21200 {07} section 9.3

2) $\dfrac{dy}{dx} = \dfrac{x}{y^4}$

$y^4\,dy = x\,dx$

$\int y^4\,dy = \int x\,dx$

$\left[\dfrac{y^5}{5}\right] = \left[\dfrac{x^2}{2}\right] + C_1$

$y^5 = \dfrac{5}{2}x^2 + 5C_1$

$y^5 = \dfrac{5}{2}x^2 + C_2$ (let $C_2 = 5C_1$)

$y = \sqrt[5]{\dfrac{5}{2}x^2 + C_2}$

$\underline{\underline{y(x) = \sqrt[5]{\dfrac{5}{2}x^2 + C_2}}}$

4) $xy' = y + 3$

$x\left(\dfrac{dy}{dx}\right) = y + 3$

$\dfrac{1}{y+3}\,dy = \dfrac{1}{x}\,dx$

$\int \dfrac{1}{y+3}\,dy = \int \dfrac{1}{x}\,dx$

$\ln|y+3| = \ln|x| + C_1$

$\Downarrow$

$y + 3 = e^{(\ln|x| + C_1)}$

$y + 3 = (e^{\ln|x|})(e^{C_1})$

$y + 3 = C_2 e^{\ln|x|}$ (let $C_2 = e^{C_1}$)

$y + 3 = C_2|x|$

$y = C_2|x| - 3$

$\underline{\underline{y(x) = C_2|x| - 3}}$

6) $y' + xe^y = 0$

$\dfrac{dy}{dx} + xe^y = 0$

$\dfrac{dy}{dx} = -xe^y$

$\dfrac{1}{e^y}\,dy = -x\,dx$

$\int \dfrac{1}{e^y}\,dy = \int -x\,dx$

$\int e^{-y}\,dy = \int -x\,dx$

$\left[-e^{-y}\right] = -1\left[\dfrac{x^2}{2}\right] + C_1$

$\dfrac{-1}{e^y} = \dfrac{-1}{2}x^2 + C_1$

$\dfrac{1}{e^y} = \dfrac{1}{2}x^2 - C_1$

$\dfrac{1}{e^y} = \dfrac{1}{2}x^2 + C_2$ (let $C_2 = -C_1$)

$e^y = \dfrac{1}{\frac{1}{2}x^2 + C_2}$

$\Downarrow$

$y = \ln\left(\dfrac{1}{\frac{1}{2}x^2 + C_2}\right)$

$\underline{\underline{y(x) = \ln\left(\dfrac{1}{\frac{1}{2}x^2 + C_2}\right)}}$

9.3/2

8) $\dfrac{dy}{dx} = 2x\,(y^2+1)$

$$\frac{1}{y^2+1}\,dy = 2x\,dx$$

$$\int \frac{1}{y^2+1^2}\,dy = \int 2x\,dx$$

$$\left[\frac{1}{1}\tan^{-1}\left(\frac{y}{1}\right)\right] = \left[x^2\right] + C_1$$

$$\tan^{-1} y = x^2 + C_1$$

$$\Downarrow$$

$$y = \tan(x^2 + C_1)$$

$$\underline{\underline{y(x) = \tan(x^2 + C_1)}}$$

12) $\dfrac{dH}{dR} = \dfrac{R H^2 \sqrt{1+R^2}}{\ln H}$

$$\frac{\ln H}{H^2}\,dH = R\sqrt{1+R^2}\,dR$$

$$\int \frac{\ln H}{H^2}\,dH = \int R\sqrt{1+R^2}\,dR$$

$\uparrow$

integration by parts needed (see 7.1)

$$\frac{-\ln H}{H} - \frac{1}{H} = \frac{1}{3}\left(\sqrt{1+R^2}\right)^3 + C_1$$

$$\frac{-\ln H - 1}{H} = \frac{1}{3}\left(\sqrt{1+R^2}\right)^3 + C_1$$

completion will be shown in 39100

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14) $\dfrac{dP}{dt} = \sqrt{Pt}$, $\quad P(1) = 2$

$$\frac{dP}{dt} = (\sqrt{P})(\sqrt{t})$$

$$\frac{1}{\sqrt{P}}\,dP = \sqrt{t}\,dt$$

$$\int \frac{1}{\sqrt{P}}\,dP = \int \sqrt{t}\,dt$$

$$\int P^{-\frac{1}{2}}\,dP = \int t^{\frac{1}{2}}\,dt$$

$$\left[\frac{P^{\frac{1}{2}}}{\frac{1}{2}}\right] = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + C_1$$

$$2\sqrt{P} = \frac{2}{3}\left(\sqrt{t}\right)^3 + C_1$$

$$\sqrt{P} = \frac{1}{3}\left(\sqrt{t}\right)^3 + \frac{C_1}{2}$$

$$\sqrt{P} = \frac{1}{3}\left(\sqrt{t}\right)^3 + C_2 \quad \left(\text{let } C_2 = \frac{C_1}{2}\right)$$

$$\sqrt{(2)} = \frac{1}{3}\left(\sqrt{(1)}\right)^3 + C_2$$

$$\sqrt{2} = \frac{1}{3} + C_2$$

$$\left(\sqrt{2} - \frac{1}{3}\right) = C_2$$

$$\sqrt{P} = \frac{1}{3}\left(\sqrt{t}\right)^3 + \left(\sqrt{2} - \frac{1}{3}\right)$$

$$P = \left(\frac{1}{3}\left(\sqrt{t}\right)^3 + \sqrt{2} - \frac{1}{3}\right)^2$$

$$\underline{\underline{P(t) = \left(\frac{1}{3}\left(\sqrt{t}\right)^3 + \sqrt{2} - \frac{1}{3}\right)^2}}$$

