$$\int u\,dv = uv - \int v\,du$$

---

2) $\int \sqrt{x}\ln x\,dx$

$u = \ln x \qquad dv = \sqrt{x}\,dx = x^{\frac{1}{2}}dx$

$du = \frac{1}{x}dx \qquad v = \frac{2}{3}x^{\frac{3}{2}} = \frac{2}{3}(\sqrt{x})^3$

$= \int(\ln x)(\sqrt{x}\,dx) = (\ln x)\left(\frac{2}{3}(\sqrt{x})^3\right) - \int\left(\frac{2}{3}x^{\frac{3}{2}}\right)\left(\frac{1}{x}dx\right)$

$= \frac{2}{3}(\sqrt{x})^3(\ln x) - \frac{2}{3}\int x^{\frac{1}{2}}dx$

$= \frac{2}{3}(\sqrt{x})^3(\ln x) - \frac{2}{3}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{2}{3}(\sqrt{x})^3(\ln x) - \frac{4}{9}(\sqrt{x})^3 + C$

---

4) $\int \sin^{-1}x\,dx$

$u = \sin^{-1}x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$

$= \int(\sin^{-1}x)(dx) = (\sin^{-1}x)(x) - \int(x)\left(\frac{1}{\sqrt{1-x^2}}dx\right)$

$= x\sin^{-1}x - \int\frac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x - \left[-\sqrt{1-x^2}\right] + C$

$= x\sin^{-1}x + \sqrt{1-x^2} + C$

$\int\frac{x}{\sqrt{1-x^2}}dx = \int\frac{1}{\sqrt{p}}\left(\frac{-1}{2}dp\right)$

$p = 1-x^2$

$dp = -2x\,dx$

$\frac{-1}{2}dp = x\,dx$

$= \frac{-1}{2}\int p^{-\frac{1}{2}}dp$

$= \frac{-1}{2}\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$

$= -\sqrt{1-x^2} + C$

6) $\int y e^{-y}\,dy$

$u = y \qquad dv = e^{-y}\,dy$
$du = dy \qquad v = -e^{-y}$

$= \int (y)(e^{-y}\,dy) = (y)(-e^{-y}) - \int (-e^{-y})(dy)$

$= -ye^{-y} + \int e^{-y}\,dy = -ye^{-y} + [-e^{-y}] + C$

$= -ye^{-y} - e^{-y} + C = \dfrac{-y}{e^{y}} - \dfrac{1}{e^{y}} + C = \dfrac{-y-1}{e^{y}} + C$

8) $\int (\pi - x)\cos \pi x\,dx$

$u = \pi - x \qquad dv = \cos \pi x\,dx$
$du = -1\,dx \qquad v = \frac{1}{\pi}\sin \pi x$

$= \int (\pi - x)(\cos \pi x\,dx) = (\pi - x)\left(\frac{1}{\pi}\sin \pi x\right) - \int \left(\frac{1}{\pi}\sin \pi x\right)(-1\,dx)$

$= \frac{1}{\pi}(\pi - x)\sin \pi x + \frac{1}{\pi}\int \sin \pi x\,dx$

$= \frac{1}{\pi}(\pi - x)\sin \pi x + \frac{1}{\pi}\left[\frac{-1}{\pi}\cos \pi x\right] + C$

$= \frac{1}{\pi}(\pi - x)\sin \pi x - \frac{1}{\pi^2}\cos \pi x + C$

7.1/3

10) $\int \frac{\ln x}{x^2}\,dx$

$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{-1}{x}$

$$= \int (\ln x)\left(\frac{1}{x^2}\,dx\right) = (\ln x)\left(\frac{-1}{x}\right) - \int \left(\frac{-1}{x}\right)\left(\frac{1}{x}\,dx\right)$$

$$= \frac{-1}{x}\ln x + \int \frac{1}{x^2}\,dx = \frac{-1}{x}\ln x + \left[\frac{-1}{x}\right] + C = \underline{\underline{\frac{-1}{x}\ln x - \frac{1}{x} + C}}$$

---

12) $\int t^2 \sin \beta t\, dt$

$u_1 = t^2 \qquad dv_1 = \sin \beta t\, dt$

$du_1 = 2t\,dt \qquad v_1 = \frac{-1}{\beta}\cos \beta t$

$$= \int (t^2)(\sin \beta t\, dt) = (t^2)\left(\frac{-1}{\beta}\cos \beta t\right) - \int \left(\frac{-1}{\beta}\cos \beta t\right)(2t\,dt)$$

$$= \frac{-1}{\beta}t^2\cos \beta t + \frac{2}{\beta}\int (t)(\cos \beta t\, dt)$$

