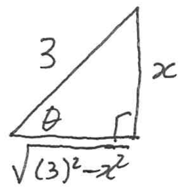
2) via trig. substitution

$$\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{x^3}{\sqrt{(3)^2-x^2}}\,dx = \int \frac{(3\sin\theta)^3}{(3\cos\theta)}(3\cos\theta\, d\theta)$$



$$\Rightarrow \quad \frac{x}{3} = \sin\theta \rightarrow x = 3\sin\theta$$

$$dx = 3\cos\theta\, d\theta$$

$$\frac{\sqrt{(3)^2-x^2}}{3} = \cos\theta \rightarrow \sqrt{(3)^2-x^2} = 3\cos\theta$$

$$= \int (3)^3 \sin^3\theta\, d\theta$$

$$= \int (3)^3 \sin^2\theta(\sin\theta\, d\theta)$$

$$= (3)^3 \int (1-\cos^2\theta)(\sin\theta\, d\theta) = (3)^3 \int (1-p^2)(-1\, dp) = -(3)^3\left\{[p] - \left[\frac{p^3}{3}\right]\right\} + C$$

$p = \cos\theta$

$dp = -\sin\theta\, d\theta$

$-1\, dp = \sin\theta\, d\theta$

$$= -(3)^3\left\{\cos\theta - \frac{1}{3}\cos^3\theta\right\} + C$$

$$= -(3)^3\left\{\left(\frac{\sqrt{(3)^2-x^2}}{3}\right) - \frac{1}{3}\left(\frac{\sqrt{(3)^2-x^2}}{3}\right)^3\right\} + C$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}\left(\sqrt{9-x^2}\right)^3 + C$$

$$= \frac{1}{3}\left(\sqrt{9-x^2}\right)^3 - 9\sqrt{9-x^2} + C$$

---

via triple substitution

$$\int \frac{x^3}{\sqrt{9-x^2}}\,dx = \int \frac{x^2}{\sqrt{9-x^2}}(x\,dx) = \int \frac{(9-p)}{\sqrt{p}}\left(\frac{-1}{2}\,dp\right)$$

$p = 9 - x^2 \rightarrow x^2 = 9 - p$

$dp = -2x\,dx$

$-\frac{1}{2}\,dp = x\,dx$

$$= \frac{-1}{2}\int\left(\frac{9}{\sqrt{p}} - \frac{p}{\sqrt{p}}\right)dp = \frac{-1}{2}\int\left(\frac{9}{\sqrt{p}} - \sqrt{p}\right)dp$$

$$= \frac{-1}{2}\int\left(9p^{-\frac{1}{2}} - p^{\frac{1}{2}}\right)dp = \frac{-1}{2}\left\{9\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] - \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right]\right\} + C$$

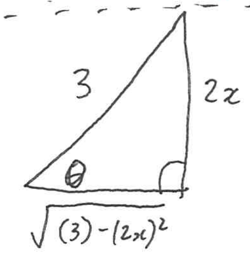
$$= \frac{-1}{2}\left\{18(9-x^2)^{\frac{1}{2}} - \frac{2}{3}(9-x^2)^{\frac{3}{2}}\right\} + C$$

$$= -9\sqrt{9-x^2} + \frac{1}{3}\left(\sqrt{9-x^2}\right)^3 + C$$

$$= \frac{1}{3}\left(\sqrt{9-x^2}\right)^3 - 9\sqrt{9-x^2} + C$$

4) via trig. substitution

$$\int \frac{x^3}{(9-4x^2)^{\frac{3}{2}}}\,dx = \int \frac{x^3}{\left(\sqrt{(3)^2-(2x)^2}\right)^3}\,dx = \int \frac{\left(\frac{3}{2}\sin\theta\right)^3}{(3\cos\theta)^3}\left(\frac{3}{2}\cos\theta\, d\theta\right)$$



$\Rightarrow$ $\frac{2x}{3} = \sin\theta \rightarrow x = \frac{3}{2}\sin\theta$

$dx = \frac{3}{2}\cos\theta\, d\theta$

$\frac{\sqrt{(3)^2-(2x)^2}}{3} = \cos\theta \rightarrow \sqrt{(3)^2-(2x)^2} = 3\cos\theta$

$$= \int \frac{\left(\frac{3}{2}\right)^4 \sin^3\theta}{(3)^3\cos^2\theta}\,d\theta$$

$$= \frac{3}{(2)^4}\int \frac{\sin^2\theta}{\cos^2\theta}\left(\sin\theta\, d\theta\right)$$

$$= \frac{3}{(2)^4}\int \frac{(1-\cos^2\theta)}{\cos^2\theta}\left(\sin\theta\, d\theta\right) = \frac{3}{(2)^4}\int\left(\frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}\right)\left(\sin\theta\, d\theta\right)$$

$\varphi = \cos\theta$

$d\varphi = -\sin\theta\, d\theta$

$-1\, d\varphi = \sin\theta\, d\theta$

$$= \frac{3}{(2)^4}\int\left(\frac{1}{\cos^2\theta} - 1\right)\left(\sin\theta\, d\theta\right) = \frac{3}{(2)^4}\int\left(\frac{1}{\varphi^2} - 1\right)\left(-1\, d\varphi\right)$$

$$= \frac{-3}{(2)^4}\int\left(\varphi^{-2} - 1\right)d\varphi = \frac{-3}{(2)^4}\left\{\left[\frac{\varphi^{-1}}{-1}\right] - \left[\varphi\right]\right\} + C$$

$$= \frac{-3}{(2)^4}\left\{\frac{-1}{\cos\theta} - \cos\theta\right\} + C = \frac{-3}{(2)^4}\left\{\frac{-1}{\left(\frac{\sqrt{(3)^2-(2x)^2}}{3}\right)} - \left(\frac{\sqrt{(3)^2-(2x)^2}}{3}\right)\right\} + C$$

$$= \frac{9}{16\sqrt{9-4x^2}} + \frac{\sqrt{9-4x^2}}{16} + C$$

via triple substitution

$$\int \frac{x^3}{(9-4x^2)^{\frac{3}{2}}}\,dx = \int \frac{x^2}{(9-4x^2)^{\frac{3}{2}}}\left(x\,dx\right) = \int \frac{\left(\frac{9-\varphi}{4}\right)}{\varphi^{\frac{3}{2}}}\left(\frac{-1}{8}\,d\varphi\right) = \frac{-1}{32}\int\left(\frac{9-\varphi}{\varphi^{\frac{3}{2}}}\right)d\varphi$$

