3\pi}{4} \le \theta \le \frac{3\pi}{2}$ $A = \int_a^b \frac{1}{2} r^2 \, d\theta$

$r^2 = (e^{\theta})^2 = e^{2\theta}$ $\int \frac{1}{2} e^{2\theta} d\theta = \frac{1}{2}\left[\frac{1}{2}e^{2\theta}\right] + C = \frac{1}{4}e^{2\theta} + C$

$$A = \int_{\frac{3\pi}{4}}^{\frac{3\pi}{2}} \frac{1}{2} e^{2\theta} d\theta = \left[\frac{1}{4}e^{2\theta} + C\right]_{\frac{3\pi}{4}}^{\frac{3\pi}{2}} = \left[\frac{1}{4}e^{2\left(\frac{3\pi}{2}\right)} + C\right] - \left[\frac{1}{4}e^{2\left(\frac{3\pi}{4}\right)} + C\right]$$

$$= \left[\frac{1}{4}e^{3\pi}\right] - \left[\frac{1}{4}e^{\frac{3\pi}{2}}\right] = \underline{\underline{\frac{1}{4}\left(e^{3\pi} - e^{\frac{3\pi}{2}}\right) \text{ units}^2}}$$

---

4) $r = \frac{1}{\theta}$, $\frac{\pi}{2} \le \theta \le 2\pi$

$r^2 = \left(\frac{1}{\theta}\right)^2 = \frac{1}{\theta^2} = \theta^{-2}$ $\int \frac{1}{2}\theta^{-2} d\theta = \frac{1}{2}\left[\frac{\theta^{-1}}{-1}\right] + C = \frac{-1}{2\theta} + C$

$$A = \int_{\frac{\pi}{2}}^{2\pi} \frac{1}{2}\theta^{-2} d\theta = \left[\frac{-1}{2\theta} + C\right]_{\frac{\pi}{2}}^{2\pi} = \left[\frac{-1}{2(2\pi)} + C\right] - \left[\frac{-1}{2\left(\frac{\pi}{2}\right)} + C\right]$$

$$= \left[\frac{-1}{4\pi}\right] - \left[\frac{-1}{\pi}\right] = \frac{-1}{4\pi} + \frac{4}{4\pi} = \underline{\underline{\frac{3}{4\pi} \text{ units}^2}}$$

---

6) $r = 2 + \cos\theta$, $\frac{\pi}{2} \le \theta \le \pi$

$$r^2 = (2+\cos\theta)^2 = 4 + 4\cos\theta + \cos^2\theta = 4 + 4\cos\theta + \left(\frac{1+\cos(2\theta)}{2}\right)$$
$$= \frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos(2\theta)$$

$$\int \frac{1}{2}\left(\frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos(2\theta)\right) d\theta = \int \left(\frac{9}{4} + 2\cos\theta + \frac{1}{4}\cos(2\theta)\right) d\theta$$
$$= \frac{9}{4}[\theta] + 2[\sin\theta] + \frac{1}{4}\left[\frac{1}{2}\sin(2\theta)\right] + C = \frac{9}{4}\theta + 2\sin\theta + \frac{1}{8}\sin(2\theta) + C$$

$$A = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}(2+\cos\theta)^2 d\theta = \left[\frac{9}{4}\theta + 2\sin\theta + \frac{1}{8}\sin(2\theta) + C\right]_{\frac{\pi}{2}}^{\pi}$$
$$= \left[\frac{9}{4}(\pi) + 2\sin(\pi) + \frac{1}{8}\sin(2(\pi)) + C\right] - \left[\frac{9}{4}\left(\frac{\pi}{2}\right) + 2\sin\left(\frac{\pi}{2}\right) + \frac{1}{8}\sin\left(2\left(\frac{\pi}{2}\right)\right) + C\right]$$
$$= \left[\frac{9\pi}{4} + 2(0) + \frac{1}{8}(0)\right] - \left[\frac{9\pi}{8} + 2(1) + \frac{1}{8}(0)\right] = \underline{\underline{\frac{9\pi}{8} - 2 \text{ units}^2}}$$