$u_2 = t \qquad dv_2 = \cos \beta t\, dt$

$du_2 = dt \qquad v_2 = \frac{1}{\beta}\sin \beta t$

$$= \frac{-1}{\beta}t^2\cos \beta t + \frac{2}{\beta}\left\{(t)\left(\frac{1}{\beta}\sin \beta t\right) - \int \left(\frac{1}{\beta}\sin \beta t\right)(dt)\right\}$$

$$= \frac{-1}{\beta}t^2\cos \beta t + \frac{2}{\beta^2}t\sin \beta t - \frac{2}{\beta^2}\int \sin \beta t\, dt$$

$$= \frac{-1}{\beta}t^2\cos \beta t + \frac{2}{\beta^2}t\sin \beta t - \frac{2}{\beta^2}\left[\frac{-1}{\beta}\cos \beta t\right] + C$$

$$= \underline{\underline{\frac{-1}{\beta}t^2\cos \beta t + \frac{2}{\beta^2}t\sin \beta t + \frac{2}{\beta^3}\cos \beta t + C}}$$

14) $\int \ln\sqrt{x}\,dx$

$u = \ln\sqrt{x} = \frac{1}{2}\ln x$ $\quad dv = dx$

$du = \frac{1}{2x}dx$ $\quad v = x$

7.1/4

$= \int(\ln\sqrt{x})(dx) = (\ln\sqrt{x})(x) - \int(x)\left(\frac{1}{2x}dx\right)$

$= x\ln\sqrt{x} - \frac{1}{2}\int dx = x\ln\sqrt{x} - \frac{1}{2}[x] + C = x\ln x - \frac{1}{2}x + C$

16) $\int \tan^{-1}(2y)\,dy$

$u = \tan^{-1}(2y)$ $\quad dv = dy$

$du = \frac{1}{1+(2y)^2}(2)dy$ $\quad v = y$

$= \int(\tan^{-1}(2y))(dy) = (\tan^{-1}(2y))(y) - \int(y)\left(\frac{2}{1+(2y)^2}dy\right)$

$= y\tan^{-1}(2y) - 2\int\frac{y}{1+4y^2}dy$

$\int\frac{y}{1+4y^2}dy = \int\frac{1}{1+4y^2}(y\,dy) = \int\frac{1}{p}\left(\frac{1}{8}dp\right) = \frac{1}{8}[\ln|p|] + C$

$p = 1+4y^2$

$dp = 8y\,dy$

$\frac{1}{8}dp = y\,dy$

$= \frac{1}{8}\ln|1+4y^2| + C$

$= y\tan^{-1}(2y) - 2\left[\frac{1}{8}\ln|1+4y^2|\right] + C$

$= y\tan^{-1}(2y) - \frac{1}{4}\ln|1+4y^2| + C$

18) $\int x \cosh ax \, dx$

$u = x \qquad dv = \cosh ax \, dx$
$du = dx \qquad v = \frac{1}{a} \sinh ax$

$= \int (x)(\cosh ax \, dx) = (x)\left(\frac{1}{a} \sinh ax\right) - \int \left(\frac{1}{a} \sinh ax\right)(dx)$

$= \frac{1}{a} x \sinh ax - \frac{1}{a} \int \sinh ax \, dx$

$= \frac{1}{a} x \sinh ax - \frac{1}{a}\left[\frac{1}{a} \cosh ax\right] + C$

$= \frac{1}{a} x \sinh ax - \frac{1}{a^2} \cosh ax + C$

---

20) $\int \frac{z}{10^z} \, dz$

$u = z \qquad dv = 10^{-z} dz$
$du = dz \qquad v = \frac{-1}{\ln 10} 10^{-z}$

$= \int (z)(10^{-z} dz) = (z)\left(\frac{-1}{\ln 10} 10^{-z}\right) - \int \left(\frac{-1}{\ln 10} 10^{-z}\right)(dz)$

$= \frac{-1}{\ln 10} z 10^{-z} + \frac{1}{\ln 10} \int 10^{-z} dz$

$= \frac{-1}{\ln 10} z 10^{-z} + \frac{1}{\ln 10}\left[\frac{-1}{\ln 10} 10^{-z}\right] + C$

$= \frac{-1}{\ln 10} z 10^{-z} - \frac{1}{(\ln 10)^2} 10^{-z} + C = \frac{-z}{(\ln 10) 10^z} - \frac{1}{(\ln 10)^2 10^z} + C$

22) $u_1 = \sin \pi x$ $\quad dv_1 = e^x dx$ $\quad u_2 = \cos \pi x$ $\quad dv_2 = e^x dx$

