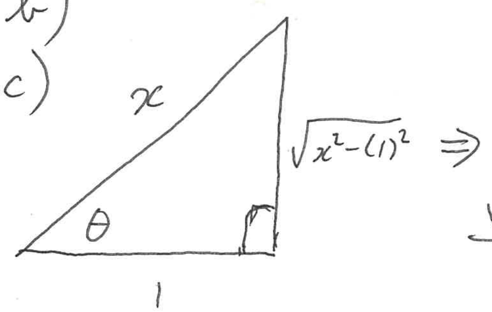
2-a) $\int x\sqrt{x^2-1}\,dx = \int \sqrt{x^2-1}\,(x\,dx) = \int \sqrt{p}\left(\frac{1}{2}dp\right) = \frac{1}{2}\int p^{\frac{1}{2}}dp$

$p = x^2-1$
$dp = 2x\,dx$
$\frac{1}{2}dp = x\,dx$

$= \frac{1}{2}\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{1}{3}\left(\sqrt{x^2-1}\right)^3 + C$

2-b)
2-c)



$\Rightarrow$

$\frac{x}{1} = \sec\theta \rightarrow x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\frac{\sqrt{x^2-(1)^2}}{1} = \tan\theta \rightarrow \sqrt{x^2-(1)^2} = \tan\theta$

2-b) $\int \frac{1}{x\sqrt{x^2-1}}dx = \int \frac{1}{x\sqrt{x^2-(1)^2}}dx = \int \frac{1}{(\sec\theta)(\tan\theta)}(\sec\theta\tan\theta\,d\theta)$

$= \int 1\,d\theta = [\theta] + C = \left(\sec^{-1}\left(\frac{x}{1}\right)\right) + C = \sec^{-1}(x) + C$

2-c) $\int \frac{\sqrt{x^2-1}}{x}dx = \int \frac{\sqrt{x^2-(1)^2}}{x}dx = \int \frac{(\tan\theta)}{(\sec\theta)}(\sec\theta\tan\theta\,d\theta)$

$= \int \tan^2\theta\,d\theta = \int(\sec^2\theta - 1)\,d\theta = \int \sec^2\theta\,d\theta - \int 1\,d\theta$

$= [\tan\theta] - [\theta] + C = \left(\frac{\sqrt{x^2-(1)^2}}{1}\right) - \left(\sec^{-1}\left(\frac{x}{1}\right)\right) + C$

$= \sqrt{x^2-1} - \sec^{-1}(x) + C$

4-a) $\int \sin^2 x\, dx = \int \left(\frac{1-\cos(2x)}{2}\right) dx$

$= \frac{1}{2}\int (1-\cos(2x))\, dx = \frac{1}{2}\left\{[x] - \left[\frac{1}{2}\sin(2x)\right]\right\} + C$

$= \underline{\underline{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}}$

4-b) $\int \sin^3 x\, dx = \int \sin^2 x(\sin x\, dx) = \int (1-\cos^2 x)(\sin x\, dx)$

$p = \cos x$
$dp = -\sin x\, dx$
$-1 dp = \sin x\, dx$

$= \int (1-p^2)(-1\, dp) = -1\left\{[p] - \left[\frac{p^3}{3}\right]\right\} + C$

$= -\left\{\cos x - \frac{1}{3}\cos^3 x\right\} + C = \underline{\underline{\frac{1}{3}\cos^3 x - \cos x + C}}$

4-c) $\int \sin 2x\, dx = \int 2\sin x \cos x\, dx = 2\int \sin x(\cos x\, dx)$

$p = \sin x$
$dp = \cos x\, dx$

$= 2\int p\, dp = 2\left[\frac{p^2}{2}\right] + C = \underline{\underline{\sin^2 x + C}}$

6-a) $\int x \cos x^2\, dx = \int \cos(x^2)(x\, dx) = \int \cos p \left(\frac{1}{2}dp\right)$

$p = x^2$
$dp = 2x\, dx$
$\frac{1}{2}dp = x\, dx$

$= \frac{1}{2}[\sin p] + C = \underline{\underline{\frac{1}{2}\sin(x^2) + C}}$

6-b) $\int x \cos^2 x\, dx = \int x\left(\frac{1+\cos(2x)}{2}\right) dx$

$= \frac{1}{2}\int x\, dx + \frac{1}{2}\int x\cos(2x)\, dx$

$= \frac{1}{2}\left[\frac{x^2}{2}\right] + \frac{1}{2}\left[\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)\right] + C$

$= \underline{\underline{\frac{1}{4}x^2 + \frac{1}{4}\sin(2x) + \frac{1}{8}\cos(2x) + C}}$

6-b) continued...

$$\int x \cos(2x)\,dx = (x)\left(\tfrac{1}{2}\sin(2x)\right) - \int \left(\tfrac{1}{2}\sin(2x)\right)(dx)$$

$u = x \qquad dv = \cos(2x)\,dx$

$du = dx \qquad v = \tfrac{1}{2}\sin(2x)$

$$= \tfrac{1}{2}x\sin(2x) - \tfrac{1}{2}\int \sin(2x)\,dx$$

$$= \tfrac{1}{2}x\sin(2x) - \tfrac{1}{2}\left[\tfrac{-1}{2}\cos(2x)\right] + C$$

$$= \tfrac{1}{2}x\sin(2x) + \tfrac{1}{4}\cos(2x) + C$$

6-c) $$\int x^2 \cos x\,dx = (x^2)(\sin x) - \int (\sin x)(2x\,dx)$$

$u_1 = x^2 \qquad dv_1 = \cos x\,dx$

$du_1 = 2x\,dx \qquad v_1 = \sin x$

$u_2 = x \qquad dv_2 = \sin x\,dx$

$du_2 = dx \qquad v_2 = -\cos x$

$$= x^2\sin x - 2\int x\sin x\,dx$$

$$= x^2\sin x - 2\left\{(x)(-\cos x) - \int(-\cos x)(dx)\right\}$$

$$= x^2\sin x + 2x\cos x - 2\int \cos x\,dx$$

$$= x^2\sin x + 2x\cos x - 2\left[\sin x\right] + C$$

$$= \underline{\underline{x^2\sin x + 2x\cos x - 2\sin x + C}}$$

8-a) $$\int e^x\sqrt{e^x-1}\,dx = \int\sqrt{e^x-1}\,(e^x\,dx) = \int\sqrt{p}\,(dp)$$

$p = e^x - 1$

$dp = e^x\,dx$

$$= \int p^{\frac{1}{2}}\,dp = \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \underline{\underline{\tfrac{2}{3}\left(\sqrt{e^x-1}\right)^3 + C}}$$

8-b) $$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int\frac{1}{\sqrt{(1)^2-(e^x)^2}}(e^x\,dx) = \int\frac{1}{\sqrt{(1)^2-p^2}}(dp)$$

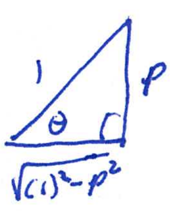
$p = e^x$

$dp = e^x\,dx$

$$= \int\frac{1}{(\cos\theta)}(\cos\theta\,d\theta) = \int 1\,d\theta = [\theta] + C$$

$$= \left(\sin^{-1}\left(\tfrac{p}{1}\right)\right) + C$$

$$= \underline{\underline{\sin^{-1}(e^x) + C}}$$



$\Rightarrow \frac{p}{1} = \sin\theta \rightarrow p = \sin\theta$

$dp = \cos\theta\,d\theta$

$\frac{\sqrt{(1)^2-p^2}}{1} = \cos\theta \rightarrow \sqrt{(1)^2-p^2} = \cos\theta$

8-c) $\int \frac{1}{\sqrt{e^x-1}}\,dx = \int \frac{1}{p}\left(\frac{2p}{p^2+1}\,dp\right) = \int \frac{2}{p^2+1}\,dp$ 7.5/4

$p = \sqrt{e^x-1}$ $\quad \frac{1}{p^2+1}(2p)\,dp = dx$ $\quad = \int \frac{2}{p^2+(1)^2}\,dp$

$p^2 = e^x - 1$

$p^2+1 = e^x$ $\quad \frac{2p}{p^2+1}\,dp = dx$ $\quad = 2\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C$