8) $r = \sqrt{\ln\theta} \qquad 1 \le \theta \le 2\pi$

$u = \ln\theta \qquad dv = \frac{1}{2}d\theta$

$du = \frac{1}{\theta}d\theta \qquad v = \frac{1}{2}\theta$

10.4/2

$r^2 = \left(\sqrt{\ln\theta}\right)^2 = \ln\theta$

$\int \frac{1}{2}\ln\theta\, d\theta = (\ln\theta)\left(\frac{1}{2}\theta\right) - \int\left(\frac{1}{2}\theta\right)\left(\frac{1}{\theta}d\theta\right)$

$= \frac{1}{2}\theta\ln\theta - \int\frac{1}{2}d\theta = \frac{1}{2}\theta\ln\theta - \frac{1}{2}\theta + C$

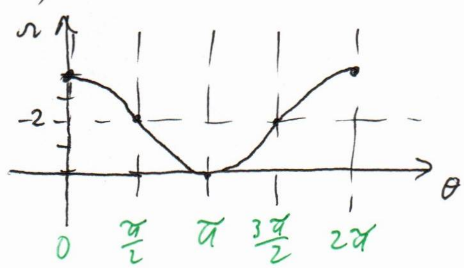
$A = \int_1^{2\pi}\frac{1}{2}\ln\theta\, d\theta = \left[\frac{1}{2}\theta\ln\theta - \frac{1}{2}\theta + C\right]_1^{2\pi} = \left[\frac{1}{2}(2\pi)\ln(2\pi) - \frac{1}{2}(2\pi) + C\right] - \left[\frac{1}{2}(1)\ln(1) - \frac{1}{2}(1) + C\right]$

$= \left[\pi\ln(2\pi) - \pi\right] - \left[\frac{1}{2}(0) - \frac{1}{2}\right] = \underline{\underline{\pi\ln(2\pi) - \pi + \frac{1}{2} \text{ units}^2}}$

---

10) $r = 2 + 2\cos\theta$

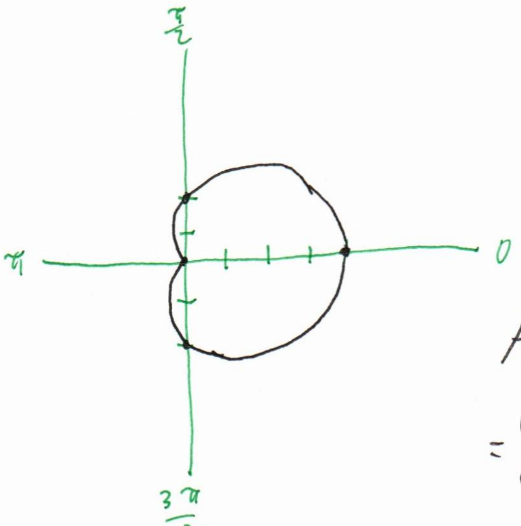
period $2\pi \rightarrow 0 \le \theta \le 2\pi$



$r^2 = (2 + 2\cos\theta)^2 = 4 + 8\cos\theta + 4\cos^2\theta$

$= 4 + 8\cos\theta + 4\left(\frac{1 + \cos(2\theta)}{2}\right)$

$= 6 + 8\cos\theta + 2\cos(2\theta)$



$\int\frac{1}{2}(6 + 8\cos\theta + 2\cos(2\theta))\,d\theta$

$= \int(3 + 4\cos\theta + \cos(2\theta)\,d\theta$

$= 3[\theta] + 4[\sin\theta] + \left[\frac{1}{2}\sin(2\theta)\right] + C$

$= 3\theta + 4\sin\theta + \frac{1}{2}\sin(2\theta) + C$

$A = \int_0^{2\pi}\frac{1}{2}(2 + 2\cos\theta)^2\,d\theta = \left[3\theta + 4\sin\theta + \frac{1}{2}\sin(2\theta) + C\right]_0^{2\pi}$