$du_1 = \pi \cos \pi x \, dx$ $\quad v_1 = e^x$ $\quad du_2 = -\pi \sin \pi x \, dx$ $\quad v_2 = e^x$

7.1/6

$$\int e^x \sin \pi x \, dx = \int (\sin \pi x)(e^x dx)$$

$$\int e^x \sin \pi x \, dx = (\sin \pi x)(e^x) - \int (e^x)(\pi \cos \pi x \, dx)$$

$$\int e^x \sin \pi x \, dx = e^x \sin \pi x - \pi \int (\cos \pi x)(e^x dx)$$

$$\int e^x \sin \pi x \, dx = e^x \sin \pi x - \pi \left\{ (\cos \pi x)(e^x) - \int (e^x)(-\pi \sin \pi x \, dx) \right\}$$

$$\int e^x \sin \pi x \, dx = e^x \sin \pi x - \pi e^x \cos \pi x + \pi^2 \int e^x \sin \pi x \, dx$$

$$-\pi^2 \int e^x \sin \pi x \, dx \qquad\qquad -\pi^2 \int e^x \sin \pi x \, dx$$

$$\frac{(1-\pi^2) \int e^x \sin \pi x \, dx}{(1-\pi^2)} = \frac{e^x \sin \pi x - \pi e^x \cos \pi x}{(1-\pi^2)}$$

$$\int e^x \sin \pi x \, dx = \frac{1}{1-\pi^2} e^x \sin \pi x - \frac{\pi}{1-\pi^2} e^x \cos \pi x + C$$

"these integrations is a little bit simpler when we choose u to be trigonometric functions"

7.1/7

24) $u_1 = \cos 2\theta$ $\quad dv_1 = e^{-\theta}d\theta$ $\quad u_2 = \sin 2\theta$ $\quad dv_2 = e^{-\theta}d\theta$

$du_1 = -2\sin 2\theta\, d\theta$ $\quad v_1 = -e^{-\theta}$ $\quad du_2 = 2\cos 2\theta\, d\theta$ $\quad v_2 = -e^{-\theta}$

$$\int e^{-\theta}\cos 2\theta\, d\theta = \int (\cos 2\theta)(e^{-\theta}d\theta)$$

$$\int e^{-\theta}\cos 2\theta\, d\theta = (\cos 2\theta)(-e^{-\theta}) - \int(-e^{-\theta})(-2\sin 2\theta\, d\theta)$$

$$\int e^{-\theta}\cos 2\theta\, d\theta = -e^{-\theta}\cos 2\theta - 2\int(\sin 2\theta)(e^{-\theta}d\theta)$$

$$\int e^{-\theta}\cos 2\theta\, d\theta = -e^{-\theta}\cos 2\theta - 2\left\{(\sin 2\theta)(-e^{-\theta}) - \int(-e^{-\theta})(2\cos 2\theta\, d\theta)\right\}$$

$$\int e^{-\theta}\cos 2\theta\, d\theta = -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta - 4\int e^{-\theta}\cos 2\theta\, d\theta$$

$$5\int e^{-\theta}\cos 2\theta\, d\theta = -e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta$$

$$\int e^{-\theta}\cos 2\theta\, d\theta = \frac{-e^{-\theta}\cos 2\theta + 2e^{-\theta}\sin 2\theta}{5}$$

$$\int e^{-\theta}\cos 2\theta = \frac{2}{5}e^{-\theta}\sin 2\theta - \frac{1}{5}e^{-\theta}\cos 2\theta + C$$

26) $u_1 = (\sin^{-1}x)^2$ $\quad dv_1 = dx$

$du_1 = 2(\sin^{-1}x)\left(\frac{1}{\sqrt{1-x^2}}\right)dx$ $\quad v_1 = x$

$u_2 = \sin^{-1}x$ $\quad dv_1 = \frac{x}{\sqrt{1-x^2}}dx$

$du_2 = \frac{1}{\sqrt{1-x^2}}dx$ $\quad v_1 = -\sqrt{1-x^2}$

$\boxed{7.1/8}$

see ex 4 of 7.1 for this integration

$$\int (\arcsin x)^2 dx = \int\left((\sin^{-1}x)^2\right)(dx)$$

$$= \left((\sin^{-1}x)^2\right)(x) - \int (x)\left(2(\sin^{-1}x)\left(\frac{1}{\sqrt{1-x^2}}\right)dx\right)$$

$$= x(\sin^{-1}x)^2 - 2\int(\sin^{-1}x)\left(\frac{x}{\sqrt{1-x^2}}dx\right)$$

$$= x(\sin^{-1}x)^2 - 2\left\{(\sin^{-1}x)\left(-\sqrt{1-x^2}\right) - \int\left(-\sqrt{1-x^2}\right)\left(\frac{1}{\sqrt{1-x^2}}dx\right)\right\}$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}(\sin^{-1}x) - 2\int dx$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}(\sin^{-1}x) - 2[x] + C$$

$$= \underline{\underline{x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}(\sin^{-1}x) - 2x + C}}$$