$\Downarrow$

$\ln(p^2+1) = x$ $\quad = 2\tan^{-1}(\sqrt{e^x-1}) + C$

---

10) $\int_0^1 (3x+1)^{\sqrt{2}}\,dx$

$\int (3x+1)^{\sqrt{2}}\,dx = \int p^{\sqrt{2}}\left(\frac{1}{3}\,dp\right) = \frac{1}{3}\left[\frac{p^{(\sqrt{2}+1)}}{\sqrt{2}+1}\right] + C$

$p = 3x+1$

$dp = 3\,dx$ $\quad = \frac{1}{3(1+\sqrt{2})}(3x+1)^{(1+\sqrt{2})} + C$

$\frac{1}{3}\,dp = dx$

$\int_0^1 (3x+1)^{\sqrt{2}}\,dx = \left[\frac{1}{3(1+\sqrt{2})}(3x+1)^{(1+\sqrt{2})} + C\right]_0^1$

$= \left[\frac{1}{3(1+\sqrt{2})}(3(1)+1)^{(1+\sqrt{2})} + C\right] - \left[\frac{1}{3(1+\sqrt{2})}(3(0)+1)^{(1+\sqrt{2})} + C\right]$

$= \left[\frac{1}{3(1+\sqrt{2})}(4)^{(1+\sqrt{2})}\right] - \left[\frac{1}{3(1+\sqrt{2})}(1)^{(1+\sqrt{2})}\right]$

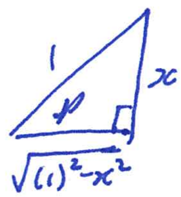
$= \frac{(4)^{(1+\sqrt{2})} - 1}{3(1+\sqrt{2})}$

12) $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx = \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx = \int e^{\sin^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}\,dx\right)$ 7.5/5

$p = \sin^{-1}x$

$\Downarrow$

$\sin p = x = \frac{x}{1} \Rightarrow$



$\left[\cos p \frac{dp}{dx}\right] = [1]$

$\frac{dp}{dx} = \frac{1}{\cos p} = \sec p = \left(\frac{1}{\sqrt{(1)^2-x^2}}\right) \Rightarrow dp = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int e^{p}(dp) = [e^{p}] + C$

$= \underline{\underline{e^{\sin^{-1}x} + C}}$

14) $\int_0^1 \frac{x}{(2x+1)^3}\,dx$

$\int \frac{x}{(2x+1)^3}\,dx = \int \frac{\left(\frac{p-1}{2}\right)}{p^3}\left(\frac{1}{2}dp\right) = \frac{1}{4}\int\left(\frac{p-1}{p^3}\right)dp$

$p = 2x+1 \longrightarrow \frac{p-1}{2} = x$

$\frac{1}{2}dp = dx$

$= \frac{1}{4}\int\left(\frac{p}{p^3} - \frac{1}{p^3}\right)dp = \frac{1}{4}\int\left(\frac{1}{p^2} - \frac{1}{p^3}\right)dp$

$= \frac{1}{4}\int\left(p^{-2} - p^{-3}\right)dp = \frac{1}{4}\left\{\left[\frac{p^{-1}}{-1}\right] - \left[\frac{p^{-2}}{-2}\right]\right\} + C$

$= \frac{1}{4}\left\{\frac{-1}{2x+1} + \frac{1}{2(2x+1)^2}\right\} + C = \frac{1}{8(2x+1)^2} - \frac{1}{4(2x+1)} + C$

$\int_0^1 \frac{x}{(2x+1)^3}\,dx = \left[\frac{1}{8(2x+1)^2} - \frac{1}{4(2x+1)} + C\right]_0^1$

$= \left[\frac{1}{8(2(1)+1)^2} - \frac{1}{4(2(1)+1)} + C\right] - \left[\frac{1}{8(2(0)+1)^2} - \frac{1}{4(2(0)+1)} + C\right]$

$= \left[\frac{1}{8(3)^2} - \frac{1}{4(3)}\right] - \left[\frac{1}{8(1)^2} - \frac{1}{4(1)}\right] = \left[\frac{1}{72} - \frac{1}{12}\right] - \left[\frac{1}{8} - \frac{1}{4}\right]$

$= \left[\frac{1}{72} - \frac{6}{72}\right] - \left[\frac{-1}{8}\right] = \frac{1}{8} - \frac{5}{72} = \frac{9}{72} - \frac{5}{72} = \frac{4}{72} = \frac{2}{36} = \underline{\underline{\frac{1}{18}}}$

16) $\int t \sin t \cos t \, dt = \int t\left(\frac{1}{2}\sin(2t)\right)dt$

$u = t \quad dv = \sin(2t)\,dt$ $= \frac{1}{2}\int t \sin(2t)\,dt$

$du = dt \quad v = \frac{-1}{2}\cos(2t)$ $= \frac{1}{2}\left\{(t)\left(\frac{-1}{2}\cos(2t)\right) - \int\left(\frac{-1}{2}\cos(2t)\right)(dt)\right\}$

$= \frac{1}{2}\left\{\frac{-1}{2}t\cos(2t) + \frac{1}{2}\int\cos(2t)\,dt\right\} = \frac{-1}{4}t\cos(2t) + \frac{1}{4}\int\cos(2t)\,dt$

$= \frac{-1}{4}t\cos(2t) + \frac{1}{4}\left[\frac{1}{2}\sin(2t)\right] + C = \underline{\underline{\frac{-1}{4}t\cos(2t) + \frac{1}{8}\sin(2t) + C}}$

---

18) $\int \frac{\cos\left(\frac{1}{x}\right)}{x^3}\,dx = \int\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\left(\frac{1}{x^2}\,dx\right) = \int p\cos p\,(-1\,dp)$

$p = \frac{1}{x} = x^{-1}$

$dp = \frac{-1}{x^2}\,dx$

$-1\,dp = \frac{1}{x^2}\,dx$

$u = p \quad dv = \cos p\,dp$

$du = dp \quad v = \sin p$

$= -1\int p\cos p\,dp$

$= -1\left\{(p)(\sin p) - \int(\sin p)(dp)\right\}$

$= -p\sin p + \int \sin p\,dp$

$= -p\sin p + [-\cos p] + C$

$= -\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) + C$

$= \underline{\underline{\frac{-\sin\left(\frac{1}{x}\right)}{x} - \cos\left(\frac{1}{x}\right) + C}}$

---

20) $\frac{2x-3}{x^3+3x} = \frac{2x-3}{(x)^1(x^2+3)^1} = \frac{A}{(x)^1} + \frac{(Bx+C)}{(x^2+3)^1}$

$2x-3 = A(x^2+3) + (Bx+C)(x)$

$2x-3 = A(x^2+3) + B(x^2) + C(x)$

const term	x-term	x^2-term
$-3 = 3A$	$\underline{2 = C}$	$0 = A + B$
$\underline{-1 = A}$		$B = -A$

$B = -(-1)$

$\underline{B = 1}$

20) continued...

$$\int \frac{2x-3}{x^3+3x}\,dx = \int \left(\frac{(-1)}{(x)^1} + \frac{(1)x+(2)}{(x^2+3)^1} \right) dx$$

$$= \int \left(\frac{-1}{x} + \frac{x}{x^2+3} + \frac{2}{x^2+(\sqrt{3})^2} \right) dx$$

$\varphi = x^2+3$

$$= -1\left[\ln|x|\right] + \left[\frac{1}{2}\ln|x^2+3|\right] + 2\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right] + C$$

$$= -\ln|x| + \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$$

---

22) $$\int \ln(1+x^2)\,dx = \left(\ln(1+x^2)\right)(x) - \int (x)\left(\frac{2x}{1+x^2}\,dx\right)$$

$u = \ln(1+x^2) \qquad dv = dx$

$du = \frac{1}{1+x^2}(2x)\,dx \qquad v = x$

$du = \frac{2x}{1+x^2}\,dx$

$$\begin{array}{r} 2 \\ x^2+0x+1 \overline{)\, 2x^2+0x+0} \\ -(2x^2+0x+2) \\ \hline -2 \end{array}$$

$$= x\ln(1+x^2) - \int \frac{2x^2}{x^2+1}\,dx$$

$$= x\ln(1+x^2) - \int \left(2 + \frac{(-2)}{x^2+1}\right) dx$$

$$= x\ln(1+x^2) - \int 2\,dx + \int \frac{2}{x^2+(1)^2}\,dx$$

$$= x\ln(1+x^2) - 2[x] + 2\left[\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right] + C$$

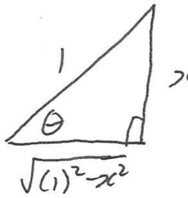
$$= x\ln(1+x^2) - 2x + 2\tan^{-1}(x) + C$$

---

24) $$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{x^2}{\sqrt{(1)^2-x^2}}\,dx = \int \frac{(\sin\theta)^2}{(\cos\theta)}(\cos\theta\,d\theta)$$

24) continued...