$\varphi = 9-4x^2 \rightarrow x^2 = \frac{9-\varphi}{4}$

$d\varphi = -8x\,dx$

$\frac{-1}{8}\,d\varphi = x\,dx$

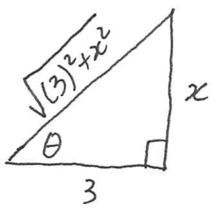
$$= \frac{-1}{32}\int\left(\frac{9}{\varphi^{\frac{3}{2}}} - \frac{\varphi}{\varphi^{\frac{3}{2}}}\right)d\varphi = \frac{-1}{32}\int\left(9\varphi^{-\frac{3}{2}} - \varphi^{-\frac{1}{2}}\right)d\varphi$$

$$= \frac{-1}{32}\left\{9\left[\frac{\varphi^{-\frac{1}{2}}}{-\frac{1}{2}}\right] - \left[\frac{\varphi^{\frac{1}{2}}}{\frac{1}{2}}\right]\right\} + C = \frac{-1}{32}\left\{\frac{-18}{(9-4x^2)^{\frac{1}{2}}} - 2\left(9-4x^2\right)^{\frac{1}{2}}\right\} + C$$

$$= \frac{9}{16\sqrt{9-4x^2}} + \frac{\sqrt{9-4x^2}}{16} + C$$

6) via trig. substitution

$$\int \frac{x^3}{\sqrt{9+x^2}}\,dx = \int \frac{x^3}{\sqrt{(3)^2+x^2}}\,dx = \int \frac{(3\tan\theta)^3}{(3\sec\theta)}\,(3\sec^2\theta\,d\theta)$$



$\Rightarrow$

$\frac{x}{3} = \tan\theta \to x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$\frac{\sqrt{(3)^2+x^2}}{3} = \sec\theta \to \sqrt{(3)^2+x^2} = 3\sec\theta$

$$= (3)^3 \int \tan^3\theta\,\sec\theta\,d\theta$$

$$= (3)^3 \int \tan^2\theta\,(\sec\theta\tan\theta\,d\theta)$$

$$= (3)^3 \int (\sec^2\theta - 1)(\sec\theta\tan\theta\,d\theta) = (3)^3 \int (p^2-1)\,dp$$

$p = \sec\theta$

$dp = \sec\theta\tan\theta\,d\theta$

$$= (3)^3 \left\{ \left[\frac{p^3}{3}\right] - \left[p\right] \right\} + C$$

$$= (3)^3 \left\{ \frac{\sec^3\theta}{3} - \sec\theta \right\} + C$$

$$= (3)^3 \left\{ \frac{\left(\frac{\sqrt{(3)^2+x^2}}{3}\right)^3}{3} - \left(\frac{\sqrt{(3)^2+x^2}}{3}\right) \right\} + C = \underline{\underline{\frac{1}{3}\left(\sqrt{9+x^2}\right)^3 - 9\sqrt{9+x^2} + C}}$$

via triple substitution

$$\int \frac{x^3}{\sqrt{9+x^2}}\,dx = \int \frac{x^2}{\sqrt{9+x^2}}\,(x\,dx) = \int \frac{(p-9)}{\sqrt{p}}\left(\frac{1}{2}\,dp\right)$$

$p = 9+x^2 \to p-9 = x^2$

$dp = 2x\,dx$

$\frac{1}{2}\,dp = x\,dx$

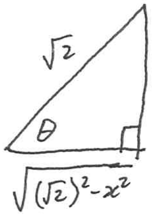
$$= \frac{1}{2}\int \left(\frac{p}{\sqrt{p}} - \frac{9}{\sqrt{p}}\right)dp = \frac{1}{2}\int \left(p^{\frac{1}{2}} - 9p^{-\frac{1}{2}}\right)dp$$

$$= \frac{1}{2}\left\{ \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] - 9\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] \right\} + C$$

$$= \frac{1}{2}\left\{ \frac{2}{3}(9+x^2)^{\frac{3}{2}} - 18(9+x^2)^{\frac{1}{2}} \right\} + C$$

$$= \underline{\underline{\frac{1}{3}\left(\sqrt{9+x^2}\right)^3 - 9\sqrt{9+x^2} + C}}$$

8) $\int \frac{\sqrt{2-x^2}}{x^2}\,dx = \int \frac{\sqrt{(\sqrt{2})^2-x^2}}{x^2}\,dx = \int \frac{(\sqrt{2}\cos\theta)}{(\sqrt{2}\sin\theta)^2}(\sqrt{2}\cos\theta\,d\theta)$



$\Rightarrow \frac{x}{\sqrt{2}} = \sin\theta \rightarrow x = \sqrt{2}\sin\theta$

$dx = \sqrt{2}\cos\theta\,d\theta$

$\frac{\sqrt{(\sqrt{2})^2-x^2}}{\sqrt{2}} = \cos\theta \rightarrow \sqrt{(\sqrt{2})^2-x^2} = \sqrt{2}\cos\theta$

$= \int \frac{(\sqrt{2})^2\cos^2\theta}{(\sqrt{2})^2\sin^2\theta}\,d\theta$

$= \int \cot^2\theta\,d\theta$

$= \int (\csc^2\theta - 1)\,d\theta = \int \csc^2\theta\,d\theta - \int 1\,d\theta = [-\cot\theta] - [\theta] + C$

use triangle above to find $\cot\theta$ and

$\frac{x}{\sqrt{2}} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) = \theta$

$= -\cot\theta - \theta + C$

$= -\left(\frac{\sqrt{(\sqrt{2})^2-x^2}}{x}\right) - \left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right) + C$

$= \frac{-\sqrt{2-x^2}}{x} - \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

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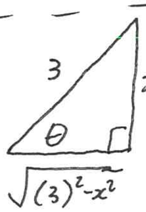
10) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \int \frac{x^2}{\sqrt{(3)^2-x^2}}\,dx = \int \frac{(3\sin\theta)^2}{(3\cos\theta)}(3\cos\theta\,d\theta)$



$\Rightarrow \frac{x}{3} = \sin\theta \rightarrow x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$\frac{\sqrt{(3)^2-x^2}}{3} = \cos\theta \rightarrow \sqrt{(3)^2-x^2} = 3\cos\theta$

$= (3)^2 \int \sin^2\theta\,d\theta$

$= (3)^2 \int \left(\frac{1-\cos(2\theta)}{2}\right) d\theta$

$= \frac{9}{2} \int (1 - \cos(\underbrace{2\theta}_{\varphi = 2\theta}))\,d\theta$

$= \frac{9}{2}\left\{[\theta] - \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C = \frac{9}{2}\left\{\theta - \frac{1}{2}(2\sin\theta\cos\theta)\right\} + C$

$\frac{x}{3} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{x}{3}\right) = \theta$