28) $\int_0^{\frac{1}{2}} \theta \sin 3\pi\theta \, d\theta$

$u = \theta \qquad dv = \sin 3\pi\theta \, d\theta$

$du = d\theta \qquad v = \frac{-1}{3\pi} \cos 3\pi\theta$

$$\int (\theta)(\sin 3\pi\theta \, d\theta) = (\theta)\left(\frac{-1}{3\pi} \cos 3\pi\theta\right) - \int \left(\frac{-1}{3\pi} \cos 3\pi\theta\right)(d\theta)$$

$$= \frac{-1}{3\pi} \theta \cos 3\pi\theta + \frac{1}{3\pi} \int \cos 3\pi\theta \, d\theta$$

$$= \frac{-1}{3\pi} \theta \cos 3\pi\theta + \frac{1}{3\pi} \left[\frac{1}{3\pi} \sin 3\pi\theta\right] + C$$

$$= \frac{-1}{3\pi} \theta \cos 3\pi\theta + \frac{1}{9\pi^2} \sin 3\pi\theta + C$$

$$\int_0^{\frac{1}{2}} \theta \sin 3\pi\theta \, d\theta = \left[\frac{-1}{3\pi} \theta \cos 3\pi\theta + \frac{1}{9\pi^2} \sin 3\pi\theta + C\right]_0^{\frac{1}{2}}$$

$$= \left[\frac{-1}{3\pi}\left(\frac{1}{2}\right) \cos 3\pi\left(\frac{1}{2}\right) + \frac{1}{9\pi^2} \sin 3\pi\left(\frac{1}{2}\right) + C\right] - \left[\frac{-1}{3\pi}(0) \cos 3\pi(0) + \frac{1}{9\pi^2} \sin 3\pi(0) + C\right]$$

$$= \left[\frac{-1}{6\pi} \cos\left(\frac{3\pi}{2}\right) + \frac{1}{9\pi^2} \sin\left(\frac{3\pi}{2}\right)\right] - \left[\frac{-1}{3\pi}(0)(1) + \frac{1}{9\pi^2}(0)\right]$$

$$= \left[\frac{-1}{6\pi}(0) + \frac{1}{9\pi^2}(-1)\right] - [0]$$

$$= \underline{\underline{\frac{-1}{9\pi^2}}}$$

30) $\int_0^1 \frac{xe^x}{(1+x)^2}\,dx$

$u = xe^x$ $\quad dv = \frac{1}{(1+x)^2}dx$

$du = xe^x + e^x dx$

$= e^x(x+1)dx$ $\quad v = \frac{-1}{1+x}$

7.1/10

$$\int (xe^x)\left(\frac{1}{(1+x)^2}dx\right) = (xe^x)\left(\frac{-1}{1+x}\right) - \int\left(\frac{-1}{1+x}\right)\left(e^x(x+1)dx\right)$$

$$= \frac{-xe^x}{1+x} + \int \frac{e^x(x+1)}{x+1}dx = \frac{-xe^x}{1+x} + \int e^x dx$$

$$= \frac{-xe^x}{1+x} + e^x + C$$

$$\int_0^1 \frac{xe^x}{(1+x)^2}dx = \left[\frac{-xe^x}{1+x} + e^x + C\right]_0^1$$

$$= \left[\frac{-(1)e^{(1)}}{1+(1)} + e^{(1)} + C\right] - \left[\frac{-(0)e^{(0)}}{1+(0)} + e^{(0)} + C\right]$$

$$= \left[\frac{-e}{2} + e\right] - \left[0+1\right]$$

$$= \left[\frac{e}{2}\right] - \left[1\right] = \underline{\underline{\frac{e}{2} - 1}}$$