$\Rightarrow$ $\frac{x}{1} = \sin\theta \rightarrow x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\frac{\sqrt{(1)^2-x^2}}{1} = \cos\theta \rightarrow \sqrt{(1)^2-x^2} = \cos\theta$

$= \int \sin^2\theta\, d\theta$

$= \int \left(\frac{1-\cos(2\theta)}{2}\right) d\theta$

$= \frac{1}{2}\int (1-\cos(2\theta))\,d\theta = \frac{1}{2}\left\{[\theta] - \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C$

$= \frac{1}{2}\{\theta - \sin\theta\cos\theta\} + C = \frac{1}{2}\left\{\left(\sin^{-1}\left(\frac{x}{1}\right)\right) - \left(\frac{x}{1}\right)\left(\frac{\sqrt{(1)^2-x^2}}{1}\right)\right\} + C$

$= \frac{1}{2}\sin^{-1}(x) - \frac{1}{2}x\sqrt{1-x^2} + C$

$\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2}{\sqrt{1-x^2}}\,dx = \left[\frac{1}{2}\sin^{-1}(x) - \frac{1}{2}x\sqrt{1-x^2} + C\right]_0^{\frac{\sqrt{2}}{2}}$

$= \left[\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\left(\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}\right) + C\right] - \left[\frac{1}{2}\sin^{-1}(0) - \frac{1}{2}(0)\left(\sqrt{1-(0)^2}\right) + C\right]$

$= \left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(\sqrt{1-\frac{2}{4}}\right)\right] - \left[\frac{1}{2}(0) - 0\right] = \left[\frac{\pi}{8} - \frac{\sqrt{2}}{4}\left(\sqrt{\frac{2}{4}}\right)\right] - [0]$

$= \frac{\pi}{8} - \frac{\sqrt{2}}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{8} - \frac{2}{8} = \frac{\pi}{8} - \frac{1}{4} = \underline{\underline{\frac{\pi - 2}{8}}}$

26) $\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$

$\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt = \int e^{\sqrt{t}}\left(\frac{1}{\sqrt{t}}\,dt\right) = \int e^{p}(2\,dp) = 2\left[e^{p}\right] + C$

$p = \sqrt{t}$

$dp = \frac{1}{2\sqrt{t}}\,dt$

$2\,dp = \frac{1}{\sqrt{t}}\,dt$

$= 2e^{\sqrt{t}} + C$

$\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt = \left[2e^{\sqrt{t}} + C\right]_1^4 = \left[2e^{\sqrt{(4)}} + C\right] - \left[2e^{\sqrt{(1)}} + C\right]$

$= \left[2e^{2}\right] - \left[2e^{1}\right] = \underline{\underline{2e^2 - 2e}}$

28) $\int \frac{e^x}{1+e^{2a}}\,dx = \frac{1}{1+e^{2a}}\int e^x\,dx = \frac{1}{1+e^{2a}}\left[e^x\right]+C$

$$= \frac{1}{1+e^{2a}}e^x + C$$

---

30) $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}\,dx = \int \frac{\ln x}{\sqrt{1+(\ln x)^2}}\left(\frac{1}{x}dx\right)$

$p = \ln x$
$dp = \frac{1}{x}dx$

$$= \int \frac{p}{\sqrt{1+p^2}}(dp) = \int \frac{1}{\sqrt{1+p^2}}(p\,dp)$$

$q = 1+p^2$
$dq = 2p\,dp$
$\frac{1}{2}dq = p\,dp$

$$= \int \frac{1}{\sqrt{q}}\left(\frac{1}{2}dq\right) = \frac{1}{2}\int q^{-\frac{1}{2}}dq = \frac{1}{2}\left[\frac{q^{\frac{1}{2}}}{\frac{1}{2}}\right]+C$$

$$= \sqrt{1+p^2}+C = \sqrt{1+(\ln x)^2}+C$$

---

32) $\int (1+\tan x)^2 \sec x\,dx = \int (1+2\tan x+\tan^2 x)\sec x\,dx$

$$= \int \left(1+2\tan x+(\sec^2 x-1)\right)\sec x\,dx = \int (2\tan x+\sec^2 x)\sec x\,dx$$

$$= 2\int \tan x \sec x\,dx + \int \sec^3 x\,dx$$

$$= 2\int \sec x \tan x\,dx + \int \sec^3 x\,dx$$

$$= 2\left[\sec x\right] + \left[\frac{1}{2}\left(\sec x\tan x + \ln|\sec x+\tan x|\right)\right]+C$$

$$= 2\sec x + \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x+\tan x| + C$$

7.5/10

34) $\dfrac{3x^2+1}{x^3+x^2+x+1} = \dfrac{3x^2+1}{x^2(x+1)+1(x+1)} = \dfrac{3x^2+1}{(x+1)(x^2+1)}$

$= \dfrac{3x^2+1}{(x+1)^1(x^2+1)^1} = \dfrac{A}{(x+1)^1} + \dfrac{(Bx+C)}{(x^2+1)^1}$

$3x^2+1 = A(x^2+1) + (Bx+C)(x+1)$

$3x^2+1 = A(x^2+1) + B(x^2+x) + C(x+1)$

const. term: $1 = A + C$, $1 = A - B$ ✳

x-term: $0 = B + C$, $C = -B$

x^2-term: $3 = A + B$ ✳

$1 = A - B$
$3 = A + B$
$4 = 2A$
$\underline{2 = A}$

$3 = (2) + B$
$\underline{1 = B}$

$\underline{C = -1}$

$\displaystyle\int \frac{3x^2+1}{x^3+x^2+x+1}\,dx = \int\left(\frac{(2)}{(x+1)^1} + \frac{(1)x+(-1)}{(x^2+1)^1}\right)dx$

$\displaystyle= \int\left(\frac{2}{x+1} + \frac{x}{x^2+1} - \frac{1}{x^2+(1)^2}\right)dx$

$= 2\left[\ln|x+1|\right] + \left[\frac{1}{2}\ln|x^2+1|\right] - \left[\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)\right] + C$

$= \underline{2\ln|x+1| + \frac{1}{2}\ln|x^2+1| - \tan^{-1}(x) + C}$

$\displaystyle\int_0^1 \frac{3x^2+1}{x^3+x^2+x+1}\,dx = \left[2\ln|x+1| + \frac{1}{2}\ln|x^2+1| - \tan^{-1}(x) + C\right]_0^1$

$= \left[2\ln|(1)+1| + \frac{1}{2}\ln|(1)^2+1| - \tan^{-1}(1) + C\right] - \left[2\ln|(0)+1| + \frac{1}{2}\ln|(0)^2+1| - \tan^{-1}(0) + C\right]$

$= \left[2\ln|2| + \frac{1}{2}\ln|2| - \left(\frac{\pi}{4}\right)\right] - \left[2\ln|1| + \frac{1}{2}\ln|1| - (0)\right]$

$= \left[2\ln(2) + \frac{1}{2}\ln(2) - \frac{\pi}{4}\right] - \left[2(0) + \frac{1}{2}(0)\right]$

$= \underline{\underline{\frac{5}{2}\ln(2) - \frac{\pi}{4}}}$

36) $\int \sin\sqrt{at}\,dt = \int \sin p \left(\frac{2}{a} p\,dp\right)$

$p = \sqrt{at}$

$p^2 = at$

$2p\,dp = a\,dt$

$\frac{2}{a} p\,dp = dt$

$= \frac{2}{a}\int p \sin p\,dp = \frac{2}{a}\left\{(p)(-\cos p) - \int(-\cos p)(dp)\right\}$