16) $x^2 y' = k \sec y$, $\quad y(1) = \frac{\pi}{6}$

$$x^2 \frac{dy}{dx} = k \sec y$$

$$\frac{1}{\sec y} dy = \frac{k}{x^2} dx$$

$$\int \cos y \, dy = \int k x^{-2} dx$$

$$[-\sin y] = [-k x^{-1}] + C_1$$

$$\sin y = \frac{k}{x} - C_1$$

$$\sin y = \frac{k}{x} + C_2 \quad (\text{let } C_2 = -C_1)$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{k}{(1)} + C_2$$

$$\frac{1}{2} = k + C_2$$

$$\left(\frac{1}{2} - k\right) = C_2$$

$$\sin y = \frac{k}{x} + \left(\frac{1}{2} - k\right)$$

$$\Downarrow$$

$$y = \sin^{-1}\left(\frac{k}{x} + \frac{1}{2} - k\right)$$

$$\underline{\underline{y(x) = \sin^{-1}\left(\frac{k}{x} + \frac{1}{2} - k\right)}}$$

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18) $x + 3y^2 \sqrt{x^2+1} \frac{dy}{dx} = 0$, $\quad y(0) = 1$

$$3y^2 \sqrt{x^2+1} \frac{dy}{dx} = -x$$

$$3y^2 dy = \frac{-x}{\sqrt{x^2+1}} dx$$

$$\int 3y^2 dy = \int \frac{-x}{\sqrt{x^2+1}} dx$$

$$[y^3] = \left[-\sqrt{x^2+1}\right] + C_1$$

$$y^3 = -\sqrt{x^2+1} + C_1$$

$$(1)^3 = -\sqrt{(0)^2+1} + C_1$$

$$1 = -\sqrt{1} + C_1$$

$$1 = -1 + C_1$$

$$2 = C_1$$

$$y^3 = -\sqrt{x^2+1} + (2)$$

$$y = \sqrt[3]{-\sqrt{x^2+1} + 2}$$

$$\underline{\underline{y(x) = \sqrt[3]{2 - \sqrt{x^2+1}}}}$$

$$\int \frac{-x}{\sqrt{x^2+1}} dx = \int \frac{-1}{\sqrt{x^2+1}} (x\,dx) = \int \frac{-1}{\sqrt{p}} \left(\frac{1}{2} dp\right) = \int \frac{-1}{2} p^{-\frac{1}{2}} dp = \frac{-1}{2}\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$$

$$= -\sqrt{p} + C = -\sqrt{x^2+1} + C$$

$p = x^2 + 1$

$dp = 2x\,dx$

$\frac{1}{2} dp = x\,dx$

20) $\frac{dy}{dx} = \frac{x \sin x}{y}$, $y(0) = -1$

$y\,dy = x \sin x\,dx$

$\int y\,dy = \int x \sin x\,dx$

integration by parts (7.1)

$\left[\frac{y^2}{2}\right] = \left[-x\cos x + \sin x\right] + C_1$

$y^2 = -2x\cos x + 2\sin x + 2c_1$

(let $C_2 = 2c_1$)

$y^2 = 2\sin x - 2x\cos x + C_2$

$(-1)^2 = 2\sin(0) - 2(0)\cos(0) + C_2$

$1 = 2(0) - 2(0)(1) + C_2$

$1 = C_2$

$y^2 = 2\sin x - 2x\cos x + (1)$

$y = \sqrt{2\sin x - 2x\cos x + 1}$

$y(x) = \sqrt{2\sin x - 2x\cos x + 1}$

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22) $f'(x) = x f(x) - x$, $f(0) = 2$; $f(x) = ?$

let $y = f(x) \rightarrow \frac{dy}{dx} = f'(x)$

$\frac{dy}{dx} = xy - x$

$\frac{dy}{dx} = x(y-1)$

$\frac{1}{y-1}\,dy = x\,dx$

$\int \frac{1}{y-1}\,dy = \int x\,dx$

$\left[\ln|y-1|\right] = \left[\frac{x^2}{2}\right] + C_1$

$\Downarrow$

$y - 1 = e^{\left(\frac{x^2}{2} + C_1\right)}$

$y - 1 = \left(e^{\frac{x^2}{2}}\right)\left(e^{C_1}\right)$

$y - 1 = C_2 e^{\frac{x^2}{2}}$ (let $C_2 = e^{C_1}$)

$(2) - 1 = C_2 e^{\frac{(0)^2}{2}}$

$1 = C_2$

$y - 1 = (1) e^{\frac{x^2}{2}}$

$y = e^{\frac{x^2}{2}} + 1$

$f(x) = e^{\frac{x^2}{2}} + 1$