32) $\int_1^2 w^2 \ln w \, dw$

$u = \ln w \qquad dv = w^2 dw$

$du = \frac{1}{w} dw \qquad v = \frac{1}{3} w^3$

7.1/11

$$\int (\ln w)(w^2 dw) = (\ln w)\left(\frac{1}{3}w^3\right) - \int \left(\frac{1}{3}w^3\right)\left(\frac{1}{w}\,dw\right)$$

$$= \frac{1}{3}w^3 \ln w - \frac{1}{3}\int w^2\,dw$$

$$= \frac{1}{3}w^3 \ln w - \frac{1}{3}\left[\frac{1}{3}w^3\right] + C$$

$$= \frac{1}{3}w^3 \ln w - \frac{1}{9}w^3 + C$$

$$\int_1^2 w^2 \ln w\,dw = \left[\frac{1}{3}w^3 \ln w - \frac{1}{9}w^3 + C\right]_1^2$$

$$= \left[\frac{1}{3}(2)^3 \ln(2) - \frac{1}{9}(2)^3 + C\right] - \left[\frac{1}{3}(1)^3 \ln(1) - \frac{1}{9}(1)^3 + C\right]$$

$$= \left[\frac{8}{3}\ln 2 - \frac{8}{9}\right] - \left[\frac{1}{3}(0) - \frac{1}{9}\right]$$

$$= \frac{8}{3}\ln 2 - \frac{8}{9} + \frac{1}{9}$$

$$= \underline{\underline{\frac{8}{3}\ln 2 - \frac{7}{9}}}$$

34) $\int_0^{2\pi} t^2 \sin 2t \, dt$ $\quad u_1 = t^2 \quad dv_1 = \sin 2t\,dt$ $\quad du_1 = 2t\,dt \quad v_1 = -\frac{1}{2}\cos 2t$

$$\int (t^2)(\sin 2t\,dt) = (t^2)\left(-\frac{1}{2}\cos 2t\right) - \int \left(-\frac{1}{2}\cos 2t\right)(2t\,dt)$$

$$= -\frac{1}{2}t^2\cos 2t + \int (t)(\cos 2t\,dt)$$

$u_2 = t \quad dv_2 = \cos 2t\,dt$

$du_2 = dt \quad v_2 = \frac{1}{2}\sin 2t$

$$= -\frac{1}{2}t^2\cos 2t + \left\{ (t)\left(\frac{1}{2}\sin 2t\right) - \int \left(\frac{1}{2}\sin 2t\right)(dt) \right\}$$

$$= -\frac{1}{2}t^2\cos 2t + \frac{1}{2}t\sin 2t - \frac{1}{2}\int \sin 2t\,dt$$

$$= -\frac{1}{2}t^2\cos 2t + \frac{1}{2}t\sin 2t - \frac{1}{2}\left[-\frac{1}{2}\cos 2t\right] + C$$

$$= -\frac{1}{2}t^2\cos 2t + \frac{1}{2}t\sin 2t + \frac{1}{4}\cos 2t + C$$

$$\int_0^{2\pi} t^2 \sin 2t\,dt = \left[-\frac{1}{2}t^2\cos 2t + \frac{1}{2}t\sin 2t + \frac{1}{4}\cos 2t + C\right]_0^{2\pi}$$

$$= \left[-\frac{1}{2}(2\pi)^2\cos 2(2\pi) + \frac{1}{2}(2\pi)\sin 2(2\pi) + \frac{1}{4}\cos 2(2\pi) + C\right] - \left[-\frac{1}{2}(0)^2\cos 2(0) + \frac{1}{2}(0)\sin 2(0) + \frac{1}{4}\cos 2(0) + C\right]$$

$$= \left[-2\pi^2(1) + \pi(0) + \frac{1}{4}(1)\right] - \left[0 + 0 + \frac{1}{4}(1)\right]$$

$$= \left[-2\pi^2 + \frac{1}{4}\right] - \left[\frac{1}{4}\right]$$

$$= \underline{\underline{-2\pi^2}}$$

36) $\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx$

$u = \tan^{-1}\left(\frac{1}{x}\right)$

$du = \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(\frac{-1}{x^2}\right) dx$

$= \frac{-1}{x^2+1} dx$

$dv = dx$

$v = x$

$\boxed{7.1/13}$

$$\int \arctan\left(\frac{1}{x}\right) dx = \int \left(\tan^{-1}\left(\frac{1}{x}\right)\right)(dx)$$

$$= \left(\tan^{-1}\left(\frac{1}{x}\right)\right)(x) - \int (x)\left(\frac{-1}{x^2+1} dx\right)$$

$$= x\tan^{-1}\left(\frac{1}{x}\right) + \int \frac{x}{x^2+1} dx$$

$$\int \frac{x}{x^2+1} dx = \int \frac{1}{x^2+1}(x\,dx) = \int \frac{1}{p}\left(\frac{1}{2} dp\right) = \frac{1}{2}\left[\ln|p|\right] + C$$

$$= \frac{1}{2}\ln|x^2+1| + C$$

$p = x^2+1$

$dp = 2x\,dx$

$\frac{1}{2} dp = x\,dx$

$$= x\tan^{-1}\left(\frac{1}{x}\right) + \left[\frac{1}{2}\ln|x^2+1|\right] + C$$