$u = p \quad dv = \sin p$

$du = dp \quad v = -\cos p$

$= \frac{2}{a}\left\{-p\cos p + \int \cos p\,dp\right\}$

$= \frac{-2}{a} p\cos p + \frac{2}{a}\int \cos p\,dp$

$= \frac{-2}{a} p\cos p + \frac{2}{a}\left[\sin p\right] + C$

$= \frac{-2}{a}\left(\sqrt{at}\right)\cos\left(\sqrt{at}\right) + \frac{2}{a}\sin\left(\sqrt{at}\right) + C$

---

38) note: $|e^x - 1| = \begin{cases} +(e^x-1) = (e^x-1) & \text{if } x \geq 0 \\ -(e^x-1) = (1-e^x) & \text{if } x < 0 \end{cases}$

$\int_{-1}^{2} |e^x - 1|\,dx = \int_{-1}^{0} |e^x-1|\,dx + \int_{0}^{2} |e^x-1|\,dx$

$= \int_{-1}^{0}(1-e^x)\,dx + \int_{0}^{2}(e^x-1)\,dx$

$= \left[x - e^x + C\right]_{-1}^{0} + \left[e^x - x + C\right]_{0}^{2}$

$= \left\{\left[(0) - e^{(0)} + C\right] - \left[(-1) - e^{(-1)} + C\right]\right\} + \left\{\left[e^{(2)} - (2) + C\right] - \left[e^{(0)} - (0) + C\right]\right\}$

$= \left\{[-1] - \left[-1 - \frac{1}{e}\right]\right\} + \left\{\left[e^2 - 2\right] - [1]\right\}$

$= \left\{\frac{1}{e}\right\} + \left\{e^2 - 3\right\} = e^2 - 3 - \frac{1}{e}$

40) $\int \frac{e^{\frac{3}{x}}}{x^2}dx = \int e^{\frac{3}{x}}\left(\frac{1}{x^2}dx\right) = \int e^{p}\left(\frac{-1}{3}dp\right)$ 7.5/12

$p = \frac{3}{x}$

$dp = \frac{-3}{x^2}dx$

$\frac{-1}{3}dp = \frac{1}{x^2}dx$

$= \frac{-1}{3}\left[e^{p}\right] + C = \frac{-1}{3}e^{\frac{3}{x}} + C$

$\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}dx = \left[\frac{-1}{3}e^{\frac{3}{x}} + C\right]_1^3 = \left[\frac{-1}{3}e^{\frac{3}{(3)}} + C\right] - \left[\frac{-1}{3}e^{\frac{3}{(1)}} + C\right]$

$= \left[\frac{-1}{3}e^{1}\right] - \left[\frac{-1}{3}e^{3}\right] = \frac{1}{3}e^3 - \frac{1}{3}e = \underline{\underline{\frac{e^3 - e}{3}}}$

42) $\int \frac{1 + 4\cot x}{4 - \cot x}dx = \int \frac{1 + \frac{4\cos x}{\sin x}}{4 - \frac{\cos x}{\sin x}}dx$

$= \int \left(\frac{\frac{1}{1} + \frac{4\cos x}{\sin x}}{\frac{4}{1} - \frac{\cos x}{\sin x}}\right)\left(\frac{\frac{\sin x}{1}}{\frac{\sin x}{1}}\right)dx = \int \frac{\sin x + 4\cos x}{4\sin x - \cos x}dx$

$p = 4\sin x - \cos x$

$dp = 4\cos x + \sin x\, dx$

$dp = (\sin x + 4\cos x)dx$

$= \int \frac{1}{4\sin x - \cos x}(\sin x + 4\cos x)dx$

$= \int \frac{1}{p}(dp) = \left[\ln|p|\right] + C$

$= \ln|4\sin x - \cos x| + C$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 + 4\cot x}{4 - \cot x}dx = \left[\ln|4\sin x - \cos x| + C\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left[\ln\left|4\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)\right| + C\right] - \left[\ln\left|4\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)\right| + C\right]$

$= \left[\ln|4(1) - (0)|\right] - \left[\ln\left|4\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right|\right] = \left[\ln|4|\right] - \left[\ln\left|\frac{3}{\sqrt{2}}\right|\right]$

$= \underline{\underline{\ln(4) - \ln\left(\frac{3}{\sqrt{2}}\right)}}$

7.5/13

44) $\int \frac{1+\sin x}{1+\cos x}\,dx = \int \left(\frac{1+\sin x}{1+\cos x}\right)\left(\frac{1-\cos x}{1-\cos x}\right) dx$

$= \int \frac{1-\cos x+\sin x-\sin x\cos x}{1-\cos^2 x}\,dx$

$= \int \frac{1-\cos x+\sin x-\sin x\cos x}{\sin^2 x}\,dx$

$= \int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} + \frac{\sin x}{\sin^2 x} - \frac{\sin x\cos x}{\sin^2 x}\right) dx$

$= \int \frac{1}{\sin^2 x}\,dx - \int \frac{\cos x}{\sin^2 x}\,dx + \int \frac{1}{\sin x}\,dx - \int \frac{\cos x}{\sin x}\,dx$

$= \int \csc^2 x\,dx - \int \frac{1}{\sin^2 x}(\cos x\,dx) + \int \csc x\,dx - \int \cot x\,dx$

$u = \sin x$

$= [-\cot x] - \left[\frac{-1}{\sin x}\right] + [\ln|\csc x-\cot x|] - [\ln|\sin x|] + C$

$= -\cot x + \frac{1}{\sin x} + \ln|\csc x-\cot x| - \ln|\sin x| + C$

---

46) $\int \frac{\sin\theta\cot\theta}{\sec\theta}\,d\theta = \int \frac{\sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta}\right)}\,d\theta = \int \cos^2\theta\,d\theta$

$= \int \left(\frac{1+\cos(2\theta)}{2}\right) dx = \frac{1}{2}\int (1+\cos(2\theta))\,d\theta$

$= \frac{1}{2}\left\{[\theta] + \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C = \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C$

$\int_{\pi/6}^{\pi/3} \frac{\sin\theta\cot\theta}{\sec\theta}\,d\theta = \left[\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C\right]_{\pi/6}^{\pi/3}$

$= \left[\frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{4}\sin\left(2\left(\frac{\pi}{3}\right)\right) + C\right] - \left[\frac{1}{2}\left(\frac{\pi}{6}\right) + \frac{1}{4}\sin\left(2\left(\frac{\pi}{6}\right)\right) + C\right] = \left[\frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{2\pi}{3}\right)\right] - \left[\frac{\pi}{12} + \frac{1}{4}\sin\left(\frac{\ }{3}\right.\right.$

$= \left[\frac{\pi}{6} + \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right] - \left[\frac{\pi}{12} + \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right] = \underline{\underline{\frac{\pi}{12}}}$

48) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) = \sin A\cos B$

$$\int \sin(6x)\cos(3x)\,dx = \int \frac{1}{2}\left(\sin\left((6x)+(3x)\right) + \sin\left((6x)-(3x)\right)\right)dx$$

$$= \frac{1}{2}\int\left(\sin(9x) + \sin(3x)\right)dx = \frac{1}{2}\left\{\left[\frac{-1}{9}\cos(9x)\right] + \left[\frac{-1}{3}\cos(3x)\right]\right\} + C$$

$$= \frac{-1}{18}\cos(9x) - \frac{1}{6}\cos(3x) + C$$

$$\int_0^{\pi} \sin 6x\cos 3x\,dx = \left[\frac{-1}{18}\cos(9x) - \frac{1}{6}\cos(3x) + C\right]_0^{\pi}$$

$$= \left[\frac{-1}{18}\cos(9(\pi)) - \frac{1}{6}\cos(3(\pi)) + C\right] - \left[\frac{-1}{18}\cos(9(0)) - \frac{1}{6}\cos(3(0)) + C\right]$$

$$= \left[\frac{-1}{18}(-1) - \frac{1}{6}(-1)\right] - \left[\frac{-1}{18}(1) - \frac{1}{6}(1)\right] = \left[\frac{1}{18} + \frac{1}{6}\right] - \left[\frac{-1}{18} - \frac{1}{6}\right]$$

$$= \left[\frac{1}{18} + \frac{3}{18}\right] - \left[\frac{-1}{18} - \frac{3}{18}\right] = \left[\frac{4}{18}\right] - \left[\frac{-4}{18}\right] = \left[\frac{2}{9}\right] - \left[\frac{-2}{9}\right] = \frac{4}{9}$$