$= \frac{9}{2}\{\theta - \sin\theta\cos\theta\} + C$

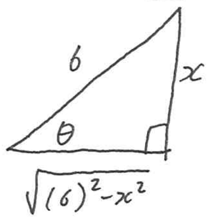
$= \frac{9}{2}\left\{\left(\sin^{-1}\left(\frac{x}{3}\right)\right) - \left(\frac{x}{3}\right)\left(\frac{\sqrt{(3)^2-x^2}}{3}\right)\right\} + C$

$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C$

12) $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$

via trig. substitution

$$\int \frac{x}{\sqrt{36-x^2}}\,dx = \int \frac{x}{\sqrt{(6)^2-x^2}}\,dx = \int \frac{(6\sin\theta)}{(6\cos\theta)}(6\cos\theta\, d\theta) = \int (6)\sin\theta\, d\theta$$



$\Rightarrow$

$\frac{x}{6} = \sin\theta \rightarrow x = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

$\frac{\sqrt{(6)^2-x^2}}{6} = \cos\theta \rightarrow \sqrt{(6)-x^2} = 6\cos\theta$

$$= 6\left[-\cos\theta\right] + C = -6\cos\theta + C$$

$$= -6\left(\frac{\sqrt{(6)^2-x^2}}{6}\right) + C = -\sqrt{36-x^2} + C$$

via substitution

$$\int \frac{x}{\sqrt{36-x^2}}\,dx = \int \frac{1}{\sqrt{36-x^2}}(x\,dx) = \int \frac{1}{\sqrt{p}}\left(\frac{-1}{2}dp\right) = \frac{-1}{2}\int p^{-\frac{1}{2}}\,dp$$

$p = 36 - x^2$

$dp = -2x\,dx$

$\frac{-1}{2}dp = x\,dx$

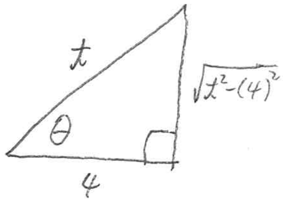
$$= \frac{-1}{2}\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = -\sqrt{36-x^2} + C$$

$$\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx = \left[-\sqrt{36-x^2} + C\right]_0^3$$

$$= \left[-\sqrt{36-(3)^2} + C\right] - \left[\sqrt{36-(0)^2} + C\right]$$

$$= \left[-\sqrt{36-9}\right] - \left[-\sqrt{36}\right] = \left[-\sqrt{27}\right] - \left[-6\right] = \underline{\underline{6-\sqrt{27}}}$$

14) $\displaystyle\int \frac{dt}{t^2\sqrt{t^2-16}} = \int \frac{dt}{t^2\sqrt{t^2-(4)^2}} = \int \frac{(4\sec\theta\tan\theta\, d\theta)}{(4\sec\theta)^2(4\tan\theta)}$



$\Rightarrow$ $\frac{t}{4} = \sec\theta \rightarrow t = 4\sec\theta$

$dt = 4\sec\theta\tan\theta\, d\theta$

$\frac{\sqrt{t^2-(4)^2}}{4} = \tan\theta \rightarrow \sqrt{t^2-(4)^2} = 4\tan\theta$

$\displaystyle = \frac{1}{(4)^2}\int \frac{1}{\sec\theta}\, d\theta$

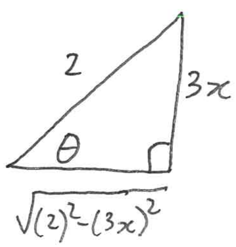
$\displaystyle = \frac{1}{16}\int \cos\theta\, d\theta$

$\displaystyle = \frac{1}{16}\left[\sin\theta\right] + C = \frac{1}{16}\left(\frac{\sqrt{t^2-(4)^2}}{t}\right) + C = \frac{\sqrt{t^2-16}}{16t} + C$

use triangle above to find $\sin\theta$

16) $\displaystyle\int_0^{\frac{2}{3}} \sqrt{4-9x^2}\, dx$

$\displaystyle\int \sqrt{4-9x^2}\, dx = \int \sqrt{(2)^2-(3x)^2}\, dx = \int (2\cos\theta)\left(\frac{2}{3}\cos\theta\, d\theta\right)$



$\Rightarrow$ $\frac{3x}{2} = \sin\theta \rightarrow x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\, d\theta$

$\frac{\sqrt{(2)^2-(3x)^2}}{2} = \cos\theta \rightarrow \sqrt{(2)^2-(3x)^2} = 2\cos\theta$

$\displaystyle = \frac{4}{3}\int \cos^2\theta\, d\theta$

$\displaystyle = \frac{4}{3}\int \left(\frac{1+\cos(2\theta)}{2}\right) d\theta$

$\displaystyle = \frac{2}{3}\int \left(1+\cos(\underset{u=2\theta}{2\theta})\right) d\theta = \frac{2}{3}\left\{\left[\theta\right] + \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C$

$\displaystyle = \frac{2}{3}\left\{\theta + \sin\theta\cos\theta\right\} + C = \frac{2}{3}\left\{\left(\sin^{-1}\left(\frac{3x}{2}\right)\right) + \left(\frac{3x}{2}\right)\left(\frac{\sqrt{(2)^2-(3x)^2}}{2}\right)\right\} + C$

$\frac{3x}{2} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{3x}{2}\right) = \theta$

$\displaystyle = \frac{2}{3}\sin^{-1}\left(\frac{3x}{2}\right) + \frac{x\sqrt{4-9x^2}}{2} + C$

16) continued...

$$\int_0^{\frac{2}{3}} \sqrt{4-9x^2}\,dx = \left[\frac{2}{3}\sin^{-1}\left(\frac{3x}{2}\right) + \frac{x\sqrt{4-9x^2}}{2} + C\right]_0^{\frac{2}{3}}$$

$$= \left[\frac{2}{3}\sin^{-1}\left(\frac{3\left(\frac{2}{3}\right)}{2}\right) + \frac{\left(\frac{2}{3}\right)\sqrt{4-9\left(\frac{2}{3}\right)^2}}{2} + C\right] - \left[\frac{2}{3}\sin^{-1}\left(\frac{3(0)}{2}\right) + \frac{(0)\sqrt{4-9(0)^2}}{2} + C\right]$$

$$= \left[\frac{2}{3}\sin^{-1}(1) + \frac{\left(\frac{2}{3}\right)\sqrt{4-4}}{2}\right] - \left[\frac{2}{3}\sin^{-1}(0) + 0\right] = \left[\frac{2}{3}\left(\frac{\pi}{2}\right) + 0\right] - \left[\frac{2}{3}(0)\right]$$

