Decoding the rules: $\Delta x = \dfrac{b-a}{n}$ $\quad x_i = a + i\Delta x \qquad \overline{x}_i = \dfrac{1}{2}(x_{i-1} + x_i)$

Midpoint Rule: $\int_a^b f(x)\,dx \approx M_n = \dfrac{\Delta x}{1}\left[f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + \cdots + f(\overline{x}_{n-1}) + f(\overline{x}_n) \right]$

Trapezoidal Rule: $\int_a^b f(x)\,dx \approx T_n = \dfrac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$

Simpson's Rule: $\int_a^b f(x)\,dx \approx S_n = \dfrac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$

The value of n must be even for Simpson's rule to work.

Error Bounds: For $a \le x \le b$

Midpoint error: $\left|E_M\right| \le \dfrac{K(b-a)^3}{24n^2} \qquad \left|f''(x)\right| \le K$

Trapezoidal error: $\left|E_T\right| \le \dfrac{K(b-a)^3}{12n^2} \qquad \left|f''(x)\right| \le K$

Simpson error: $\left|E_S\right| \le \dfrac{K(b-a)^5}{180n^4} \qquad \left|f^{(4)}(x)\right| \le K$

Additional examples (Examples created from the text starts on page 6):

8) $\int_0^2 \dfrac{1}{1+x^6}\,dx \qquad n = 8$

$\Delta x = \dfrac{(2)-(0)}{(8)} = \dfrac{2}{8} = \dfrac{1}{4} \qquad \dfrac{\Delta x}{2} = \dfrac{1}{8}$

$\frac{0}{8}$	$\frac{1}{8}$		$\frac{3}{8}$		$\frac{5}{8}$		$\frac{7}{8}$		$\frac{9}{8}$		$\frac{11}{8}$		$\frac{13}{8}$		$\frac{15}{8}$	
x_0	$\overline{x}_1$	x_1	$\overline{x}_2$	x_2	$\overline{x}_3$	x_3	$\overline{x}_4$	x_4	$\overline{x}_5$	x_5	$\overline{x}_6$	x_6	$\overline{x}_7$	x_7	$\overline{x}_8$	x_8
$0=\frac{0}{4}$		$\frac{1}{4}$		$\frac{2}{4}$		$\frac{3}{4}$		$\frac{4}{4}$		$\frac{5}{4}$		$\frac{6}{4}$		$\frac{7}{4}$		$\frac{8}{4}$

$$M_8 = \frac{\left(\frac{1}{4}\right)}{1}\left[\frac{1}{1+\left(\frac{1}{8}\right)^6} + \frac{1}{1+\left(\frac{3}{8}\right)^6} + \frac{1}{1+\left(\frac{5}{8}\right)^6} + \frac{1}{1+\left(\frac{7}{8}\right)^6} + \frac{1}{1+\left(\frac{9}{8}\right)^6} + \frac{1}{1+\left(\frac{11}{8}\right)^6} + \frac{1}{1+\left(\frac{13}{8}\right)^6} + \frac{1}{1+\left(\frac{15}{8}\right)^6} \right]$$

$$T_8 = \frac{\left(\frac{1}{4}\right)}{2}\left[\frac{1}{1+\left(\frac{1}{4}\right)^6} + 2\left(\frac{1}{1+\left(\frac{2}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{3}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{4}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{5}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{6}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{7}{4}\right)^6}\right) + \frac{1}{1+\left(\frac{8}{4}\right)^6}\right]$$

$$S_8 = \frac{\left(\frac{1}{4}\right)}{3}\left[\frac{1}{1+\left(\frac{1}{4}\right)^6} + 4\left(\frac{1}{1+\left(\frac{2}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{3}{4}\right)^6}\right) + 4\left(\frac{1}{1+\left(\frac{4}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{5}{4}\right)^6}\right) + 4\left(\frac{1}{1+\left(\frac{6}{4}\right)^6}\right) + 2\left(\frac{1}{1+\left(\frac{7}{4}\right)^6}\right) + \frac{1}{1+\left(\frac{8}{4}\right)^6}\right]$$

12) $\int_0^1 \sin(x^3)\, dx \qquad n = 10$

$$\Delta x = \frac{(1)-(0)}{(10)} = \frac{1}{10} = \frac{2}{20} \qquad \frac{\Delta x}{2} = \frac{1}{20}$$

$\frac{0}{20}$	$\frac{1}{20}$		$\frac{3}{20}$		$\frac{5}{20}$		$\frac{7}{20}$		$\frac{9}{20}$	
x_0	$\bar{x}_1$	x_1	$\bar{x}_2$	x_2	$\bar{x}_3$	x_3	$\bar{x}_4$	x_4	$\bar{x}_5$	x_5
$0 = \frac{0}{10}$		$\frac{1}{10}$		$\frac{2}{10}$		$\frac{3}{10}$		$\frac{4}{10}$		$\frac{5}{10}$