$$= x\tan^{-1}\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2+1| + C$$

$$\int_1^{\sqrt{3}} \arctan\left(\frac{1}{x}\right) dx = \left[x\tan^{-1}\left(\frac{1}{x}\right) + \frac{1}{2}\ln|x^2+1| + C\right]_1^{\sqrt{3}}$$

$$= \left[(\sqrt{3})\tan^{-1}\left(\frac{1}{(\sqrt{3})}\right) + \frac{1}{2}\ln|(\sqrt{3})^2+1| + C\right] - \left[(1)\tan^{-1}\left(\frac{1}{(1)}\right) + \frac{1}{2}\ln|(1)^2+1| + C\right]$$

$$= \left[(\sqrt{3})\left(\frac{\pi}{6}\right) + \frac{1}{2}\ln|4|\right] - \left[(1)\left(\frac{\pi}{4}\right) + \frac{1}{2}\ln|2|\right]$$

$$= \frac{\pi\sqrt{3}}{6} + \frac{1}{2}\ln(4) - \frac{\pi}{4} - \frac{1}{2}\ln(2) = \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}\ln(4) - \frac{1}{2}\ln(2)$$

$$= \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}(\ln(4) - \ln(2)) = \frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}\left(\ln\left(\frac{4}{2}\right)\right) = \underline{\underline{\frac{\pi\sqrt{3}}{6} - \frac{\pi}{4} + \frac{1}{2}\ln 2}}$$

38) $\displaystyle\int_1^2 \frac{(\ln x)^2}{x^3}\,dx$

$u_1 = (\ln x)^2$ $\qquad dv_1 = \frac{1}{x^3}dx$

$du_1 = 2\ln x\left(\frac{1}{x}\right)dx$ $\qquad v_1 = \frac{-1}{2x^2}$

7.1/14

$$\int\left((\ln x)^2\right)\left(\frac{1}{x^3}dx\right) = \left((\ln x)^2\right)\left(\frac{-1}{2x^2}\right) - \int\left(\frac{-1}{2x^2}\right)\left(2\ln x\left(\frac{1}{x}\right)dx\right)$$

$$= \frac{-(\ln x)^2}{2x^2} + \int(\ln x)\left(\frac{1}{x^3}dx\right)$$

$u_2 = \ln x$ $\qquad dv_2 = \frac{1}{x^3}dx$

$du_2 = \frac{1}{x}dx$ $\qquad v_2 = \frac{-1}{2x^2}$

$$= \frac{-(\ln x)^2}{2x^2} + \left\{(\ln x)\left(\frac{-1}{2x^2}\right) - \int\left(\frac{-1}{2x^2}\right)\left(\frac{1}{x}dx\right)\right\}$$

$$= \frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} + \frac{1}{2}\int\frac{1}{x^3}dx = \frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} + \frac{1}{2}\left[\frac{-1}{2x^2}\right] + C$$

$$= \frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} - \frac{1}{4x^2} + C$$

$$\int_1^2 \frac{(\ln x)^2}{x^3}dx = \left[\frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} - \frac{1}{4x^2} + C\right]_1^2$$

$$= \left[\frac{-(\ln(2))^2}{2(2)^2} - \frac{\ln(2)}{2(2)^2} - \frac{1}{4(2)^2} + C\right] - \left[\frac{-(\ln(1))^2}{2(1)^2} - \frac{\ln(1)}{2(1)^2} - \frac{1}{4(1)^2} + C\right]$$

$$= \left[\frac{-(\ln 2)^2}{8} - \frac{\ln 2}{8} - \frac{1}{16}\right] - \left[0 - 0 - \frac{1}{4}\right]$$

$$= \frac{-(\ln 2)^2}{8} - \frac{\ln 2}{8} - \frac{1}{16} + \frac{1}{4}$$

$$= \frac{-(\ln 2)^2}{8} - \frac{\ln 2}{8} + \frac{3}{16}$$

40) $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$ it is easier to use triple substitution