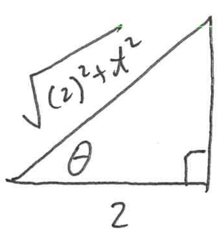
$$= \underline{\underline{\frac{\pi}{3}}}$$

---

18) $\displaystyle\int_0^2 \frac{dt}{\sqrt{4+t^2}}$

$$\int \frac{dt}{\sqrt{4+t^2}} = \int \frac{dt}{\sqrt{(2)^2+t^2}} = \int \frac{(2\sec^2\theta\, d\theta)}{(2\sec\theta)} = \int \sec\theta\, d\theta$$

$$= \left[\ln\left|\sec\theta + \tan\theta\right|\right] + C$$



$\Rightarrow$

$\frac{t}{2} = \tan\theta \rightarrow t = 2\tan\theta$

$dt = 2\sec^2\theta\, d\theta$

$\frac{\sqrt{(2)^2+t^2}}{2} = \sec\theta \rightarrow \sqrt{(2)^2+t^2} = 2\sec\theta$

$$= \ln\left|\left(\frac{\sqrt{(2)^2+t^2}}{2}\right) + \left(\frac{t}{2}\right)\right| + C$$

$$= \ln\left|\frac{t+\sqrt{4+t^2}}{2}\right| + C$$

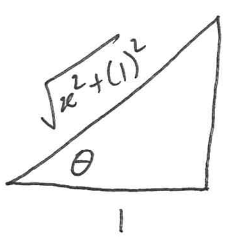
$$\int_0^2 \frac{dt}{\sqrt{4+t^2}} = \left[\ln\left|\frac{t+\sqrt{4+t^2}}{2}\right| + C\right]_0^2 = \left[\ln\left|\frac{(2)+\sqrt{4+(2)^2}}{2}\right| + C\right] - \left[\ln\left|\frac{(0)+\sqrt{4+(0)^2}}{2}\right| + C\right]$$

$$= \left[\ln\left|\frac{2+\sqrt{8}}{2}\right|\right] - \left[\ln\left|\frac{2}{2}\right|\right] = \left[\ln\left|\frac{2+2\sqrt{2}}{2}\right|\right] - \left[\ln|1|\right]$$

$$= \left[\ln\left|\frac{2(1+\sqrt{2})}{2}\right|\right] - [0] = \underline{\underline{\ln\left|1+\sqrt{2}\right| = \ln\left(1+\sqrt{2}\right)}}$$

20) $\displaystyle\int_0^1 \frac{dx}{(x^2+1)^2}$

$$\int \frac{dx}{(x^2+1)^2} = \int \frac{dx}{\left(\sqrt{x^2+(1)^2}\right)^4} = \int \frac{(\sec^2\theta\, d\theta)}{(\sec\theta)^4} = \int \frac{1}{\sec^2\theta}\, d\theta$$



$\Rightarrow$ $\frac{x}{1} = \tan\theta \rightarrow x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\frac{\sqrt{x^2+(1)^2}}{1} = \sec\theta \rightarrow \sqrt{x^2+(1)^2} = \sec\theta$

$$= \int \cos^2\theta\, d\theta$$

$$= \int \left(\frac{1+\cos(2\theta)}{2}\right) d\theta$$

$$= \frac{1}{2}\int \left(1 + \cos\underset{\varphi = 2\theta}{\underbrace{(2\theta)}}\right) d\theta = \frac{1}{2}\left\{[\theta] + \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C$$

$$= \frac{1}{2}\{\theta + \sin\theta\cos\theta\} + C = \frac{1}{2}\left\{\left(\tan^{-1}\left(\frac{x}{1}\right)\right) + \left(\frac{x}{\sqrt{x^2+(1)^2}}\right)\left(\frac{1}{\sqrt{x^2+(1)^2}}\right)\right\} + C$$

use triangle above to find $\sin\theta$ and $\cos\theta$

$\frac{x}{1} = \tan\theta \Rightarrow \tan^{-1}\left(\frac{x}{1}\right) = \theta$

$$= \frac{1}{2}\tan^{-1}(x) + \frac{x}{2(x^2+1)} + C$$

$$\int_0^1 \frac{dx}{(x^2+1)^2} = \left[\frac{1}{2}\tan^{-1}(x) + \frac{x}{2(x^2+1)} + C\right]_0^1$$

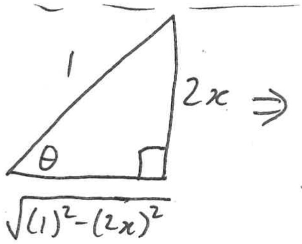
$$= \left[\frac{1}{2}\tan^{-1}(1) + \frac{(1)}{2((1)^2+1)} + C\right] - \left[\frac{1}{2}\tan^{-1}(0) + \frac{(0)}{2((0)^2+1)} + C\right]$$

$$= \left[\frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{4}\right] - \left[\frac{1}{2}(0) + 0\right] = \frac{\pi}{8} + \frac{1}{4} = \underline{\underline{\frac{\pi+2}{8}}}$$

22) $\int_{\frac{1}{4}}^{\frac{\sqrt{3}}{4}} \sqrt{1-4x^2}\,dx$

7.3/9

$$\int \sqrt{1-4x^2}\,dx = \int \sqrt{(1)^2-(2x)^2}\,dx = \int (\cos\theta)\left(\tfrac{1}{2}\cos\theta\,d\theta\right)$$



$\Rightarrow$ $\frac{2x}{1} = \sin\theta \rightarrow x = \frac{1}{2}\sin\theta$

$dx = \frac{1}{2}\cos\theta\,d\theta$

$\frac{\sqrt{(1)^2-(2x)^2}}{1} = \cos\theta \rightarrow \sqrt{(1)^2-(2x)^2} = \cos\theta$

$$= \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int \left(\frac{1+\cos(2\theta)}{2}\right)d\theta$$

$$= \frac{1}{4}\int \left(1+\cos(2\theta)\right)d\theta = \frac{1}{4}\left\{[\theta] + \left[\tfrac{1}{2}\sin(2\theta)\right]\right\} + C$$

$p = 2\theta$

$$= \frac{1}{4}\{\theta + \sin\theta\cos\theta\} + C = \frac{1}{4}\left\{\left(\sin^{-1}\left(\frac{2x}{1}\right)\right) + \left(\frac{2x}{1}\right)\left(\frac{\sqrt{(1)^2-(2x)^2}}{1}\right)\right\} + C$$