$\frac{11}{20}$		$\frac{13}{20}$		$\frac{15}{20}$		$\frac{17}{20}$		$\frac{19}{20}$	
$\bar{x}_6$	x_6	$\bar{x}_7$	x_7	$\bar{x}_8$	x_8	$\bar{x}_9$	x_9	$\bar{x}_{10}$	x_{10}
	$\frac{6}{10}$		$\frac{7}{10}$		$\frac{8}{10}$		$\frac{9}{10}$		$\frac{10}{10}$

$$M_{10} = \frac{\left(\frac{1}{10}\right)}{1}\left[\sin\left(\frac{1}{20}\right) + \sin\left(\frac{3}{20}\right) + \sin\left(\frac{5}{20}\right) + \sin\left(\frac{7}{20}\right) + \sin\left(\frac{9}{20}\right) + \sin\left(\frac{11}{20}\right) + \sin\left(\frac{13}{20}\right) + \sin\left(\frac{15}{20}\right) + \sin\left(\frac{17}{20}\right) + \sin\left(\frac{19}{20}\right)\right]$$

$$T_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\begin{aligned} &\sin\left(\frac{0}{10}\right) + 2\left(\sin\left(\frac{1}{10}\right)\right) + 2\left(\sin\left(\frac{2}{10}\right)\right) + 2\left(\sin\left(\frac{3}{10}\right)\right) + 2\left(\sin\left(\frac{4}{10}\right)\right) + 2\left(\sin\left(\frac{5}{10}\right)\right) \\ &\qquad + 2\left(\sin\left(\frac{6}{10}\right)\right) + 2\left(\sin\left(\frac{7}{10}\right)\right) + 2\left(\sin\left(\frac{8}{10}\right)\right) + 2\left(\sin\left(\frac{9}{10}\right)\right) + \sin\left(\frac{10}{10}\right) \end{aligned}\right]$$

$$S_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\begin{aligned} &\sin\left(\frac{0}{10}\right) + 4\left(\sin\left(\frac{1}{10}\right)\right) + 2\left(\sin\left(\frac{2}{10}\right)\right) + 4\left(\sin\left(\frac{3}{10}\right)\right) + 2\left(\sin\left(\frac{4}{10}\right)\right) + 4\left(\sin\left(\frac{5}{10}\right)\right) \\ &\qquad + 2\left(\sin\left(\frac{6}{10}\right)\right) + 4\left(\sin\left(\frac{7}{10}\right)\right) + 2\left(\sin\left(\frac{8}{10}\right)\right) + 4\left(\sin\left(\frac{9}{10}\right)\right) + \sin\left(\frac{10}{10}\right) \end{aligned}\right]$$

14) $\int_4^6 \ln(x^3+2)\,dx \qquad n=10$

$$\Delta x = \frac{(6)-(4)}{(10)} = \frac{2}{10} = \frac{1}{5} \qquad \frac{\Delta x}{2} = \frac{1}{10}$$

x_0	$\bar{x}_1$	x_1	$\bar{x}_2$	x_2	$\bar{x}_3$	x_3	$\bar{x}_4$	x_4	$\bar{x}_5$	x_5
$4=\frac{20}{5}$ ($\frac{40}{10}$)	$\frac{41}{10}$	$\frac{21}{5}$	$\frac{43}{10}$	$\frac{22}{5}$	$\frac{45}{10}$	$\frac{23}{5}$	$\frac{47}{10}$	$\frac{24}{5}$	$\frac{49}{10}$	$\frac{25}{5}$

$\bar{x}_6$	x_6	$\bar{x}_7$	x_7	$\bar{x}_8$	x_8	$\bar{x}_9$	x_9	$\bar{x}_{10}$	x_{10}
$\frac{51}{10}$	$\frac{26}{5}$	$\frac{53}{10}$	$\frac{27}{5}$	$\frac{55}{10}$	$\frac{28}{5}$	$\frac{57}{10}$	$\frac{29}{5}$	$\frac{59}{10}$	$\frac{30}{5}$

$$M_{10} = \frac{\left(\frac{1}{5}\right)}{1}\left[\begin{aligned} &\ln\left(\left(\frac{41}{10}\right)^3+2\right) + \ln\left(\left(\frac{43}{10}\right)^3+2\right) + \ln\left(\left(\frac{45}{10}\right)^3+2\right) + \ln\left(\left(\frac{47}{10}\right)^3+2\right) + \ln\left(\left(\frac{49}{10}\right)^3+2\right) \\ &\qquad + \ln\left(\left(\frac{51}{10}\right)^3+2\right) + \ln\left(\left(\frac{53}{10}\right)^3+2\right) + \ln\left(\left(\frac{55}{10}\right)^3+2\right) + \ln\left(\left(\frac{57}{10}\right)^3+2\right) + \ln\left(\left(\frac{59}{10}\right)^3+2\right) \end{aligned}\right]$$