7.1/15

$$\int \frac{r^3}{\sqrt{4+r^2}}\,dr = \int \frac{r^2}{\sqrt{4+r^2}}(r\,dr)$$

$p = 4+r^2 \rightarrow (p-4) = r^2$

$dp = 2r\,dr$

$\frac{1}{2}dp = r\,dr$

$$= \int \frac{(p-4)}{\sqrt{p}}\left(\frac{1}{2}dp\right) = \frac{1}{2}\int\left(\frac{p}{\sqrt{p}} - \frac{4}{\sqrt{p}}\right)dp$$

$$= \frac{1}{2}\int\left(p^{\frac{1}{2}} - 4p^{-\frac{1}{2}}\right)dp = \frac{1}{2}\left\{\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] - 4\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right]\right\} + C$$

$$= \frac{1}{3}p^{\frac{3}{2}} - 4p^{\frac{1}{2}} + C = \frac{1}{3}(\sqrt{p})^3 - 4\sqrt{p} + C$$

$$= \frac{1}{3}\left(\sqrt{4+r^2}\right)^3 - 4\sqrt{4+r^2} + C$$

$$\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr = \left[\frac{1}{3}\left(\sqrt{4+r^2}\right)^3 - 4\sqrt{4+r^2} + C\right]_0^1$$

$$= \left[\frac{1}{3}\left(\sqrt{4+(1)^2}\right)^3 - 4\sqrt{4+(1)^2} + C\right] - \left[\frac{1}{3}\left(\sqrt{4+(0)^2}\right)^3 - 4\sqrt{4+(0)^2} + C\right]$$

$$= \left[\frac{1}{3}(\sqrt{5})^3 - 4\sqrt{5}\right] - \left[\frac{1}{3}(2)^3 - 4(2)\right]$$

$$= \left[\frac{5\sqrt{5}}{3} - 4\sqrt{5}\right] - \left[\frac{8}{3} - 8\right] = \left[\frac{5\sqrt{5}}{3} - \frac{12\sqrt{5}}{3}\right] - \left[\frac{8}{3} - \frac{24}{3}\right]$$

$$= \left[\frac{-7\sqrt{5}}{3}\right] - \left[\frac{-16}{3}\right] = \frac{-7\sqrt{5}}{3} + \frac{16}{3} = \underline{\underline{\frac{16-7\sqrt{5}}{3}}}$$

42) $\int_0^t e^s \sin(t-s)\,ds$ 7.1/16

$u_1 = \sin(t-s)$ $dv_1 = e^s ds$ $u_2 = \cos(t-s)$ $dv_2 = e^s ds$

$du_1 = -\cos(t-s)ds$ $v_1 = e^s$ $du_2 = \sin(t-s)ds$ $v_2 = e^s$

$$\int e^s \sin(t-s)ds = \int (\sin(t-s))(e^s ds)$$

$$\int e^s \sin(t-s)ds = (\sin(t-s))(e^s) - \int (e^s)(-\cos(t-s)ds)$$

$$\int e^s \sin(t-s)ds = e^s \sin(t-s) + \int (\cos(t-s))(e^s ds)$$

$$\int e^s \sin(t-s)ds = e^s \sin(t-s) + \left\{(\cos(t-s))(e^s) - \int (e^s)(\sin(t-s)ds)\right\}$$

$$\int e^s \sin(t-s)ds = e^s \sin(t-s) + e^s\cos(t-s) - \int e^s \sin(t-s)ds$$

$$2\int e^s \sin(t-s)ds = e^s \sin(t-s) + e^s\cos(t-s)$$

$$\int e^s \sin(t-s)ds = \frac{1}{2}e^s \sin(t-s) + \frac{1}{2}e^s\cos(t-s) + C$$

$$\int_0^t e^s \sin(t-s)ds = \left[\frac{1}{2}e^s \sin(t-s) + \frac{1}{2}e^s\cos(t-s) + C\right]_0^t$$

$$= \left[\frac{1}{2}e^{(t)}\sin(t-(t)) + \frac{1}{2}e^{(t)}\cos(t-(t)) + C\right] - \left[\frac{1}{2}e^{(0)}\sin(t-(0)) + \frac{1}{2}e^{(0)}\cos(t-(0)) + C\right]$$

$$= \left[\frac{1}{2}e^t \sin(0) + \frac{1}{2}e^t\cos(0)\right] - \left[\frac{1}{2}(1)\sin t + \frac{1}{2}(1)\cos t\right]$$

$$= \left[\frac{1}{2}e^t(0) + \frac{1}{2}e^t(1)\right] - \left[\frac{1}{2}\sin t + \frac{1}{2}\cos t\right]$$

$$= \underline{\underline{\frac{1}{2}e^t - \frac{1}{2}\sin t - \frac{1}{2}\cos t}}$$

44) $\int \cos(\ln x)\,dx$

$= \int \cos(p)(e^p\,dp)$

$p = \ln x$
$\Downarrow$
$e^p = x$
$e^p\,dp = dx$

7.1/17

$u_1 = \cos p$ $\quad dv_1 = e^p\,dp$ $\quad u_2 = \sin p$ $\quad dv_2 = e^p\,dp$