50) $$\int \frac{1}{x\sqrt{x-1}}\,dx = \int \frac{1}{(p^2+1)p}(2p\,dp) = \int \frac{2}{p^2+1}\,dp$$

$p = \sqrt{x-1}$

$p^2 = x-1 \rightarrow p^2+1 = x$

$2p\,dp = dx$

$$= 2\int \frac{1}{p^2+(1)^2}\,dp = 2\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C$$

$$= 2\tan^{-1}\left(\sqrt{x-1}\right) + C$$

$$52)\ \int \sqrt{1+e^x}\,dx = \int (p)\left(\frac{2p}{p^2-1}\,dp\right) = \int \frac{2p^2}{p^2-1}\,dp$$

7.5/15

$p = \sqrt{1+e^x}$

$p^2 = 1+e^x$

$p^2 - 1 = e^x$

$\Downarrow$

$\ln(p^2-1) = x$

$\frac{1}{p^2-1}(2p)\,dp = dx$

$\frac{2p}{p^2-1}\,dp = dx$

$$\begin{array}{r} 2 \\ p^2+0p-1\ \overline{)\ 2p^2+0p+0} \\ -(2p^2+0p-2) \\ \hline +2 \end{array}$$

$$= \int \left(2 + \frac{(+2)}{p^2-1}\right)dp$$

$$\frac{2}{p^2-1} = \frac{2}{(p+1)^1(p-1)^1} = \frac{A}{(p+1)^1} + \frac{B}{(p-1)^1}$$

$2 = A(p-1) + B(p+1)$

const term:

$2 = -A + B$

$2 = B + B$

$2 = 2B$

$\underline{1 = B}$

p-term

$0 = A + B$

$-A = B$

$A = -B$

$\underline{A = -1}$

$$= \int \left(2 + \frac{(-1)}{(p+1)^1} + \frac{(1)}{(p-1)^1}\right)dp$$

$$= 2[p] - 1[\ln|p+1|] + [\ln|p-1|] + C$$

$$= 2\sqrt{1+e^x} - \ln\left|\sqrt{1+e^x}+1\right| + \ln\left|\sqrt{1+e^x}-1\right| + C$$

$$= 2\sqrt{1+e^x} - \ln\left|\sqrt{1+e^x}+1\right| + \ln\left|\sqrt{1+e^x}-1\right| + C$$

$$54)\ \int \frac{(x-1)e^x}{x^2}\,dx = \int \left((x-1)e^x\right)\left(\frac{1}{x^2}\,dx\right)$$

$u = (x-1)e^x$

$du = (x-1)[e^x] + (e^x)[1]\,dx$

$du = xe^x - e^x + e^x\,dx$

$du = xe^x\,dx$

$dv = \frac{1}{x^2}\,dx$

$v = \frac{-1}{x}$

$$= \left((x-1)e^x\right)\left(\frac{-1}{x}\right) - \int \left(\frac{-1}{x}\right)\left(xe^x\,dx\right)$$

$$= \frac{-(x-1)e^x}{x} + \int e^x\,dx$$

$$= \frac{-(x-1)e^x}{x} + [e^x] + C$$

$$= \left(\frac{-x+1}{x}\right)e^x + e^x + C = \left(\frac{-x}{x} + \frac{1}{x}\right)e^x + e^x + C = \left(-1 + \frac{1}{x}\right)e^x + e^x + C$$

$$= -e^x + \frac{1}{x}e^x + e^x + C = \underline{\frac{e^x}{x} + C}$$

$$56)\ \int x\sqrt{2-\sqrt{1-x^2}}\,dx = \int \sqrt{2-\sqrt{1-x^2}}\,(x\,dx)$$

7.5/16

$p=\sqrt{1-x^2}$

$p^2 = 1-x^2$

$2p\,dp = -2x\,dx$

$-1p\,dp = x\,dx$

$$= \int \sqrt{2-p}\,(-1p\,dp) = \int -p\sqrt{2-p}\,dp$$

$q = 2-p \rightarrow p = 2-q$

$dq = -1\,dp$

$$= \int p\sqrt{2-p}\,(-1\,dp)$$

$$= \int (2-q)\sqrt{q}\,(dq)$$

$$= \int \left(2q^{\frac{1}{2}} - q^{\frac{3}{2}}\right)dq = 2\left[\frac{q^{\frac{3}{2}}}{\frac{3}{2}}\right] - \left[\frac{q^{\frac{5}{2}}}{\frac{5}{2}}\right] + C = \frac{4}{3}\left(\sqrt{2-p}\right)^3 - \frac{2}{5}\left(\sqrt{2-p}\right)^5 + C$$

$$= \frac{4}{3}\left(\sqrt{2-\sqrt{1-x^2}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{1-x^2}}\right)^5 + C$$

$$\int_0^1 x\sqrt{2-\sqrt{1-x^2}}\,dx = \left[\frac{4}{3}\left(\sqrt{2-\sqrt{1-x^2}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{1-x^2}}\right)^5 + C\right]_0^1$$

$$= \left[\frac{4}{3}\left(\sqrt{2-\sqrt{1-(1)^2}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{1-(1)^2}}\right)^5 + C\right] - \left[\frac{4}{3}\left(\sqrt{2-\sqrt{1-(0)^2}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{1-(0)^2}}\right)^5 + C\right]$$

$$= \left[\frac{4}{3}\left(\sqrt{2-\sqrt{0}}\right)^3 - \frac{2}{5}\left(\sqrt{2-\sqrt{0}}\right)^5\right] - \left[\frac{4}{3}\left(\sqrt{2-1}\right)^3 - \frac{2}{5}\left(\sqrt{2-1}\right)^5\right]$$

$$= \left[\frac{4}{3}\left(\sqrt{2}\right)^3 - \frac{2}{5}\left(\sqrt{2}\right)^5\right] - \left[\frac{4}{3}\left(\sqrt{1}\right)^3 - \frac{2}{5}\left(\sqrt{1}\right)^5\right]$$

$$= \left[\frac{4}{3}\left(2\sqrt{2}\right) - \frac{2}{5}\left(4\sqrt{2}\right)\right] - \left[\frac{4}{3} - \frac{2}{5}\right] = \left[\frac{40\sqrt{2}}{15} - \frac{24\sqrt{2}}{15}\right] - \left[\frac{20}{15} - \frac{6}{15}\right]$$

$$= \left[\frac{16\sqrt{2}}{15}\right] - \left[\frac{14}{15}\right] = \underline{\underline{\frac{16\sqrt{2}-14}{15}}}$$

$$58)\ \int \frac{1}{x^2\sqrt{4x+1}}\,dx = \int \frac{1}{\left(\frac{p^2-1}{4}\right)^2(p)}\left(\frac{1}{2}p\,dp\right)$$

$p = \sqrt{4x+1}$

$p^2 = 4x+1 \rightarrow \frac{p^2-1}{4} = x$

$2p\,dp = 4\,dx \rightarrow \frac{1}{2}p\,dp = dx$

$$= \int \frac{(4)^2}{(p^2-1)^2}\left(\frac{1}{2}\,dp\right)$$

$$= 8\int \frac{1}{(p^2-1)^2}\,dp$$

58) continued... (part 1)