$\frac{2x}{1} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{2x}{1}\right) = \theta$

$$= \frac{1}{4}\sin^{-1}(2x) + \frac{x\sqrt{1-4x^2}}{2} + C$$

$$\int_{\frac{1}{4}}^{\frac{\sqrt{3}}{4}} \sqrt{1-4x^2}\,dx = \left[\frac{1}{4}\sin^{-1}(2x) + \frac{x\sqrt{1-4x^2}}{2} + C\right]_{\frac{1}{4}}^{\frac{\sqrt{3}}{4}}$$

$$= \left[\frac{1}{4}\sin^{-1}\left(2\left(\frac{\sqrt{3}}{4}\right)\right) + \frac{\left(\frac{\sqrt{3}}{4}\right)\sqrt{1-4\left(\frac{\sqrt{3}}{4}\right)^2}}{2} + C\right] - \left[\frac{1}{4}\sin^{-1}\left(2\left(\frac{1}{4}\right)\right) + \frac{\left(\frac{1}{4}\right)\sqrt{1-4\left(\frac{1}{4}\right)^2}}{2} + C\right]$$

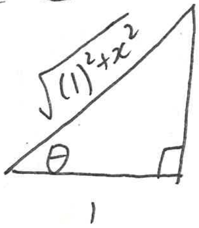
$$= \left[\frac{1}{4}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{(\sqrt{3})}{8}\sqrt{1-\frac{3}{4}}\right] - \left[\frac{1}{4}\sin^{-1}\left(\frac{1}{2}\right) + \frac{1}{8}\sqrt{1-\frac{1}{4}}\right]$$

$$= \left[\frac{1}{4}\left(\frac{\pi}{3}\right) + \left(\frac{\sqrt{3}}{8}\right)\sqrt{\frac{1}{4}}\right] - \left[\frac{1}{4}\left(\frac{\pi}{6}\right) + \left(\frac{1}{8}\right)\sqrt{\frac{3}{4}}\right] = \left[\frac{\pi}{12} + \left(\frac{\sqrt{3}}{8}\right)\left(\frac{\sqrt{1}}{\sqrt{4}}\right)\right] - \left[\frac{\pi}{24} + \left(\frac{1}{8}\right)\left(\frac{\sqrt{3}}{\sqrt{4}}\right)\right]$$

$$= \left[\frac{\pi}{12} + \left(\frac{\sqrt{3}}{8}\right)\left(\frac{1}{2}\right)\right] - \left[\frac{\pi}{24} + \left(\frac{1}{8}\right)\left(\frac{\sqrt{3}}{2}\right)\right] = \left[\frac{\pi}{12} + \frac{\sqrt{3}}{16}\right] - \left[\frac{\pi}{24} + \frac{\sqrt{3}}{16}\right] = \underline{\underline{\frac{\pi}{24}}}$$

24) via trig. substitution

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{x}{\sqrt{(1)^2+x^2}}\,dx = \int \frac{(\tan\theta)}{(\sec\theta)}(\sec^2\theta\,d\theta)$$



$\Rightarrow$ $\frac{x}{1} = \tan\theta \rightarrow x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{(1)^2+x^2}}{1} = \sec\theta \rightarrow \sqrt{(1)^2+x^2} = \sec\theta$

$$= \int \tan\theta\sec\theta\,d\theta$$

$$= \int \sec\theta\tan\theta\,d\theta$$

$$= [\sec\theta] + C = \left(\frac{\sqrt{(1)^2+x^2}}{1}\right) + C = \underline{\underline{\sqrt{1+x^2}+C}}$$

via substitution

$$\int \frac{x}{\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{1+x^2}}(x\,dx) = \int \frac{1}{\sqrt{p}}\left(\frac{1}{2}\,dp\right)$$

$p = 1+x^2$

$dp = 2x\,dx$

$\frac{1}{2}dp = x\,dx$

$$= \frac{1}{2}\int p^{-\frac{1}{2}}\,dp = \frac{1}{2}\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$$

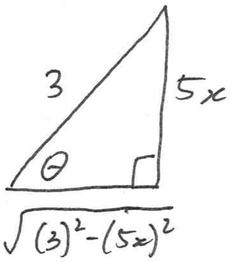
$$= \sqrt{p} + C = \underline{\underline{\sqrt{1+x^2}+C}}$$

26) $\displaystyle\int_0^{0.3} \frac{x}{(9-25x^2)^{\frac{3}{2}}}\,dx$

via trig. substitution

$$\int \frac{x}{(9-25x^2)^{\frac{3}{2}}}\,dx = \int \frac{x}{\left(\sqrt{(3)^2-(5x)^2}\right)^3}\,dx = \int \frac{\left(\frac{3}{5}\sin\theta\right)}{(3\cos\theta)^3}\left(\frac{3}{5}\cos\theta\,d\theta\right)$$

26) continued...



$\Rightarrow$

$$\frac{5x}{3} = \sin\theta \rightarrow x = \frac{3}{5}\sin\theta$$

$$dx = \frac{3}{5}\cos\theta\, d\theta$$

$$\frac{\sqrt{(3)^2-(5x)^2}}{3} = \cos\theta \rightarrow \sqrt{(3)^2-(5x)^2} = 3\cos\theta$$

$$= \frac{1}{3(5)^2}\int \frac{\sin\theta}{\cos^2\theta}\, d\theta$$

$$= \frac{1}{75}\int \frac{1}{\cos^2\theta}(\sin\theta\, d\theta)$$

$p = \cos\theta$

$dp = -\sin\theta\, d\theta$

$-1\, dp = \sin\theta\, d\theta$

$$= \frac{1}{75}\int \frac{1}{p^2}(-1\, dp) = \frac{-1}{75}\int p^{-2}\, dp = \frac{-1}{75}\left[\frac{p^{-1}}{-1}\right] + C$$

$$= \frac{1}{75\cos\theta} + C = \frac{1}{75\left(\frac{\sqrt{(3)^2-(5x)^2}}{3}\right)} + C = \frac{1}{25\sqrt{9-25x^2}} + C$$

via substitution

$$\int \frac{x}{(9-25x^2)^{\frac{3}{2}}}\, dx = \int \frac{1}{(9-25x^2)^{\frac{3}{2}}}(x\, dx) = \int \frac{1}{p^{\frac{3}{2}}}\left(\frac{-1}{50}\, dp\right)$$

$p = 9-25x^2$

$dp = -50x\, dx$

$\frac{-1}{50}\, dp = x\, dx$

$$= \frac{-1}{50}\int p^{\frac{-3}{2}}\, dp = \frac{-1}{50}\left[\frac{p^{\frac{-1}{2}}}{\frac{-1}{2}}\right] + C$$

$$= \frac{1}{25(9-25x^2)^{\frac{1}{2}}} + C = \frac{1}{25\sqrt{9-25x^2}} + C$$

$$\int_0^{0.3} \frac{x}{(9-25x^2)^{\frac{3}{2}}}\, dx = \left[\frac{1}{25\sqrt{9-25x^2}} + C\right]_0^{0.3} = \left[\frac{1}{25\sqrt{9-25(0.3)^2}} + C\right] - \left[\frac{1}{25\sqrt{9-25(0)^2}} + C\right]$$