$$T_{10}=\frac{\left(\frac{1}{5}\right)}{2}\left[\begin{aligned}&\ln\left(\left(\frac{20}{5}\right)^3+2\right)+2\left(\ln\left(\left(\frac{21}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{22}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{23}{5}\right)^3+2\right)\right)\\&+2\left(\ln\left(\left(\frac{24}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{25}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{26}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{27}{5}\right)^3+2\right)\right)\\&+2\left(\ln\left(\left(\frac{28}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{29}{5}\right)^3+2\right)\right)+\ln\left(\left(\frac{30}{5}\right)^3+2\right)\end{aligned}\right]$$

$$S_{10}=\frac{\left(\frac{1}{5}\right)}{2}\left[\begin{aligned}&\ln\left(\left(\frac{20}{5}\right)^3+2\right)+4\left(\ln\left(\left(\frac{21}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{22}{5}\right)^3+2\right)\right)+4\left(\ln\left(\left(\frac{23}{5}\right)^3+2\right)\right)\\&+2\left(\ln\left(\left(\frac{24}{5}\right)^3+2\right)\right)+4\left(\ln\left(\left(\frac{25}{5}\right)^3+2\right)\right)+2\left(\ln\left(\left(\frac{26}{5}\right)^3+2\right)\right)+4\left(\ln\left(\left(\frac{27}{5}\right)^3+2\right)\right)\\&+2\left(\ln\left(\left(\frac{28}{5}\right)^3+2\right)\right)+4\left(\ln\left(\left(\frac{29}{5}\right)^3+2\right)\right)+\ln\left(\left(\frac{30}{5}\right)^3+2\right)\end{aligned}\right]$$

16) $\int_0^1 \sqrt{z}e^{-z}\,dx=\int_0^1\frac{\sqrt{z}}{e^z}\,dx$ $\qquad n=10$

$$\Delta x=\frac{(1)-(0)}{(10)}=\frac{1}{10}=\frac{2}{20}\qquad\frac{\Delta x}{2}=\frac{1}{20}$$

$\frac{0}{20}$	$\frac{1}{20}$		$\frac{3}{20}$		$\frac{5}{20}$		$\frac{7}{20}$		$\frac{9}{20}$	
z_0	$\bar{z}_1$	z_1	$\bar{z}_2$	z_2	$\bar{z}_3$	z_3	$\bar{z}_4$	z_4	$\bar{z}_5$	z_5
$0=\frac{0}{10}$		$\frac{1}{10}$		$\frac{2}{10}$		$\frac{3}{10}$		$\frac{4}{10}$		$\frac{5}{10}$

$\frac{11}{20}$		$\frac{13}{20}$		$\frac{15}{20}$		$\frac{17}{20}$		$\frac{19}{20}$	
$\bar{z}_6$	z_6	$\bar{z}_7$	z_7	$\bar{z}_8$	z_8	$\bar{z}_9$	z_9	$\bar{z}_{10}$	z_{10}
	$\frac{6}{10}$		$\frac{7}{10}$		$\frac{8}{10}$		$\frac{9}{10}$		$\frac{10}{10}$

$$M_{10} = \frac{\left(\frac{1}{10}\right)}{1}\left[\frac{\sqrt{\left(\frac{1}{20}\right)}}{e^{\left(\frac{1}{20}\right)}} + \frac{\sqrt{\left(\frac{3}{20}\right)}}{e^{\left(\frac{3}{20}\right)}} + \frac{\sqrt{\left(\frac{5}{20}\right)}}{e^{\left(\frac{5}{20}\right)}} + \frac{\sqrt{\left(\frac{7}{20}\right)}}{e^{\left(\frac{7}{20}\right)}} + \frac{\sqrt{\left(\frac{9}{20}\right)}}{e^{\left(\frac{9}{20}\right)}} + \frac{\sqrt{\left(\frac{11}{20}\right)}}{e^{\left(\frac{11}{20}\right)}} + \frac{\sqrt{\left(\frac{13}{20}\right)}}{e^{\left(\frac{13}{20}\right)}} + \frac{\sqrt{\left(\frac{15}{20}\right)}}{e^{\left(\frac{15}{20}\right)}} + \frac{\sqrt{\left(\frac{17}{20}\right)}}{e^{\left(\frac{17}{20}\right)}} + \frac{\sqrt{\left(\frac{19}{20}\right)}}{e^{\left(\frac{19}{20}\right)}}\right]$$

$$T_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\frac{\sqrt{\left(\frac{0}{10}\right)}}{e^{\left(\frac{0}{10}\right)}} + 2\left(\frac{\sqrt{\left(\frac{1}{10}\right)}}{e^{\left(\frac{1}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{2}{10}\right)}}{e^{\left(\frac{2}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{3}{10}\right)}}{e^{\left(\frac{3}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{4}{10}\right)}}{e^{\left(\frac{4}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{5}{10}\right)}}{e^{\left(\frac{5}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{6}{10}\right)}}{e^{\left(\frac{6}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{7}{10}\right)}}{e^{\left(\frac{7}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{8}{10}\right)}}{e^{\left(\frac{8}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{9}{10}\right)}}{e^{\left(\frac{9}{10}\right)}}\right) + \frac{\sqrt{\left(\frac{10}{10}\right)}}{e^{\left(\frac{10}{10}\right)}}\right]$$