$du_1 = -\sin p\,dp$ $\quad v_1 = e^p$ $\quad du_2 = \cos p\,dp$ $\quad v_2 = e^p$

$$\int \cos(p)e^p\,dp = (\cos p)(e^p) - \int (e^p)(-\sin p\,dp)$$

$$\int \cos p\, e^p\,dp = \cos p\, e^p + \int \sin p\, e^p\,dp$$

$$\int \cos p\, e^p\,dp = \cos p\, e^p + \{(\sin p)(e^p) - \int (e^p)(\cos p\,dp)\}$$

$$\int \cos p\, e^p\,dp = \cos p\, e^p + \sin p\, e^p - \int \cos p\, e^p\,dp$$

$$2\int \cos p\, e^p\,dp = \cos p\, e^p + \sin p\, e^p$$

$$\int \cos p\, e^p\,dp = \frac{1}{2}\cos p\, e^p + \frac{1}{2}\sin p\, e^p + C$$

$$\int \cos(\ln x)\,dx = \int \cos p\, e^p\,dp$$

$$= \frac{1}{2}\cos p\, e^p + \frac{1}{2}\sin p\, e^p + C$$

$$= \frac{1}{2}\cos(\ln x)(x) + \frac{1}{2}\sin(\ln x)(x) + C$$

$$= \frac{1}{2}x\cos(\ln x) + \frac{1}{2}x\sin(\ln x) + C$$

$$46)\ \int_0^{\pi} e^{\cos t} \sin 2t\, dt$$

7.1/18

$$\int e^{\cos t} \sin 2t\, dt = \int e^{\cos t} (2 \sin t \cos t)\, dt$$

$$= \int e^{\cos t} (2 \cos t)(\sin t\, dt) \qquad p = \cos t$$

$$dp = -\sin t\, dt$$

$$= \int e^{p} (2p)(-1\, dp) = \int (-2p)(e^{p} dp) \qquad -1\, dp = \sin t\, dt$$

$$\int (-2p)(e^{p} dp) \qquad u = 2p \quad dv = e^{p} dp$$

$$du = -2\, dp \quad v = e^{p}$$

$$= (-2p)(e^{p}) - \int (e^{p})(-2\, dp) = -2pe^{p} + 2\int e^{p} dp$$

$$= -2pe^{p} + 2[e^{p}] + C = -2pe^{p} + 2e^{p} + C$$

$$\int e^{\cos t} \sin 2t\, dt = \int (-2p)(e^{p} dp) = -2pe^{p} + 2e^{p} + C$$

$$= -2(\cos t)e^{\cos t} + 2e^{\cos t} + C$$

$$\int_0^{\pi} e^{\cos t} \sin 2t\, dt = \left[-2(\cos t)e^{\cos t} + 2e^{\cos t} + C\right]_0^{\pi}$$

$$= \left[-2(\cos(\pi))e^{\cos(\pi)} + 2e^{\cos(\pi)} + C\right] - \left[-2(\cos(0))e^{\cos(0)} + 2\cos^{(0)} + C\right]$$

$$= \left[-2(-1)e^{(-1)} + 2e^{(-1)}\right] - \left[-2(1)e^{(1)} + 2e^{(1)}\right]$$

$$= \left[2e^{-1} + 2e^{-1}\right] - \left[-2e + 2e\right]$$

$$= \left[\frac{2}{e} + \frac{2}{e}\right] - [0] = \underline{\underline{\frac{4}{e}}}$$

48) $\int \frac{\arcsin(\ln x)}{x}\,dx$ $\qquad p = \ln x$, $\quad dp = \frac{1}{x}\,dx$ $\qquad$ 7.1/19

$= \int \sin^{-1}(\ln x)\left(\frac{1}{x}\,dx\right) = \int \sin^{-1} p\ dp$

$\int (\sin^{-1} p)(dp) = (\sin^{-1} p)(p) - \int (p)\left(\frac{1}{\sqrt{1-p^2}}\,dp\right)$ $\qquad u = \sin^{-1} p \quad dv = dp$, $\quad du = \frac{1}{\sqrt{1-p^2}}\,dp \quad v = p$

$= p\sin^{-1} p - \int \frac{p}{\sqrt{1-p^2}}\,dp$ see ex 4 of 7.1 for integration

$= p\sin^{-1} p - \left[-\sqrt{1-p^2}\right] + C$

$= p\sin^{-1} p + \sqrt{1-p^2} + C$

$\int \frac{\arcsin(\ln x)}{x}\,dx = \int \sin^{-1} p\ dp$

$= p\sin^{-1} p + \sqrt{1-p^2} + C$

$= (\ln x)\sin^{-1}(\ln x) + \sqrt{1-(\ln x)^2} + C$