$= \left[3(2\pi) + 4\sin(2\pi) + \frac{1}{2}\sin(2(2\pi)) + C\right] - \left[3(0) + 4\sin(0) + \frac{1}{2}\sin(2(0)) + C\right]$

$= \left[6\pi + 4(0) + \frac{1}{2}(0)\right] - \left[0 + 4(0) + \frac{1}{2}(0)\right]$

$= \underline{\underline{6\pi \text{ units}^2}}$

12) $r = 2\sin 3\theta$ rose $n = 3$ odd 3 petals period $= \pi$

use $0 \le \theta \le \pi$

or get angles that generates a single petal by setting $r = 0$ and solve for θ

Q IV & Q I Q II & Q III

$(0) = r = 2\sin 3\theta \Rightarrow$ $3\theta = 0 \Rightarrow 3\theta = 0$ $3\theta = \pi$

$0 = \sin 3\theta$ $\theta = \frac{0}{3} = 0$ $\theta = \frac{\pi}{3}$

use $0 \le \theta \le \frac{\pi}{3}$ for area of 1 petal then mult by 3

"method 2 shown"

$$r^2 = (2\sin 3\theta)^2 = 4\sin^2(3\theta) = 4\left(\frac{1-\cos(6\theta)}{2}\right) = 2 - 2\cos(6\theta)$$

$$\int \frac{1}{2}(2 - 2\cos(6\theta))\,d\theta = \int (1 - \cos(6\theta))\,d\theta = [\theta] - \left[\frac{1}{6}\sin(6\theta)\right] + C$$

$$= \theta - \frac{1}{6}\sin(6\theta) + C$$

$$A_{\frac{1}{3}} = \int_0^{\frac{\pi}{3}} \frac{1}{2}(2\sin 3\theta)^2\,d\theta = \left[\theta - \frac{1}{6}\sin(6\theta) + C\right]_0^{\frac{\pi}{3}}$$

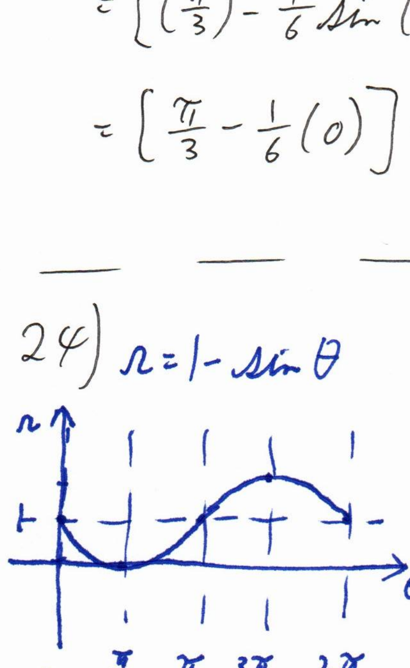
$$= \left[\left(\frac{\pi}{3}\right) - \frac{1}{6}\sin\left(6\left(\frac{\pi}{3}\right)\right) + C\right] - \left[(0) - \frac{1}{6}\sin(6(0)) + C\right]$$

$$= \left[\frac{\pi}{3} - \frac{1}{6}(0)\right] - \left[0 - \frac{1}{6}(0)\right] = \frac{\pi}{3}$$

$$A = 3A_{\frac{1}{3}} = 3\left(\frac{\pi}{3}\right) = \underline{\pi \text{ units}^2}$$