7.5/17

$$\frac{1}{(p^2-1)^2}=\frac{1}{((p+1)(p-1))^2}=\frac{1}{(p+1)^2(p-1)^2}=\frac{A}{(p+1)^1}+\frac{B}{(p+1)^2}+\frac{C}{(p-1)^1}+\frac{D}{(p-1)^2}$$

$$1=A(p+1)(p-1)^2+B(p-1)^2+C(p+1)^2(p-1)+D(p+1)^2$$

$$1=A(p^2-1)(p-1)+B(p^2-2p+1)+C(p^2-1)(p+1)+D(p^2+2p+1)$$

$$1=A(p^3-p^2-p+1)+B(p^2-2p+1)+C(p^3+p^2-p-1)+D(p^2+2p+1)$$

const. term

$1=A+B-C+D$

$1=-C+B-C+D$

$1=B-2C+D$

$1=(+D)-2C+D$

$*\ 1=-2C+2D$

$$\begin{aligned}1&=-2C+2D\\0&=2C+2D\\\hline 1&=4D\\\tfrac{1}{4}&=D\end{aligned}$$

p-term

$0=-A-2B-C+2D$

$0=C-2B-C+2D$

$0=-2B+2D$

$2B=+2D$

$B=+D$

$\underline{B=\frac{1}{4}}$

p^2-term

$0=-A+B+C+D$

$0=C+B+C+D$

$0=B+2C+D$

$0=D+2C+D$

$*\ 0=2C+2D$

$2C=-2D$

$C=-D$

$\underline{C=\frac{-1}{4}}$

p^3-term

$0=A+C$

$-A=C$

$A=-C$

$A=-\left(\frac{-1}{4}\right)$

$\underline{A=\frac{1}{4}}$

$$=8\int\frac{1}{(p^2-1)^2}\,dp=8\int\left(\frac{\left(\frac{1}{4}\right)}{(p+1)^1}+\frac{\left(\frac{1}{4}\right)}{(p+1)^2}+\frac{\left(\frac{-1}{4}\right)}{(p-1)^1}+\frac{\left(\frac{1}{4}\right)}{(p-1)^2}\right)dp$$

$$=\int\left(\frac{2}{p+1}+\frac{2}{(p+1)^2}-\frac{2}{p-1}+\frac{2}{(p-1)^2}\right)dp$$

$$=2\left[\ln|p+1|\right]+2\left[\frac{(p+1)^{-1}}{-1}\right]-2\left[\ln|p-1|\right]+2\left[\frac{(p-1)^{-1}}{-1}\right]+C$$

$$=2\ln|p+1|-\frac{2}{p+1}-2\ln|p-1|-\frac{2}{p-1}+C$$

58) continued... (part 2)

7.5/18

$$= 2\ln\left|\sqrt{4x+1}+1\right| - \frac{2}{\sqrt{4x+1}+1} - 2\ln\left|\sqrt{4x+1}-1\right| - \frac{2}{\sqrt{4x+1}-1} + C$$

---

60) $$\int \frac{dx}{x(x^4+1)} = \int \frac{1}{x((x^2)^2+1)}\,dx = \int \frac{1}{x((x^2)^2+1)}\left(\frac{x}{x}\right)dx$$

$p = x^2$
$dp = 2x\,dx$
$\frac{1}{2}dp = x\,dx$

$$= \int \frac{1}{x^2((x^2)^2+1)}(x\,dx) = \int \frac{1}{p(p^2+1)}\left(\frac{1}{2}dp\right)$$

$$= \frac{1}{2}\int \frac{1}{(p)^1(p^2+1)^1}\,dp = \frac{1}{2}\int\left(\frac{(1)}{(p)^1} + \frac{(-1)p+(0)}{(p^2+1)^1}\right)dp$$

$$\frac{1}{(p)^1(p^2+1)^1} = \frac{A}{(p)^1} + \frac{(Bp+C)}{(p^2+1)^1}$$

$$1 = A(p^2+1) + (Bp+C)(p)$$

$$1 = A(p^2+1) + B(p^2) + C(p)$$

const term: $1 = A$

p-term: $0 = C$

p^2-term: $0 = A+B$, $B = -A$, $B = -1$

$$= \frac{1}{2}\int\left(\frac{1}{p} - \frac{p}{p^2+1}\right)dp = \frac{1}{2}\left\{\left[\ln|p|\right] - \left[\frac{1}{2}\ln|p^2+1|\right]\right\} + C$$

$$= \frac{1}{2}\ln|x^2| - \frac{1}{4}\ln|(x^2)^2+1| + C = \frac{1}{2}\ln|x^2| - \frac{1}{4}\ln|x^4+1| + C$$

---

62) $$\int (x+\sin x)^2\,dx = \int (x^2 + 2x\sin x + \sin^2 x)\,dx$$

$$= \int x^2\,dx + 2\int x\sin x\,dx + \int \sin^2 x\,dx$$

$$= \left[\frac{x^3}{3}\right] + 2\left[-x\cos x + \sin x\right] + \left[\frac{1}{2}x - \frac{1}{4}\sin(2x)\right] + C$$

$$= \frac{1}{3}x^3 - 2x\cos x + 2\sin x + \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

62) continued...

$$\int x \sin x \, dx = (x)(-\cos x) - \int (-\cos x)(dx)$$

$u = x \quad dv = \sin x \, dx$

$du = dx \quad v = -\cos x$

$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + [\sin x] + C_*$$

$$= \underline{-x\cos x + \sin x} + C_*$$

$$\int \sin^2 x \, dx = \int \left(\frac{1-\cos(2x)}{2}\right) dx = \frac{1}{2}\int (1-\cos(2x))\,dx$$

$$= \frac{1}{2}\left\{[x] - \left[\frac{1}{2}\sin(2x)\right]\right\} + C_* = \underline{\frac{1}{2}x - \frac{1}{4}\sin(2x)} + C_*$$

---

64) $$\int \frac{dx}{\sqrt{x} + x\sqrt{x}} = \int \frac{(2p\,dp)}{(p)+(p^2)(p)} = \int \frac{2p}{p(1+p^2)}\,dp$$

$p = \sqrt{x}$

$p^2 = x$

$2p\,dp = dx$

$$= \int \frac{2}{1+p^2}\,dp = \int \frac{2}{p^2+(1)^2}\,dp$$

$$= 2\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C = \underline{\underline{2\tan^{-1}(\sqrt{x}) + C}}$$

---

66) $$\int \frac{x \ln x}{\sqrt{x^2-1}}\,dx = \int \ln x \left(\frac{x}{\sqrt{x^2-1}}\,dx\right)$$

$p = \sqrt{x^2-1} \rightarrow p^2 = x^2 - 1$

$p^2 + 1 = x^2$

$\sqrt{p^2+1} = x$

$dp = \frac{1}{2\sqrt{x^2-1}}(2x)\,dx$

$dp = \frac{x}{\sqrt{x^2-1}}\,dx$

$$= \int \ln\sqrt{p^2+1}\,(dp)$$

$$= \int \ln(p^2+1)^{\frac{1}{2}}\,dp$$

$$= \int \frac{1}{2}\ln(p^2+1)\,dp$$

$$= \frac{1}{2}\int \ln(p^2+1)\,dp$$

66) continued...

$$u = \ln(p^2+1) \qquad dv = dp$$
$$du = \frac{2p}{p^2+1}\,dp \qquad v = p$$

$$= \frac{1}{2}\left\{\left(\ln(p^2+1)\right)(p) - \int (p)\left(\frac{2p}{p^2+1}dp\right)\right\}$$

$$= \frac{1}{2}p\ln(p^2+1) - \int \frac{p^2}{p^2+1}\,dp$$

$$= \frac{1}{2}p\ln(p^2+1) - \int \frac{p^2+1-1}{p^2+1}\,dp = \frac{1}{2}p\ln(p^2+1) - \int\left(\frac{p^2+1}{p^2+1} - \frac{1}{p^2+1}\right)dp$$

$$= \frac{1}{2}p\ln(p^2+1) - \int\left(1 - \frac{1}{p^2+(1)^2}\right)dp$$

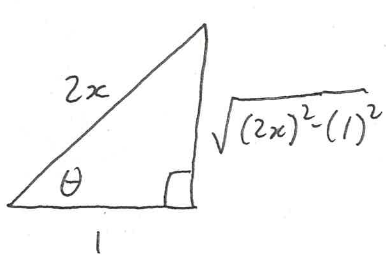
$$= \frac{1}{2}p\ln(p^2+1) - \left\{[p] - \left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right]\right\} + C$$

$$= \frac{1}{2}\left(\sqrt{x^2-1}\right)\ln\left(x^2\right) - \left(\sqrt{x^2-1}\right) + \tan^{-1}\left(\sqrt{x^2-1}\right) + C$$

$$= \frac{1}{2}\sqrt{x^2-1}\left(2\ln x\right) - \sqrt{x^2-1} + \tan^{-1}\left(\sqrt{x^2-1}\right) + C$$

$$= \sqrt{x^2-1}\,\ln x - \sqrt{x^2-1} + \tan^{-1}\sqrt{x^2-1} + C$$

68) $$\int \frac{dx}{x^2\sqrt{4x^2-1}} = \int \frac{1}{x^2\sqrt{(2x)^2-(1)^2}}\,dx$$