$$S_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\frac{\sqrt{\left(\frac{0}{10}\right)}}{e^{\left(\frac{0}{10}\right)}} + 4\left(\frac{\sqrt{\left(\frac{1}{10}\right)}}{e^{\left(\frac{1}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{2}{10}\right)}}{e^{\left(\frac{2}{10}\right)}}\right) + 4\left(\frac{\sqrt{\left(\frac{3}{10}\right)}}{e^{\left(\frac{3}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{4}{10}\right)}}{e^{\left(\frac{4}{10}\right)}}\right) + 4\left(\frac{\sqrt{\left(\frac{5}{10}\right)}}{e^{\left(\frac{5}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{6}{10}\right)}}{e^{\left(\frac{6}{10}\right)}}\right) + 4\left(\frac{\sqrt{\left(\frac{7}{10}\right)}}{e^{\left(\frac{7}{10}\right)}}\right) + 2\left(\frac{\sqrt{\left(\frac{8}{10}\right)}}{e^{\left(\frac{8}{10}\right)}}\right) + 4\left(\frac{\sqrt{\left(\frac{9}{10}\right)}}{e^{\left(\frac{9}{10}\right)}}\right) + \frac{\sqrt{\left(\frac{10}{10}\right)}}{e^{\left(\frac{10}{10}\right)}}\right]$$

20) $\int_0^1 e^{x^2}\,dx \qquad |E_S| \le 0.00001$

$y = e^{x^2} \qquad f^{(4)}(x) = \frac{d^4y}{dx^4} = e^{x^2}(12 + 48x^2 + 16x^4) \qquad |E_S| \le \frac{K(b-a)^5}{180n^4} \qquad \left|f^{(4)}(x)\right| \le K$

$\left|f^{(4)}(x)\right| \le K = \left|e^{(1)^2}(12 + 48(1)^2 + 16(1)^4)\right| = 76e$

$$\frac{1}{100000} = \frac{(76e)(1)^5}{180n^4} \qquad n = \sqrt[4]{\frac{380000e}{9}}$$

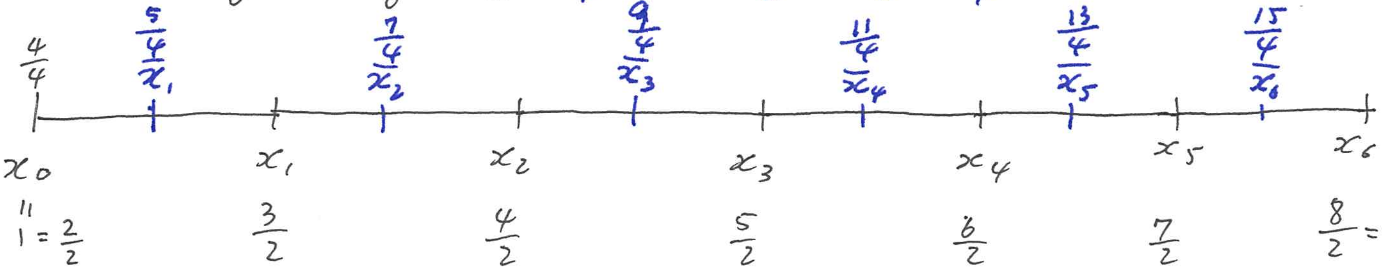
$$0.00001 \le \frac{(76e)((1)-(0))^5}{180n^4} \quad \Rightarrow \quad n^4 = \frac{(76e)(100000)}{180} \quad \Rightarrow \quad n = 18.40597773$$

$$n^4 = \frac{380000e}{9} \qquad n \approx 20$$

$\boxed{7.7/6}$

8) $\int_1^4 \sin\sqrt{x}\, dx$, $n = 6$ $\quad f(x_i) = \sin\sqrt{x_i}$

$\Delta x = \frac{(4)-(1)}{6} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4}$ $\qquad \frac{\Delta x}{2} = \frac{\left(\frac{1}{2}\right)}{2} = \frac{1}{4}$



b) $M_6 = \frac{\left(\frac{1}{2}\right)}{1}\left[\sin\sqrt{\left(\frac{5}{4}\right)} + \sin\sqrt{\left(\frac{7}{4}\right)} + \sin\sqrt{\left(\frac{9}{4}\right)} + \sin\sqrt{\left(\frac{11}{4}\right)} + \sin\sqrt{\left(\frac{13}{4}\right)} + \sin\sqrt{\left(\frac{15}{4}\right)}\right]$