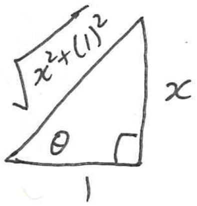
$$= \left[\frac{1}{25\sqrt{9-25(0.09)}}\right] - \left[\frac{1}{25\sqrt{9-25(0)}}\right] = \left[\frac{1}{25\sqrt{9-25\left(\frac{9}{100}\right)}}\right] - \left[\frac{1}{25\sqrt{9}}\right]$$

$$= \left[\frac{1}{25\sqrt{9-\frac{9}{4}}}\right] - \left[\frac{1}{25(3)}\right] = \left[\frac{1}{25\sqrt{\frac{36}{4}-\frac{9}{4}}}\right] - \left[\frac{1}{75}\right] = \left[\frac{1}{25\sqrt{\frac{27}{4}}}\right] - \left[\frac{1}{75}\right]$$

$$= \left[\frac{1}{25\left(\frac{\sqrt{27}}{2}\right)}\right] - \left[\frac{1}{75}\right] = \left[\frac{2}{25(3\sqrt{3})}\right] - \left[\frac{1}{75}\right] = \frac{2}{75\sqrt{3}} - \frac{1}{75} = \underline{\frac{2-\sqrt{3}}{75\sqrt{3}}}$$

28) $\int_0^1 \sqrt{x^2+1}\,dx$

$$\int \sqrt{x^2+1}\,dx = \int \sqrt{x^2+(1)^2}\,dx = \int (\sec\theta)(\sec^2\theta\,d\theta)$$



$\Rightarrow$ $\frac{x}{1} = \tan\theta \rightarrow x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{x^2+(1)^2}}{1} = \sec\theta \rightarrow \sqrt{x^2+(1)^2} = \sec\theta$

$$= \int \sec^3\theta\,d\theta$$

formula from Integration Notes

$$= \boxed{\frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right)} + C$$

$$= \frac{1}{2}\left(\left(\frac{\sqrt{x^2+(1)^2}}{1}\right)\left(\frac{x}{1}\right) + \ln\left|\left(\frac{\sqrt{x^2+(1)^2}}{1}\right) + \left(\frac{x}{1}\right)\right|\right) + C$$

$$= \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln\left|x + \sqrt{x^2+1}\right| + C$$

$$\int_0^1 \sqrt{x^2+1}\,dx = \left[\frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln\left|x+\sqrt{x^2+1}\right| + C\right]_0^1$$

$$= \left[\frac{1}{2}(1)\sqrt{(1)^2+1} + \frac{1}{2}\ln\left|(1)+\sqrt{(1)^2+1}\right| + C\right] - \left[\frac{1}{2}(0)\sqrt{(0)^2+1} + \frac{1}{2}\ln\left|(0)+\sqrt{(0)^2+1}\right| + C\right]$$

$$= \left[\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln|1+\sqrt{2}|\right] - \left[0 - \ln|0+\sqrt{1}|\right] = \left[\frac{\sqrt{2}}{2} + \frac{1}{2}\ln|1+\sqrt{2}\right] - [0]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{2}\ln|1+\sqrt{2}|$$

30) $\int_0^1 \sqrt{x-x^2}\,dx$

we must use completing square so inside the radical will be sum or difference of squares.

30) continued... (part 1)

$$x - x^2 = -(-x + x^2) = -\left(\frac{-1}{4} + \frac{1}{4} - x + x^2\right)$$

b_x

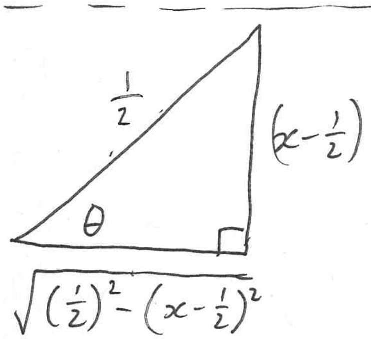
$$= -\left(\frac{-1}{4}\right) - \left(\frac{1}{4} - x + x^2\right)$$

$$\frac{b_x}{2} = \frac{-1}{2}$$

$$\left(\frac{b_x}{2}\right)^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$$

$$= \frac{1}{4} - \left(\frac{-1}{2} + x\right)^2 = \left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2$$

$$\int \sqrt{x - x^2}\, dx = \int \sqrt{\left(\tfrac{1}{2}\right)^2 - \left(x - \tfrac{1}{2}\right)^2}\, dx = \int \left(\tfrac{1}{2}\cos\theta\right)\left(\tfrac{1}{2}\cos\theta\, d\theta\right)$$



$$\Rightarrow \quad \frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}} = \sin\theta \longrightarrow x = \frac{1}{2} + \frac{1}{2}\sin\theta$$

$$dx = \frac{1}{2}\cos\theta\, d\theta$$

$$\frac{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}{\frac{1}{2}} = \cos\theta \rightarrow \sqrt{\left(\tfrac{1}{2}\right)^2 - \left(x - \tfrac{1}{2}\right)^2} = \frac{1}{2}\cos\theta$$

$$= \frac{1}{4}\int \cos^2\theta\, d\theta = \frac{1}{4}\int \left(\frac{1 + \cos(2\theta)}{2}\right) d\theta$$

$$= \frac{1}{8}\int \left(1 + \cos(2\theta)\right) d\theta = \frac{1}{8}\left\{\left[\theta\right] + \left[\frac{1}{2}\sin(2\theta)\right]\right\} + C$$

$\phi = 2\theta$

$$= \frac{1}{8}\left\{\theta + \sin\theta\cos\theta\right\} + C = \frac{1}{8}\left\{\left(\sin^{-1}\left(\frac{2x-1}{2}\right)\right) + \left(\frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}}\right)\left(\frac{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}{\frac{1}{2}}\right)\right\}$$

$$\frac{\left(x - \frac{1}{2}\right)}{\frac{1}{2}} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right) = \theta$$

$$= \frac{1}{8}\left\{\sin^{-1}\left(\frac{2x-1}{2}\right) + \left(\frac{2x-1}{2}\right)\left(\frac{\sqrt{x - x^2}}{\frac{1}{2}}\right)\right\} + C$$

$$\theta = \sin^{-1}\left(\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right)\left(\frac{\frac{2}{1}}{\frac{2}{1}}\right)\right) = \sin^{-1}\left(\frac{2x-1}{2}\right)$$

$$= \frac{1}{8}\left\{\sin^{-1}\left(\frac{2x-1}{2}\right) + (2x-1)\sqrt{x - x^2}\right\} + C$$

$$= \frac{1}{8}\sin^{-1}\left(\frac{2x-1}{2}\right) + \frac{(2x-1)\sqrt{x - x^2}}{8} + C$$