---

24) $r = 1 - \sin\theta$



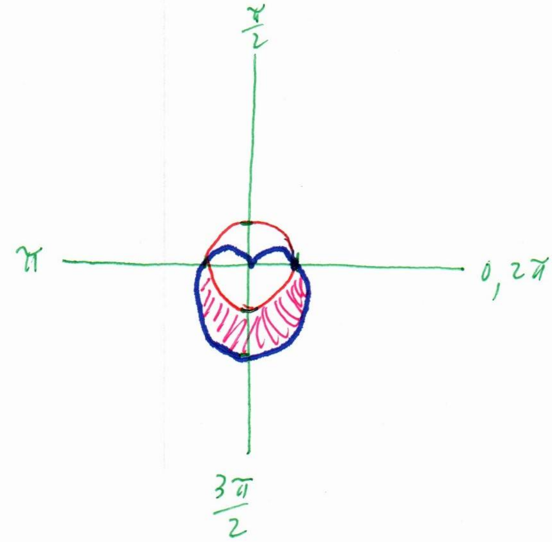
$r = 1$

circle

center origin

$(0,0)$

$r = 1$



24) continued.

for intersection points (angles)

$1 - \sin\theta = 1$ QII & QIII QIV & QI

$0 = \sin\theta$ $\underline{\underline{\theta = \pi}}$ $\underline{\underline{\theta = 2\pi}}$

$\theta = 0$ interval: $\pi \le \theta \le 2\pi$

$$r^2 = (1-\sin\theta)^2 = 1 - 2\sin\theta + \sin^2\theta = 1 - 2\sin\theta + \left(\frac{1-\cos(2\theta)}{2}\right)$$

$$= \frac{3}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)$$

$$r^2 = (1)^2 = 1$$

$$r^2 = r^2 - r^2 = \left(\frac{3}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)\right) - (1) = \frac{1}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)$$

$$\int \frac{1}{2}\left(\frac{1}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)\right)d\theta = \int \left(\frac{1}{4} - \sin\theta - \frac{1}{4}\cos(2\theta)\right)d\theta$$

$$= \frac{1}{4}[\theta] - [-\cos\theta] - \frac{1}{4}\left[\frac{1}{2}\sin(2\theta)\right] + C = \frac{1}{4}\theta + \cos\theta - \frac{1}{8}\sin(2\theta) + C$$

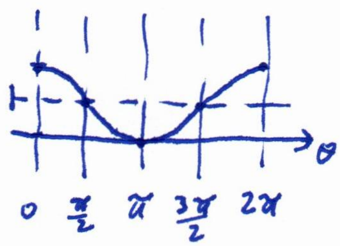
$$A = \int_{\pi}^{2\pi} \frac{1}{2} r^2\, d\theta = \int_{\pi}^{2\pi} \frac{1}{2}\left(\frac{1}{2} - 2\sin\theta - \frac{1}{2}\cos(2\theta)\right)d\theta = \left[\frac{1}{4}\theta + \cos\theta - \frac{1}{8}\sin(2\theta) + C\right]_{\pi}^{2\pi}$$

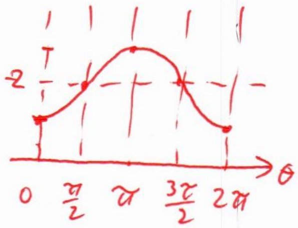
$$= \left[\frac{1}{4}(2\pi) + \cos(2\theta) - \frac{1}{8}\sin(2(2\pi)) + C\right] - \left[\frac{1}{4}(\pi) + \cos(\pi) - \frac{1}{8}\sin(2(\pi)) + C\right]$$

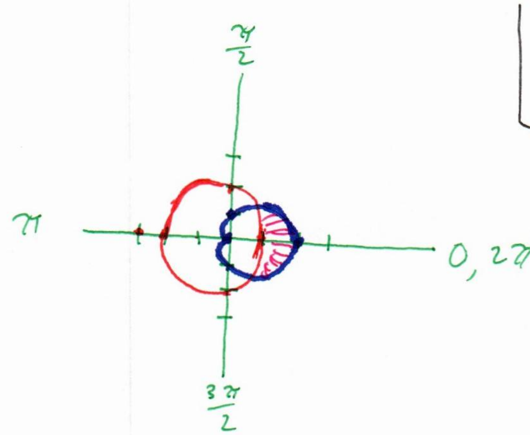
$$= \left[\frac{\pi}{2} + (1) - \frac{1}{8}(0)\right] - \left[\frac{\pi}{4} + (-1) - \frac{1}{8}(0)\right] = \left[\frac{\pi}{2} + 1\right] - \left[\frac{\pi}{4} - 1\right]$$

$$= \underline{\underline{\frac{\pi}{4} + 2 \text{ units}^2}}$$

26) $r = 1 + \cos\theta$ $\quad$ $r = 2 - \cos\theta$







for intersection points

$$1 + \cos\theta = 2 - \cos\theta$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