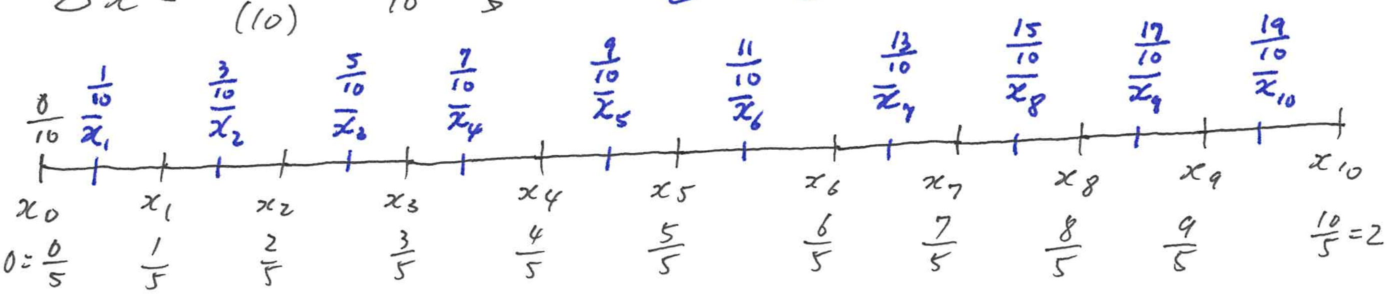
a) $T_6 = \frac{\left(\frac{1}{2}\right)}{2}\left[\sin\sqrt{\left(\frac{2}{2}\right)} + 2\sin\sqrt{\left(\frac{3}{2}\right)} + 2\sin\sqrt{\left(\frac{4}{2}\right)} + 2\sin\sqrt{\left(\frac{5}{2}\right)} + 2\sin\sqrt{\left(\frac{6}{2}\right)} + 2\sin\sqrt{\left(\frac{7}{2}\right)} + \sin\sqrt{\left(\frac{8}{2}\right)}\right]$

c) $S_6 = \frac{\left(\frac{1}{2}\right)}{3}\left[\sin\sqrt{\left(\frac{2}{2}\right)} + 4\sin\sqrt{\left(\frac{3}{2}\right)} + 2\sin\sqrt{\left(\frac{4}{2}\right)} + 4\sin\sqrt{\left(\frac{5}{2}\right)} + 2\sin\sqrt{\left(\frac{6}{2}\right)} + 4\sin\sqrt{\left(\frac{7}{2}\right)} + \sin\sqrt{\left(\frac{8}{2}\right)}\right]$

---

10) $\int_0^2 \sqrt[3]{1-x^2}\, dx$, $n = 10$ $\quad f(x_i) = \sqrt[3]{1-x_i^2}$

$\Delta x = \frac{(2)-(0)}{(10)} = \frac{2}{10} = \frac{1}{5}$ $\qquad \frac{\Delta x}{2} = \frac{\left(\frac{1}{5}\right)}{2} = \frac{1}{10}$



b) $M_{10} = \frac{\left(\frac{1}{5}\right)}{1}\left[\sqrt[3]{1-\left(\frac{1}{10}\right)^2} + \sqrt[3]{1-\left(\frac{3}{10}\right)^2} + \sqrt[3]{1-\left(\frac{5}{10}\right)^2} + \sqrt[3]{1-\left(\frac{7}{10}\right)^2} + \sqrt[3]{1-\left(\frac{9}{10}\right)^2}\right.$

$\left. + \sqrt[3]{1-\left(\frac{11}{10}\right)^2} + \sqrt[3]{1-\left(\frac{13}{10}\right)^2} + \sqrt[3]{1-\left(\frac{15}{10}\right)^2} + \sqrt[3]{1-\left(\frac{17}{10}\right)^2} + \sqrt[3]{1-\left(\frac{19}{10}\right)^2}\right]$

10) continued...

7.7/7

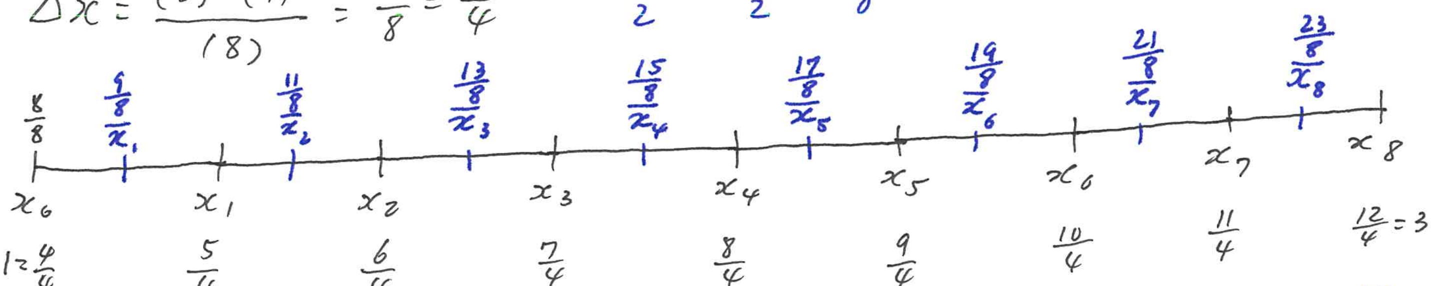
a) $$T_{10} = \frac{\left(\frac{1}{5}\right)}{2}\left[\sqrt[3]{1-\left(\tfrac{0}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{1}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{2}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{3}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{4}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{5}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{6}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{7}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{8}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{9}{5}\right)^2} + \sqrt[3]{1-\left(\tfrac{10}{5}\right)^2}\right]$$