30) continued... (part 2)

$$\int_0^1 \sqrt{x-x^2}\,dx = \left[\frac{1}{8}\sin^{-1}\left(\frac{2x-1}{2}\right) + \frac{(2x-1)\sqrt{x-x^2}}{8} + C\right]_0^1$$

$$= \left[\frac{1}{8}\sin^{-1}\left(\frac{2(1)-1}{2}\right) + \frac{(2(1)-1)\sqrt{(1)-(1)^2}}{8} + C\right] - \left[\frac{1}{8}\sin^{-1}\left(\frac{2(0)-1}{2}\right) + \frac{(2(0)-1)\sqrt{(0)-(0)^2}}{8} + C\right]$$

$$= \left[\frac{1}{8}\sin^{-1}\left(\frac{1}{2}\right) + 0\right] - \left[\frac{1}{8}\sin^{-1}\left(\frac{-1}{2}\right) + 0\right] = \left[\frac{1}{8}\left(\frac{\pi}{6}\right)\right] - \left[\frac{1}{8}\left(\frac{-\pi}{6}\right)\right] = \underline{\underline{\frac{\pi}{24}}}$$

---

32) completing square needed

$$3+4x-4x^2 = 3-4(\underset{\uparrow\, b_x}{-x}+x^2) = 3-4\left(\frac{-1}{4}+\frac{1}{4}-x+x^2\right)$$

$$\frac{b_x}{2} = \frac{-1}{2}$$

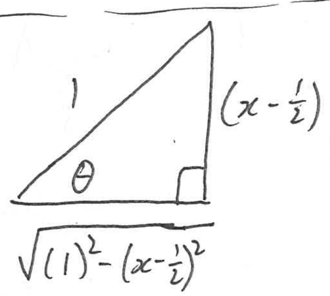
$$\left(\frac{b_x}{2}\right)^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$$

$$= 3-4\left(\frac{-1}{4}\right) - 4\left(\frac{1}{4}-x+x^2\right)$$

$$= 3+1-4\left(x^2-x+\frac{1}{4}\right)$$

$$= 4-4\left(x-\frac{1}{2}\right)^2$$

$$\int \frac{x^2}{(3+4x-4x^2)^{3/2}}\,dx = \int \frac{x^2}{\left(\sqrt{4-4(x-\frac{1}{2})^2}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{4(1-(x-\frac{1}{2})^2)}\right)^3}\,dx$$



$\Rightarrow$

$$\frac{(x-\frac{1}{2})}{1} = \sin\theta \rightarrow x = \frac{1}{2}+\sin\theta$$

$$dx = \cos\theta\,d\theta$$

$$\frac{\sqrt{(1)^2-(x-\frac{1}{2})^2}}{1} = \cos\theta \rightarrow \sqrt{(1)^2-(x-\frac{1}{2})^2} = \cos\theta$$

$$= \int \frac{x^2}{\left(2\sqrt{1-(x-\frac{1}{2})^2}\right)^3}\,dx$$

$$= \frac{1}{8}\int \frac{x^2}{\left(\sqrt{(1)^2-(x-\frac{1}{2})^2}\right)^3}\,dx$$

$$= \frac{1}{8}\int \frac{(\frac{1}{2}+\sin\theta)^2}{(\cos\theta)^3}(\cos\theta\,d\theta) = \frac{1}{8}\int \frac{\frac{1}{4}+\sin\theta+\sin^2\theta}{\cos^2\theta}\,d\theta$$

32) continued...

$$= \frac{1}{8}\int \frac{\frac{1}{4}}{\cos^2\theta}\,d\theta + \frac{1}{8}\int \frac{\sin\theta}{\cos^2\theta}\,d\theta + \frac{1}{8}\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{32}\int \sec^2\theta\,d\theta + \frac{1}{8}\int \frac{1}{\cos^2\theta}(\sin\theta\,d\theta) + \frac{1}{8}\int \tan^2\theta\,d\theta$$

$$\int \frac{1}{\cos^2\theta}(\sin\theta\,d\theta) = \int \frac{1}{p^2}(-1\,dp) = -1\int p^{-2}\,dp = -1\left[\frac{p^{-1}}{-1}\right] + C_*$$

$p = \cos\theta$

$dp = -\sin\theta\,d\theta$

$-1\,dp = \sin\theta\,d\theta$

$$= \underbrace{\frac{1}{\cos\theta}} + C_*$$

$$\int \tan^2\theta\,d\theta = \int (\sec^2\theta - 1)\,d\theta = \int \sec^2\theta\,d\theta - \int 1\,d\theta$$

$$= [\tan\theta] - [\theta] + C_* = \underbrace{\tan\theta - \theta} + C_*$$

$$= \frac{1}{32}\left[\tan\theta\right] + \frac{1}{8}\left[\frac{1}{\cos\theta}\right] + \frac{1}{8}\left[\tan\theta - \theta\right] + C$$

$$= \frac{1}{32}\tan\theta + \frac{1}{8}\sec\theta + \frac{1}{8}\tan\theta + \frac{1}{8}\theta + C$$

$$= \frac{1}{32}\tan\theta + \frac{1}{8}\sec\theta + \frac{4}{32}\tan\theta + \frac{1}{8}\theta + C$$

$$= \frac{5}{32}\tan\theta + \frac{1}{8}\sec\theta + \frac{1}{8}\theta + C$$

use triangle to obtain $\tan\theta$ and $\sec\theta$

$\frac{(x-\frac{1}{2})}{1} = \sin\theta \Rightarrow \sin^{-1}\left(\frac{x-\frac{1}{2}}{1}\right) = \theta$

$\theta = \sin^{-1}(x - \frac{1}{2})$

$$= \frac{5}{32}\left(\frac{(x-\frac{1}{2})}{\sqrt{(1)^2 - (x-\frac{1}{2})^2}}\right) + \frac{1}{8}\left(\frac{1}{\sqrt{(1)^2-(x-\frac{1}{2})^2}}\right) + \frac{1}{8}\left(\sin^{-1}\left(x-\frac{1}{2}\right)\right) + C$$