Q I: $\theta = \frac{\pi}{3}$ $\qquad$ Q IV: $\theta = \frac{-\pi}{3}$

use interval: $\frac{-\pi}{3} \leq \theta \leq \frac{\pi}{3}$

$$r^2 = (1+\cos\theta)^2 = 1 + 2\cos\theta + \cos^2\theta = 1 + 2\cos\theta + \left(\frac{1+\cos(2\theta)}{2}\right)$$
$$= \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta)$$

$$r^2 = (2-\cos\theta)^2 = 4 - 4\cos\theta + \cos^2\theta = 4 - 4\cos\theta + \left(\frac{1+\cos(2\theta)}{2}\right)$$
$$= \frac{9}{2} - 4\cos\theta + \frac{1}{2}\cos(2\theta)$$

$$r^2 = r^2 - r^2 = \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta)\right) - \left(\frac{9}{2} - 4\cos\theta + \frac{1}{2}\cos(2\theta)\right)$$
$$= \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos(2\theta) - \frac{9}{2} + 4\cos\theta - \frac{1}{2}\cos(2\theta) = \frac{-6}{2} + 6\cos\theta = 6\cos\theta - 3$$

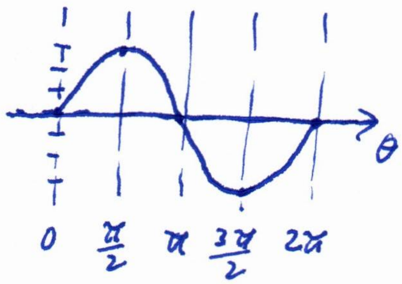
$$\int \frac{1}{2}(6\cos\theta - 3)\,d\theta = \int \left(3\cos\theta - \frac{3}{2}\right)d\theta = 3[\sin\theta] - \frac{3}{2}[\theta] + C$$
$$= 3\sin\theta - \frac{3}{2}\theta + C$$

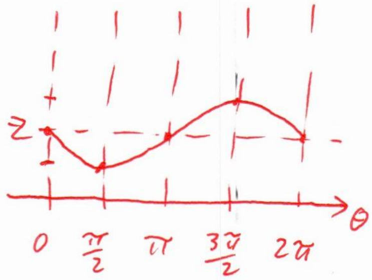
$$A = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} r^2\,d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}(6\cos\theta - 3)\,d\theta = \left[3\sin\theta - \frac{3}{2}\theta + C\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

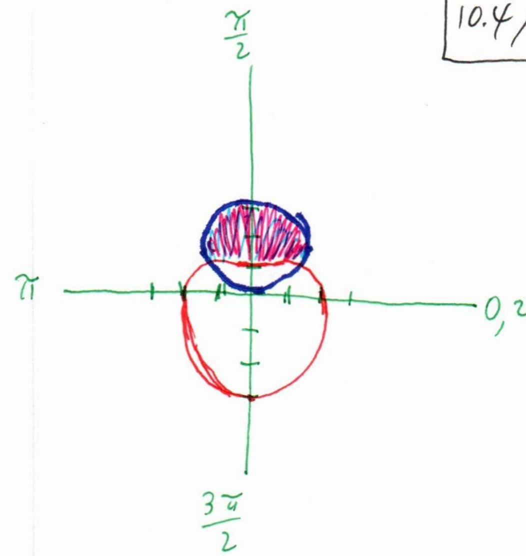
$$= \left[3\sin\left(\frac{\pi}{3}\right) - \frac{3}{2}\left(\frac{\pi}{3}\right) + C\right] - \left[3\sin\left(\frac{-\pi}{3}\right) - \frac{3}{2}\left(\frac{-\pi}{3}\right) + C\right]$$

$$= \left[3\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{2}\right] - \left[3\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{2}\right] = \frac{6\sqrt{3}}{2} - \frac{2\pi}{2} = \underline{\underline{3\sqrt{3} - \pi \text{ units}^2}}$$