c) $$S_{10} = \frac{\left(\frac{1}{5}\right)}{3}\left[\sqrt[3]{1-\left(\tfrac{0}{5}\right)^2} + 4\sqrt[3]{1-\left(\tfrac{1}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{2}{5}\right)^2} + 4\sqrt[3]{1-\left(\tfrac{3}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{4}{5}\right)^2} + 4\sqrt[3]{1-\left(\tfrac{5}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{6}{5}\right)^2} + 4\sqrt[3]{1-\left(\tfrac{7}{5}\right)^2} + 2\sqrt[3]{1-\left(\tfrac{8}{5}\right)^2} + 4\sqrt[3]{1-\left(\tfrac{9}{5}\right)^2} + \sqrt[3]{1-\left(\tfrac{10}{5}\right)^2}\right]$$

---

12) $\int_1^3 e^{\frac{1}{x}}\,dx$, $n = 8$ $\qquad f(x_i) = e^{\frac{1}{x_i}}$

$\Delta x = \frac{(3)-(1)}{(8)} = \frac{2}{8} = \frac{1}{4} \qquad \frac{\Delta x}{2} = \frac{\left(\frac{1}{4}\right)}{2} = \frac{1}{8}$



b) $$M_8 = \frac{\left(\frac{1}{4}\right)}{1}\left[e^{\frac{1}{(9/8)}} + e^{\frac{1}{(11/8)}} + e^{\frac{1}{(13/8)}} + e^{\frac{1}{(15/8)}} + e^{\frac{1}{(17/8)}} + e^{\frac{1}{(19/8)}} + e^{\frac{1}{(21/8)}} + e^{\frac{1}{(23/8)}}\right]$$

a) $$T_8 = \frac{\left(\frac{1}{4}\right)}{2}\left[e^{\frac{1}{(4/4)}} + 2e^{\frac{1}{(5/4)}} + 2e^{\frac{1}{(6/4)}} + 2e^{\frac{1}{(7/4)}} + 2e^{\frac{1}{(8/4)}} + 2e^{\frac{1}{(9/4)}} + 2e^{\frac{1}{(10/4)}} + 2e^{\frac{1}{(11/4)}} + e^{\frac{1}{(12/4)}}\right]$$

c) $$S_8 = \frac{\left(\frac{1}{4}\right)}{3}\left[e^{\frac{1}{(4/4)}} + 4e^{\frac{1}{(5/4)}} + 2e^{\frac{1}{(6/4)}} + 4e^{\frac{1}{(7/4)}} + 2e^{\frac{1}{(8/4)}} + 4e^{\frac{1}{(9/4)}} + 2e^{\frac{1}{(10/4)}} + 4e^{\frac{1}{(11/4)}} + e^{\frac{1}{(12/4)}}\right]$$

14) $\int_2^3 \frac{1}{\ln t}\,dt$, $n = 10$ $\quad f(t_i) = \frac{1}{\ln t_i}$

$\Delta t = \frac{(3)-(2)}{(10)} = \frac{1}{10} = \frac{2}{20} \qquad \frac{\Delta t}{2} = \frac{\left(\frac{1}{10}\right)}{2} = \frac{1}{20}$

$\frac{40}{20}$	$\frac{41}{20}$		$\frac{43}{20}$		$\frac{45}{20}$		$\frac{47}{20}$		$\frac{49}{20}$		$\frac{51}{20}$		$\frac{53}{20}$		$\frac{55}{20}$		$\frac{57}{20}$		$\frac{59}{20}$	
	t_1		t_2		t_3		t_4		t_5		t_6		t_7		t_8		t_9		t_{10}	
t_0		t_1		t_2		t_3		t_4		t_5		t_6		t_7		t_8		t_9		t_{10}
$2 = \frac{20}{10}$		$\frac{21}{10}$		$\frac{22}{10}$		$\frac{23}{10}$		$\frac{24}{10}$		$\frac{25}{10}$		$\frac{26}{10}$		$\frac{27}{10}$		$\frac{28}{10}$		$\frac{29}{10}$		$\frac{30}{10} = 3$

b) $M_{10} = \frac{\left(\frac{1}{10}\right)}{1}\left[\frac{1}{\ln\left(\frac{41}{20}\right)} + \frac{1}{\ln\left(\frac{43}{20}\right)} + \frac{1}{\ln\left(\frac{45}{20}\right)} + \frac{1}{\ln\left(\frac{47}{20}\right)} + \frac{1}{\ln\left(\frac{49}{20}\right)}\right.$

$\left. + \frac{1}{\ln\left(\frac{51}{20}\right)} + \frac{1}{\ln\left(\frac{53}{20}\right)} + \frac{1}{\ln\left(\frac{55}{20}\right)} + \frac{1}{\ln\left(\frac{57}{20}\right)} + \frac{1}{\ln\left(\frac{59}{20}\right)}\right]$

a) $T_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\frac{1}{\ln\left(\frac{20}{10}\right)} + 2\frac{1}{\ln\left(\frac{21}{10}\right)} + 2\frac{1}{\ln\left(\frac{22}{10}\right)} + 2\frac{1}{\ln\left(\frac{23}{10}\right)} + 2\frac{1}{\ln\left(\frac{24}{10}\right)}\right.$