$\Longrightarrow$

$$\frac{2x}{1} = \sec\theta \rightarrow x = \frac{1}{2}\sec\theta$$
$$dx = \frac{1}{2}\sec\theta\tan\theta\,d\theta$$
$$\frac{\sqrt{(2x)^2-(1)^2}}{1} = \tan\theta \rightarrow \sqrt{(2x)^2-(1)^2} = \tan\theta$$

$$= \int \frac{1}{\left(\frac{1}{2}\sec\theta\right)^2(\tan\theta)}\left(\frac{1}{2}\sec\theta\tan\theta\,d\theta\right) = \int \frac{1}{\frac{1}{2}\sec\theta}\,d\theta$$

68) continued...

$$= 2\int \cos\theta \, d\theta = 2\left[\sin\theta\right] + C = 2\left(\frac{\sqrt{(2x)^2-(1)^2}}{2x}\right) + C$$

$$= \frac{\sqrt{4x^2-1}}{x} + C$$

70) $$\int \frac{d\theta}{1+\cos^2\theta} = \int \frac{1}{1+\cos^2\theta}\, d\theta = \int \left(\frac{1}{1+\cos^2\theta}\right)\left(\frac{\frac{1}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}\right) d\theta$$

$$= \int \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}}\, d\theta = \int \frac{\sec^2\theta}{\sec^2\theta+1}\, d\theta = \int \frac{1}{(1+\tan^2\theta)+1}(\sec^2\theta\, d\theta)$$

$p = \tan\theta$
$dp = \sec^2\theta\, d\theta$

$$= \int \frac{1}{\tan^2\theta+2}(\sec^2\theta\, d\theta) = \int \frac{1}{p^2+2}(dp)$$

$$= \int \frac{1}{p^2+(\sqrt{2})^2}\, dp = \left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{p}{\sqrt{2}}\right)\right] + C = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\theta}{\sqrt{2}}\right) + C$$

72) $$\int \frac{1}{\sqrt{\sqrt{x}+1}}\, dx = \int \frac{1}{\sqrt{p}}\left(2(p-1)\, dp\right) = 2\int \frac{p-1}{\sqrt{p}}\, dp$$

$p = \sqrt{x}+1$
$p-1 = \sqrt{x}$
$(p-1)^2 = x$
$2(p-1)\, dp = dx$

$$= 2\int\left(\frac{p}{\sqrt{p}} - \frac{1}{\sqrt{p}}\right) dp = 2\int\left(p^{\frac{1}{2}} - p^{-\frac{1}{2}}\right) dp$$

$$= 2\left\{\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] - \left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right]\right\} + C$$

$$= 2\left\{\frac{2}{3}(\sqrt{x}+1)^{\frac{3}{2}} - 2\sqrt{\sqrt{x}+1}\right\} + C$$

$$= \frac{4}{3}\left(\sqrt{\sqrt{x}+1}\right)^3 - 4\sqrt{\sqrt{x}+1} + C$$

74) $\int \frac{\ln(\tan x)}{\sin x \cos x} dx = \int \frac{\ln(\tan x)}{\sin x \cos x \left(\frac{\cos x}{\cos x}\right)} dx$ 7.5/22

$= \int \frac{\ln(\tan x)}{\frac{\sin x}{\cos x}} \left(\frac{1}{\cos x(\cos x)} dx\right) = \int \frac{\ln(\tan x)}{\tan x} \left(\frac{1}{\cos^2 x} dx\right)$

$p = \tan x$
$dp = \sec^2 x\, dx$

$= \int \frac{\ln(\tan x)}{\tan x} (\sec^2 x\, dx) = \int \frac{\ln p}{p} (dp)$

$q = \ln p$
$dq = \frac{1}{p} dp$

$= \int \ln p \left(\frac{1}{p} dp\right) = \int q (dq) = \left[\frac{q^2}{2}\right] + C$

$= \frac{1}{2}(\ln p)^2 + C = \frac{1}{2}(\ln(\tan x))^2 + C$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(\tan x)}{\sin x \cos x} dx = \left[\frac{1}{2}(\ln(\tan x))^2 + C\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left[\frac{1}{2}\left(\ln\left(\tan\left(\frac{\pi}{3}\right)\right)\right)^2 + C\right] - \left[\frac{1}{2}\left(\ln\left(\tan\left(\frac{\pi}{4}\right)\right)\right)^2 + C\right]$

$= \left[\frac{1}{2}\left(\ln\left(\frac{\sqrt{3}}{1}\right)\right)^2\right] - \left[\frac{1}{2}(\ln(1))^2\right] = \left[\frac{1}{2}\left(\ln\left(3^{\frac{1}{2}}\right)\right)^2\right] - \left[\frac{1}{2}(0)^2\right]$

$= \left[\frac{1}{2}\left(\frac{1}{2}\ln(3)\right)^2\right] - [0] = \frac{1}{8}(\ln 3)^2$

---

76) $\int \frac{x^2}{x^6 + 3x^3 + 2} dx = \int \frac{1}{(x^3)^2 + 3(x^3) + 2} (x^2 dx)$

$p = x^3$
$dp = 3x^2 dx$
$\frac{1}{3} dp = x^2 dx$

$= \int \frac{1}{p^2 + 3p + 2} \left(\frac{1}{3} dp\right) = \frac{1}{3} \int \frac{1}{(p+1)(p+2)} dp$

$= \frac{1}{3} \int \left(\frac{(1)}{(p+1)'} + \frac{(-1)}{(p+2)'}\right) dp$

76) continued...

$$\frac{1}{(p+1)^1(p+2)^1} = \frac{A}{(p+1)^1} + \frac{B}{(p+2)^1}$$

$$1 = A(p+2) + B(p+1)$$

const term

$1 = 2A + B$

$1 = 2A + (-A)$

$\underline{1 = A}$

p-term

$0 = A + B$

$B = -A$

$\underline{B = -1}$

$$= \frac{1}{3}\int\left(\frac{1}{p+1} - \frac{1}{p+2}\right)dp = \frac{1}{3}\left\{\left[\ln|p+1|\right] - \left[\ln|p+2|\right]\right\} + C$$

$$= \frac{1}{3}\ln|x^3+1| - \frac{1}{3}\ln|x^3+2| + C$$

78) $\displaystyle\int \frac{1}{1+2e^x - e^{-x}}\,dx = \int \frac{1}{1+2e^x - \frac{1}{e^x}}\,dx$

$p = e^x \rightarrow \ln p = x$

$\frac{1}{p}\,dp = dx$

$$= \int \frac{1}{1+2p-\frac{1}{p}}\left(\frac{1}{p}\,dp\right)$$

$$= \int \frac{1}{\left(1+2p-\frac{1}{p}\right)(p)}\,dp = \int \frac{1}{p+2p^2-1}\,dp$$

$$= \int \frac{1}{2p^2+p-1}\,dp = \int \frac{1}{(p+1)(2p-1)}\,dp = \int\left(\frac{\left(\frac{-1}{3}\right)}{(p+1)^1} + \frac{\left(\frac{2}{3}\right)}{(2p-1)^1}\right)dp$$

$$\frac{1}{(p+1)^1(2p-1)^1} = \frac{A}{(p+1)^1} + \frac{B}{(2p-1)^1}$$

$$1 = A(2p-1) + B(p+1)$$

const term

$1 = -A + B$

$1 = -A + (-2A)$

$1 = -3A$

$\underline{\frac{-1}{3} = A}$

p-term

$0 = 2A + B$

$-2A = B$

$-2\left(\frac{-1}{3}\right) = B$

$\underline{\frac{2}{3} = B}$

$$= \frac{-1}{3}\left[\ln|p+1|\right] + \frac{2}{3}\left[\frac{1}{2}\ln|2p-1|\right] + C$$

$$= \frac{-1}{3}\ln|e^x+1| + \frac{1}{3}\ln|2e^x-1| + C$$

$$= \frac{1}{3}\ln|2e^x-1| - \frac{1}{3}\ln|e^x+1| + C$$

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80) $\displaystyle\int \frac{\ln(x+1)}{x^2}\,dx = \int \ln(x+1)\left(\frac{1}{x^2}\,dx\right)$