34) completing square needed

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$$x^2-2x+2 = x^2-2x+1-1+2$$

$\uparrow$ b_x

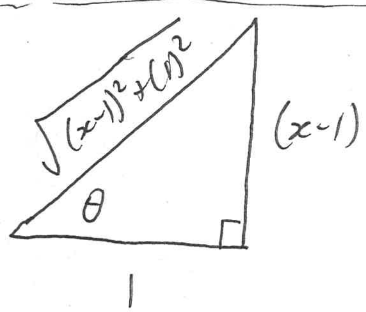
$$= x^2-2x+1 + (-1+2)$$

$\frac{b_x}{2} = \frac{-2}{2} = -1$

$$= (x-1)^2+1$$

$\left(\frac{b_x}{2}\right)^2 = (-1)^2 = 1$

$$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx = \int \frac{x^2+1}{((x-1)^2+1)^2}\,dx = \int \frac{x^2+1}{\left(\sqrt{(x-1)^2+(1)^2}\right)^4}\,dx$$



$\Rightarrow$

$\frac{(x-1)}{1} = \tan\theta \rightarrow x = 1+\tan\theta$

$dx = \sec^2\theta\,d\theta$

$\frac{\sqrt{(x-1)^2+(1)^2}}{1} = \sec\theta \rightarrow \sqrt{(x-1)^2+(1)^2} = \sec\theta$

$$= \int \frac{(1+\tan\theta)^2+1}{(\sec\theta)^4}(\sec^2\theta\,d\theta$$

$$= \int \frac{(1+2\tan\theta+\tan^2\theta)+1}{\sec^2\theta}\,d\theta = \int \frac{\tan^2\theta+2\tan\theta+2}{\sec^2\theta}\,d\theta$$

$$= \int \left(\frac{\tan^2\theta}{\sec^2\theta} + \frac{2\tan\theta}{\sec^2\theta} + \frac{2}{\sec^2\theta}\right)d\theta = \int (\sin^2\theta + 2\sin\theta\cos\theta + 2\cos^2\theta)\,d\theta$$

$$= \int (\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \cos^2\theta)\,d\theta$$

$$= \int (\underbrace{\sin^2\theta+\cos^2\theta}_{\text{identity}} + \underbrace{2\sin\theta\cos\theta}_{\text{double angle}} + \cos^2\theta)\,d\theta =$$

$$= \int \left(1 + \sin(2\theta) + \left(\frac{1+\cos(2\theta)}{2}\right)\right)d\theta$$

$$= \int \left(\frac{3}{2} + \sin(2\theta) + \frac{1}{2}\cos(2\theta)\right)d\theta$$

$\downarrow$ $p = 2\theta$ $\qquad$ $\downarrow$ $p = 2\theta$

34) continued...

$$= \frac{3}{2}\left[\theta\right] + \left[\frac{-1}{2}\cos(2\theta)\right] + \frac{1}{2}\left[\frac{1}{2}\sin(2\theta)\right] + C$$

$$= \frac{3}{2}\theta - \frac{1}{2}(\cos^2\theta - \sin^2\theta) + \frac{1}{2}\sin\theta\cos\theta + C$$

use triangle above to find $\sin\theta$ and $\cos\theta$

$$\frac{(x-1)}{1} = \tan\theta \Rightarrow \tan^{-1}\left(\frac{x-1}{1}\right) = \theta \to \theta = \tan^{-1}(x-1)$$

$$= \frac{3}{2}\left(\tan^{-1}(x-1)\right) - \frac{1}{2}\left(\left(\frac{\sqrt{(x-1)^2+(1)^2}}{1}\right)^2 - \left(\frac{x-1}{1}\right)^2\right) + \frac{1}{2}\left(\left(\frac{x-1}{1}\right)\left(\frac{\sqrt{(x-1)^2+(1)^2}}{1}\right)\right) + C$$

$$= \frac{3}{2}\tan^{-1}(x-1) - \frac{1}{2}\left(\left(\sqrt{x^2-2x+2}\right)^2 - (x^2-2x+1)\right) + \frac{1}{2}(x-1)\sqrt{x^2-2x+2} + C$$

$$= \frac{3}{2}\tan^{-1}(x-1) - \frac{1}{2}\left((x^2-2x+2) - (x^2-2x+1)\right) + \frac{1}{2}(x-1)\sqrt{x^2-2x+2} + C$$

$$= \frac{3}{2}\tan^{-1}(x-1) - \frac{1}{2}(4x-1) + \frac{1}{2}(x-1)\sqrt{x^2-2x+2} + C$$

---

36) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt$

$$\int \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt = \int \frac{1}{\sqrt{1+\sin^2 t}}(\cos t\,dt) = \int \frac{1}{\sqrt{1+p^2}}(dp)$$

$p = \sin t$

$dp = \cos t\,dt$

$$= \int \frac{1}{\sqrt{(1)^2+p^2}}\,dp = \int \frac{1}{(\sec\theta)}(\sec^2\theta\,d\theta)$$

36) continued...

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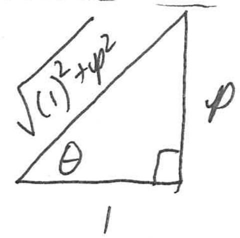

$\Rightarrow$ $\frac{\varphi}{1} = \tan\theta \rightarrow \varphi = \tan\theta$

$d\varphi = \sec^2\theta \, d\theta$

$\frac{\sqrt{(1)^2+\varphi^2}}{1} = \sec\theta \rightarrow \sqrt{(1)^2+\varphi^2} = \sec\theta$

$$= \int \sec\theta \, d\theta$$

$$= \left[\ln\left|\sec\theta + \tan\theta\right|\right] + C$$

$$= \ln\left|\left(\frac{\sqrt{(1)^2+\varphi^2}}{1}\right) + \left(\frac{\varphi}{1}\right)\right| + C = \ln\left|\varphi + \sqrt{1+\varphi^2}\right| + C$$

$$= \ln\left|\sin t + \sqrt{1+\sin^2 t}\right| + C$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos t}{\sqrt{1+\sin^2 t}} \, dt = \left[\ln\left|\sin t + \sqrt{1+\sin^2 t}\right| + C\right]_0^{\frac{\pi}{2}}$$

$$= \left[\ln\left|\sin\left(\tfrac{\pi}{2}\right) + \sqrt{1+\sin^2\left(\tfrac{\pi}{2}\right)}\right| + C\right] - \left[\ln\left|\sin(0) + \sqrt{1+\sin^2(0)}\right| + C\right]$$

$$= \left[\ln\left|(1) + \sqrt{1+(1)^2}\right|\right] - \left[\ln\left|(0) + \sqrt{1+(0)^2}\right|\right]$$

$$= \left[\ln\left|1+\sqrt{2}\right|\right] - \left[\ln|1|\right]$$

$$= \left[\ln\left(1+\sqrt{2}\right)\right] - [0]$$

$$= \ln\left(1+\sqrt{2}\right)$$