$\left. + 2\frac{1}{\ln\left(\frac{25}{10}\right)} + 2\frac{1}{\ln\left(\frac{26}{10}\right)} + 2\frac{1}{\ln\left(\frac{27}{10}\right)} + 2\frac{1}{\ln\left(\frac{28}{10}\right)} + 2\frac{1}{\ln\left(\frac{29}{10}\right)} + \ln\frac{1}{\ln\left(\frac{30}{10}\right)}\right]$

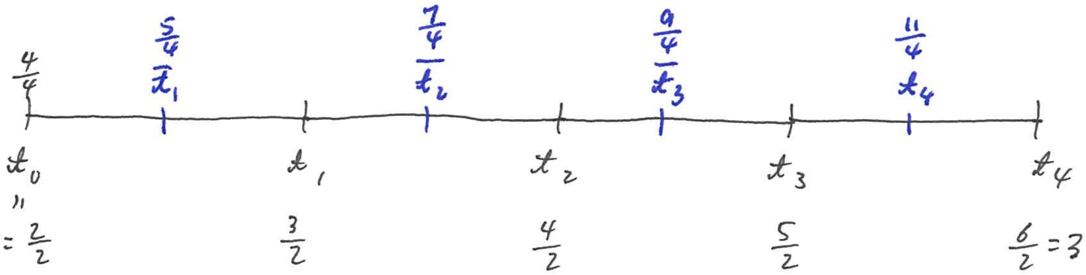
c) $S_{10} = \frac{\left(\frac{1}{10}\right)}{3}\left[\frac{1}{\ln\left(\frac{20}{10}\right)} + 4\frac{1}{\ln\left(\frac{21}{10}\right)} + 2\frac{1}{\ln\left(\frac{22}{10}\right)} + 4\frac{1}{\ln\left(\frac{23}{10}\right)} + 2\frac{1}{\ln\left(\frac{24}{10}\right)}\right.$

$\left. + 4\frac{1}{\ln\left(\frac{25}{10}\right)} + 2\frac{1}{\ln\left(\frac{26}{10}\right)} + 4\frac{1}{\ln\left(\frac{27}{10}\right)} + 2\frac{1}{\ln\left(\frac{28}{10}\right)} + 4\frac{1}{\ln\left(\frac{29}{10}\right)} + \frac{1}{\ln\left(\frac{30}{10}\right)}\right]$

7.7/9

16) $\int_1^3 \frac{\sin t}{t}\, dt$, $n=4$ $\qquad f(t_i) = \frac{\sin t_i}{t_i}$

$\Delta t = \frac{(3)-(1)}{(4)} = \frac{2}{4} = \frac{1}{2}$ $\qquad \frac{\Delta t}{2} = \frac{\left(\frac{1}{2}\right)}{2} = \frac{1}{4}$



b) $M_4 = \frac{\left(\frac{1}{2}\right)}{1}\left[\frac{\sin\left(\frac{5}{4}\right)}{\left(\frac{5}{4}\right)} + \frac{\sin\left(\frac{7}{4}\right)}{\left(\frac{7}{4}\right)} + \frac{\sin\left(\frac{9}{4}\right)}{\left(\frac{9}{4}\right)} + \frac{\sin\left(\frac{11}{4}\right)}{\left(\frac{11}{4}\right)}\right]$

a) $T_4 = \frac{\left(\frac{1}{2}\right)}{2}\left[\frac{\sin\left(\frac{2}{2}\right)}{\left(\frac{2}{2}\right)} + 2\frac{\sin\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)} + 2\frac{\sin\left(\frac{4}{2}\right)}{\left(\frac{4}{2}\right)} + 2\frac{\sin\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)} + \frac{\sin\left(\frac{6}{2}\right)}{\left(\frac{6}{2}\right)}\right]$

c) $S_4 = \frac{\left(\frac{1}{2}\right)}{3}\left[\frac{\sin\left(\frac{2}{2}\right)}{\left(\frac{2}{2}\right)} + 4\frac{\sin\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)} + 2\frac{\sin\left(\frac{4}{2}\right)}{\left(\frac{4}{2}\right)} + 4\frac{\sin\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)} + \frac{\sin\left(\frac{6}{2}\right)}{\left(\frac{6}{2}\right)}\right]$