$u = \ln(x+1) \qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x+1}\,dx \qquad v = \frac{-1}{x}$

$= \left(\ln(x+1)\right)\left(\frac{-1}{x}\right) - \int\left(\frac{-1}{x}\right)\left(\frac{1}{x+1}\,dx\right)$

$= \frac{-1}{x}\ln(x+1) + \int \frac{1}{(x)^1(x+1)^1}\,dx$

$\frac{1}{(x)^1(x+1)^1} = \frac{A}{(x)^1} + \frac{B}{(x+1)^1}$

$= \frac{-1}{x}\ln(x+1) + \int\left(\frac{(1)}{(x)^1} + \frac{(-1)}{(x+1)^1}\right)dx$

$1 = A(x+1) + B(x)$

$= \frac{-1}{x}\ln(x+1) + \left[\ln|x|\right] - \left[\ln|x+1|\right] + C$

const term $\qquad$ x-term

$1 = A \qquad 0 = A + B$

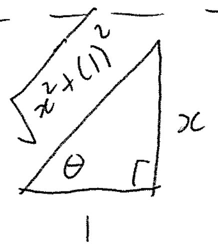
$B = -1$

$= \frac{-1}{x}\ln(x+1) + \ln|x| - \ln|x+1| + C$

82) $\displaystyle\int \frac{4^x + 10^x}{2^x}\,dx = \int\left(\frac{4^x}{2^x} + \frac{10^x}{2^x}\right)dx = \int\left(\left(\frac{4}{2}\right)^x + \left(\frac{10}{2}\right)^x\right)dx$

$= \int\left(2^x + 5^x\right)dx = \left[\frac{2^x}{\ln 2}\right] + \left[\frac{5^x}{\ln 5}\right] + C = \frac{1}{\ln 2}2^x + \frac{1}{\ln 5}5^x + C$

84) $\displaystyle\int \frac{x^2}{\sqrt{x^2+1}}\,dx = \int \frac{x^2}{\sqrt{x^2+(1)^2}}\,dx = \int \frac{(\tan\theta)^2}{\sec\theta}\left(\sec^2\theta\,d\theta\right)$



$\Rightarrow$

$\frac{x}{1} = \tan\theta \rightarrow x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{x^2+(1)^2}}{1} = \sec\theta \rightarrow \sqrt{x^2+(1)^2} = \sec\theta$

$= \int \tan^2\theta\sec\theta\,d\theta$

$= \int\left(\sec^2\theta - 1\right)\sec\theta\,d\theta$

$= \int\left(\sec^3\theta - \sec\theta\right)d\theta$

$= \left[\frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)\right] - \left[\ln|\sec\theta + \tan\theta|\right] + C$

$= \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C = \frac{1}{2}\left(\frac{x}{1}\right)\left(\frac{\sqrt{x^2+(1)^2}}{1}\right) - \frac{1}{2}\ln\left|\left(\frac{x}{1}\right) + \left(\frac{\sqrt{x^2+(1)^2}}{1}\right)\right| + C$

$= \frac{1}{2}x\sqrt{x^2+1} - \frac{1}{2}\ln\left|x + \sqrt{x^2+1}\right| + C$

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86) $\int \frac{1+\sin x}{1-\sin x}\,dx = \int \left(\frac{1+\sin x}{1-\sin x}\right)\left(\frac{1+\sin x}{1+\sin x}\right)dx$

$= \int \frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}\,dx = \int \frac{1+2\sin x+\sin^2 x}{\cos^2 x}\,dx$

$= \int \left(\frac{1}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right)dx$

$= \int \left(\sec^2 x + \frac{2\sin x}{\cos^2 x} + \tan^2 x\right)dx = \int\left(\sec^2 x + \frac{2\sin x}{\cos^2 x} + (\sec^2 x - 1)\right)dx$

$= \int \left(2\sec^2 x + \frac{2\sin x}{\cos^2 x} - 1\right)dx = 2\int \sec^2 x\,dx + 2\int \frac{\sin x}{\cos^2 x}\,dx - \int 1\,dx$

$p = \cos x$

$= 2\left[\tan x\right] + 2\left[\frac{-1}{\cos x}\right] - \left[x\right] + C$

$= 2\tan x - 2\sec x - x + C$

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88) $\int \frac{\sec x\,\cos(2x)}{\sin x + \sec x}\,dx = \int \frac{\left(\frac{1}{\cos x}\right)\cos(2x)}{\sin x + \left(\frac{1}{\cos x}\right)}\,dx$

$= \int \left(\frac{\left(\frac{1}{\cos x}\right)\cos(2x)}{\sin x + \left(\frac{1}{\cos x}\right)}\right)\left(\frac{\frac{\cos x}{1}}{\frac{\cos x}{1}}\right)dx = \int \frac{\cos(2x)}{\sin x\cos x + 1}\,dx$

$= \int \frac{\cos(2x)}{\frac{1}{2}\sin(2x)+1}\,dx = \int \frac{1}{\frac{1}{2}\sin(2x)+1}\left(\cos(2x)\,dx\right)$

$p = \frac{1}{2}\sin(2x)+1$

$dp = \frac{1}{4}\cos(2x)\,dx$

$= 4\,dp = \cos(2x)\,dx$

$= \int \frac{1}{p}(4\,dp) = 4\left[\ln|p|\right] + C$

$= 4\ln\left|\frac{1}{2}\sin(2x)+1\right| + C$

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90) $\displaystyle\int \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\,dx = \int \left(\frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\right)\left(\frac{\frac{1}{\cos^4 x}}{\frac{1}{\cos^4 x}}\right)dx$

$$= \int \frac{\frac{\sin x}{\cos^3 x}}{\frac{\sin^4 x}{\cos^4 x} + \frac{\cos^4 x}{\cos^4 x}}\,dx = \int \frac{\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos^2 x}\right)}{\tan^4 x + 1}\,dx$$

$$= \int \frac{\tan x \sec^2 x}{\tan^4 x + 1}\,dx = \int \frac{1}{(\tan^2 x)^2 + 1}\left(\tan x \sec^2 x\, dx\right)$$

$p = \tan^2 x$
$dp = 2\tan x \sec^2 x\, dx$
$\frac{1}{2}dp = \tan x \sec^2 x\, dx$

$$= \int \frac{1}{p^2+1}\left(\frac{1}{2}dp\right) = \frac{1}{2}\int \frac{1}{p^2+(1)^2}\,dp$$

$$= \frac{1}{2}\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C = \frac{1}{2}\tan^{-1}(\tan^2 x) + C$$

92) $\displaystyle\int \frac{1}{(\sin x + \cos x)^2}\,dx = \int \frac{1}{\sin^2 x + 2\sin x \cos x + \cos^2 x}\,dx$

$$= \int \frac{1}{\sin^2 x\left(\frac{\cos^2 x}{\cos^2 x}\right) + 2\sin x \cos x\left(\frac{\cos x}{\cos x}\right) + \cos^2 x}\,dx$$

$$= \int \frac{1}{\left(\left(\frac{\sin^2 x}{\cos^2 x}\right) + \frac{2\sin x}{\cos x} + 1\right)\cos^2 x}\,dx = \int \frac{1}{\tan^2 x + 2\tan x + 1}\left(\frac{1}{\cos^2 x}dx\right)$$

$$= \int \frac{1}{(\tan x + 1)^2}\left(\sec^2 x\, dx\right) = \int \frac{1}{p^2}\,dp = \int p^{-2}\,dp = \left[\frac{p^{-1}}{-1}\right] + C$$

$p = \tan x + 1$
$dp = \sec^2 x\, dx$

$$= \frac{-1}{\tan x + 1} + C$$