28) $r = 3\sin\theta$ $\qquad$ $r = 2 - \sin\theta$







for intersection points

$3\sin\theta = 2 - \sin\theta$ $\qquad$ QI $\qquad$ QII

$4\sin\theta = 2$ $\qquad$ $\theta = \frac{\pi}{6}$ $\qquad$ $\theta = \frac{5\pi}{6}$

$\sin\theta = \frac{2}{4} = \frac{1}{2}$ $\qquad$ interval: $\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}$

$$r^2 = (3\sin\theta)^2 = 9\sin^2\theta = 9\left(\frac{1-\cos(2\theta)}{2}\right) = \frac{9}{2} - \frac{9}{2}\cos(2\theta)$$

$$r^2 = (2-\sin\theta)^2 = 4 - 4\sin\theta + \sin^2\theta = 4 - 4\sin\theta + \left(\frac{1-\cos(2\theta)}{2}\right)$$

$$= \frac{9}{2} - 4\sin\theta - \frac{1}{2}\cos(2\theta)$$

$$r^2 = r^2 - r^2 = \left(\frac{9}{2} - \frac{9}{2}\cos(2\theta)\right) - \left(\frac{9}{2} - 4\sin\theta - \frac{1}{2}\cos(2\theta)\right)$$

$$= \frac{9}{2} - \frac{9}{2}\cos(2\theta) - \frac{9}{2} + 4\sin\theta + \frac{1}{2}\cos(2\theta) = 4\sin\theta - 4\cos(2\theta)$$

$$\int \frac{1}{2}\left(4\sin\theta - 4\cos(2\theta)\right)d\theta = \int \left(2\sin\theta - 2\cos(2\theta)\right)d\theta = 2\left[-\cos\theta\right] - 2\left[\frac{1}{2}\sin(2\theta)\right] + C$$

$$= -2\cos\theta - \sin(2\theta) + C$$

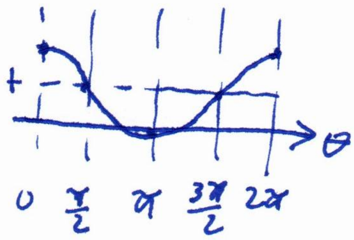
$$A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2} r^2\, d\theta = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}\left(4\sin\theta - 4\cos(2\theta)\right)d\theta = \left[-2\cos\theta - \sin(2\theta) + C\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$= \left[-2\cos\left(\frac{5\pi}{6}\right) - \sin\left(2\left(\frac{5\pi}{6}\right)\right) + C\right] - \left[-2\cos\left(\frac{\pi}{6}\right) - \sin\left(2\left(\frac{\pi}{6}\right)\right) + C\right]$$

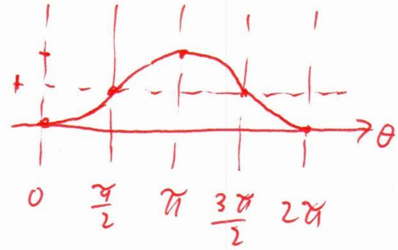
$$= \left[-2\left(\frac{-\sqrt{3}}{2}\right) - \left(\frac{-\sqrt{3}}{2}\right)\right] - \left[-2\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\right] = \left[+\sqrt{3} + \frac{\sqrt{3}}{2}\right] - \left[-\sqrt{3} - \frac{\sqrt{3}}{2}\right]$$