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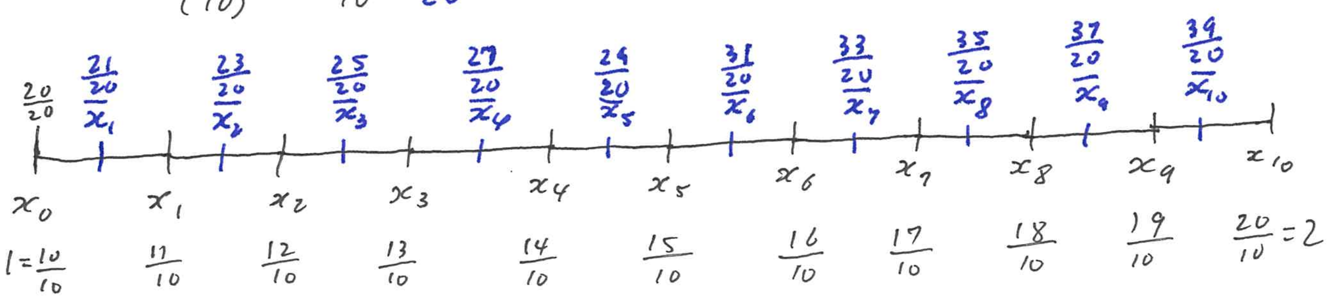
18) $\int_0^1 \sqrt{x+x^3}\, dx$, $n=10$ $\qquad f(x_i) = \sqrt{x_i + x_i^{\,3}}$

$\Delta x = \frac{(1)-(0)}{(10)} = \frac{1}{10} = \frac{2}{20}$ $\qquad \frac{\Delta x}{2} = \frac{\left(\frac{1}{10}\right)}{2} = \frac{1}{20}$



18) continued...

7.7/10

b) $$M_{10} = \frac{\left(\frac{1}{10}\right)}{1}\left[\sqrt{\left(\frac{21}{20}\right)+\left(\frac{21}{20}\right)^3} + \sqrt{\left(\frac{23}{20}\right)+\left(\frac{23}{20}\right)^3} + \sqrt{\left(\frac{25}{20}\right)+\left(\frac{25}{20}\right)^3} + \sqrt{\left(\frac{27}{20}\right)+\left(\frac{27}{20}\right)^3} + \sqrt{\left(\frac{29}{20}\right)+\left(\frac{29}{20}\right)^3}\right.$$
$$\left. + \sqrt{\left(\frac{31}{20}\right)+\left(\frac{31}{20}\right)^3} + \sqrt{\left(\frac{33}{20}\right)+\left(\frac{33}{20}\right)^3} + \sqrt{\left(\frac{35}{20}\right)+\left(\frac{35}{20}\right)^3} + \sqrt{\left(\frac{37}{20}\right)+\left(\frac{37}{20}\right)^3} + \sqrt{\left(\frac{39}{20}\right)+\left(\frac{39}{20}\right)^3}\right]$$

a) $$T_{10} = \frac{\left(\frac{1}{10}\right)}{2}\left[\sqrt{\left(\frac{10}{10}\right)+\left(\frac{10}{10}\right)^3} + 2\sqrt{\left(\frac{11}{10}\right)+\left(\frac{11}{10}\right)^3} + 2\sqrt{\left(\frac{12}{10}\right)+\left(\frac{12}{10}\right)^3} + 2\sqrt{\left(\frac{13}{10}\right)+\left(\frac{13}{10}\right)^3}\right.$$
$$+2\sqrt{\left(\frac{14}{10}\right)+\left(\frac{14}{10}\right)^3} + 2\sqrt{\left(\frac{15}{10}\right)+\left(\frac{15}{10}\right)^3} + 2\sqrt{\left(\frac{16}{10}\right)+\left(\frac{16}{10}\right)^3} + 2\sqrt{\left(\frac{17}{10}\right)+\left(\frac{17}{10}\right)^3}$$
$$\left. +2\sqrt{\left(\frac{18}{10}\right)+\left(\frac{18}{10}\right)^3} + 2\sqrt{\left(\frac{19}{10}\right)+\left(\frac{19}{10}\right)^3} + \sqrt{\left(\frac{20}{10}\right)+\left(\frac{20}{10}\right)^3}\right]$$

c) $$S_{10} = \frac{\left(\frac{1}{10}\right)}{3}\left[\sqrt{\left(\frac{10}{10}\right)+\left(\frac{10}{10}\right)^3} + 4\sqrt{\left(\frac{11}{10}\right)+\left(\frac{11}{10}\right)^3} + 2\sqrt{\left(\frac{12}{10}\right)+\left(\frac{12}{10}\right)^3} + 4\sqrt{\left(\frac{13}{10}\right)+\left(\frac{13}{10}\right)^3}\right.$$
$$+ 2\sqrt{\left(\frac{14}{10}\right)+\left(\frac{14}{10}\right)^3} + 4\sqrt{\left(\frac{15}{10}\right)+\left(\frac{15}{10}\right)^3} + 2\sqrt{\left(\frac{16}{10}\right)+\left(\frac{16}{10}\right)^3} + 4\sqrt{\left(\frac{17}{10}\right)+\left(\frac{17}{10}\right)^3}$$
$$\left. + 2\sqrt{\left(\frac{18}{10}\right)+\left(\frac{18}{10}\right)^3} + 4\sqrt{\left(\frac{19}{10}\right)+\left(\frac{19}{10}\right)^3} + \sqrt{\left(\frac{20}{10}\right)+\left(\frac{20}{10}\right)^3}\right]$$