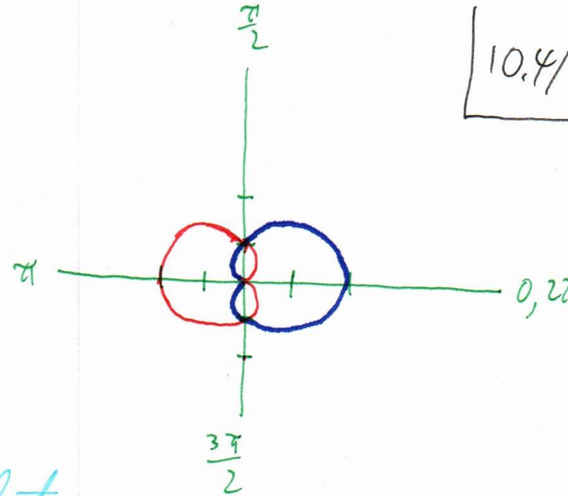
$$= 2\sqrt{3} + \frac{2\sqrt{3}}{2} = 2\sqrt{3} + \sqrt{3} = \underline{3\sqrt{3}\ \text{units}^2}$$

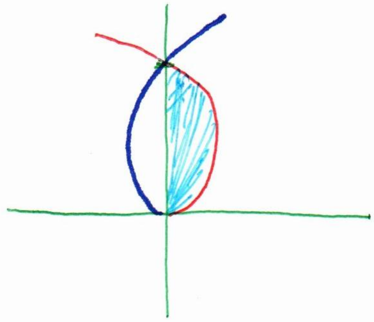
30) $r = 1 + \cos\theta$



$r = 1 - \cos\theta$







easier to calculate just $\frac{1}{2}$ of a single slit

because this is only the area under $r = 1 - \cos\theta$ and the interval is $0 \le \theta \le \frac{\pi}{2}$

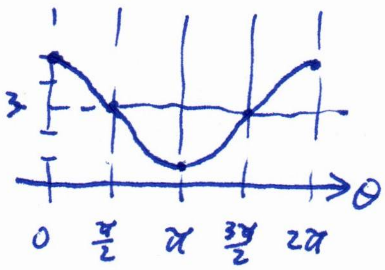
$$r^2 = (1-\cos\theta)^2 = 1 - 2\cos\theta + \cos^2\theta = 1 - 2\cos\theta + \left(\frac{1+\cos(2\theta)}{2}\right)$$
$$= \frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos(2\theta)$$

$$\int \frac{1}{2}\left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos(2\theta)\right)d\theta = \int\left(\frac{3}{4} - \cos\theta + \frac{1}{4}\cos(2\theta)\right)d\theta$$
$$= \frac{3}{4}[\theta] - [\sin\theta] + \frac{1}{4}\left[\frac{1}{2}\sin(2\theta)\right] + C = \frac{3}{4}\theta - \sin\theta + \frac{1}{8}\sin(2\theta) + C$$

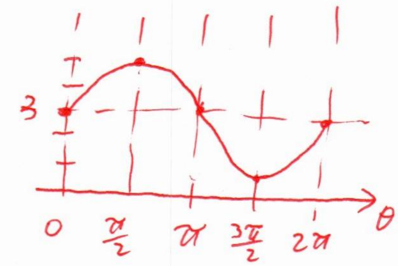
$$A_{\frac{1}{4}} = \int_0^{\frac{\pi}{2}} \frac{1}{2} r^2\, d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos\theta)^2 d\theta = \left[\frac{3}{4}\theta - \sin\theta + \frac{1}{8}\sin(2\theta) + C\right]_0^{\frac{\pi}{2}}$$
$$= \left[\frac{3}{4}\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) + \frac{1}{8}\sin\left(2\left(\frac{\pi}{2}\right)\right) + C\right] - \left[\frac{3}{4}(0) - \sin(0) + \frac{1}{8}\sin(2(0)) + C\right]$$
$$= \left[\frac{3\pi}{8} - (1) + \frac{1}{8}(0)\right] - \left[0 - (0) + \frac{1}{8}(0)\right] = \frac{3\pi}{8} - 1$$

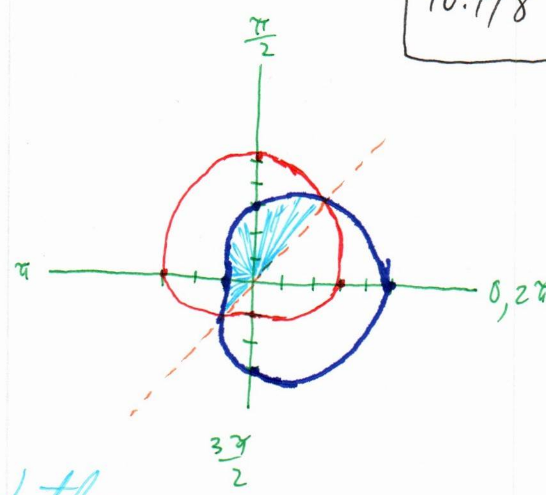
$$A = 4A_{\frac{1}{4}} = 4\left(\frac{3\pi}{8} - 1\right) = \underline{\underline{\frac{3\pi}{2} - 4 \text{ units}^2}}$$

32) $r = 3 + 2\cos\theta$



$r = 3 + 2\sin\theta$





easier to calulate $\frac{1}{2}$ of the area and the shaded region will have a slight easier angles

for intersection points (angles)

$$3 + 2\cos\theta = 3 + 2\sin\theta$$

$$2\cos\theta = 2\sin\theta$$

$$1 = \frac{2\sin\theta}{2\cos\theta}$$

$$1 = \tan\theta$$

QI $\theta = \frac{\pi}{4}$ QIII $\theta = \frac{5\pi}{4}$

use interval $\frac{\pi}{4} \le \theta \le \frac{5\pi}{4}$ and inside both curves is just inside $r = 3 + 2\cos\theta$

$$r^2 = (3 + 2\cos\theta)^2 = 9 + 12\cos\theta + 4\cos^2\theta = 9 + 12\cos\theta + 4\left(\frac{1 + \cos(2\theta)}{2}\right)$$

$$= 11 + 12\cos\theta + 2\cos(2\theta)$$

$$\int \frac{1}{2}\left(11 + 12\cos\theta + 2\cos(2\theta)\right) d\theta = \int \left(\frac{11}{2} + 6\cos\theta + \cos(2\theta)\right) d\theta$$

$$= \frac{11}{2}[\theta] + 6[\sin\theta] + \left[\frac{1}{2}\sin(2\theta)\right] + C = \frac{11}{2}\theta + 6\sin\theta + \frac{1}{2}\sin(2\theta) + C$$

$$A_{\frac{1}{2}} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \frac{1}{2}(3 + 2\cos\theta)^2 d\theta = \left[\frac{11}{2}\theta + 6\sin\theta + \frac{1}{2}\sin(2\theta) + C\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= \left[\frac{11}{2}\left(\frac{5\pi}{4}\right) + 6\sin\left(\frac{5\pi}{4}\right) + \frac{1}{2}\sin\left(2\left(\frac{5\pi}{4}\right)\right) + C\right] - \left[\frac{11}{2}\left(\frac{\pi}{4}\right) + 6\sin\left(\frac{\pi}{4}\right) + \frac{1}{2}\sin\left(2\left(\frac{\pi}{4}\right)\right) + C\right]$$

$$= \left[\frac{55\pi}{8} + 6\left(\frac{-1}{\sqrt{2}}\right) + \frac{1}{2}(1)\right] - \left[\frac{11\pi}{8} + 6\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(1)\right] = \frac{44\pi}{8} - \frac{12}{\sqrt{2}} = \frac{11\pi}{2} - \frac{12}{\sqrt{2}}$$

$$A = 2A_{\frac{1}{2}} = 2\left(\frac{11\pi}{2} - \frac{12}{\sqrt{2}}\right) = 11\pi - \frac{24}{\sqrt{2}} = 11\pi - \frac{24\sqrt{2}}{2} = \underline{11\pi - 12\sqrt{2} \text{ unit}^2}$$