2) $\int x e^{-x^2} dx$

$p = -x^2$

$dp = -2x\,dx$

$\frac{-1}{2} dp = x\,dx$

$= \int e^{-x^2} (x\,dx)$

$= \int e^{p} (dp)$

$= [e^{p}] + C = \underline{\underline{e^{-x^2} + C}}$

---

4) $\int \sin^2\theta \cos\theta\, d\theta$

$p = \sin\theta$

$dp = \cos\theta\, d\theta$

$= \int p^2\, dp$

$= \left[\frac{p^3}{3}\right] + C = \underline{\underline{\frac{1}{3}\sin^3\theta + C}}$

---

6) $\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\, dx$

$p = 1 + \frac{1}{x} = 1 + x^{-1}$

$dp = -1x^{-2}\, dx$

$dp = \frac{-1}{x^2}\, dx$

$-1\, dp = \frac{1}{x^2}\, dx$

$= \int \sqrt{1+\frac{1}{x}} \left(\frac{1}{x^2}\, dx\right)$

$= \int \sqrt{p}\, (-1\, dp)$

$= \int -p^{\frac{1}{2}}\, dp = -\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{-2}{3}\left(1+\frac{1}{x}\right)^{\frac{3}{2}} + C$

$= \underline{\underline{\frac{-2}{3}\left(\sqrt{1+\frac{1}{x}}\right)^3 + C}}$

5.5/2

8) $\int z\sqrt{z-1}\,dz$

$p = z-1 \rightarrow (p+1) = z$

$dp = dz$

*-triple substitution

$= \int (p+1)\sqrt{p}\,dp$

$= \int (p+1)p^{\frac{1}{2}}\,dp = \int \left(p^{\frac{3}{2}} + p^{\frac{1}{2}}\right)dp = \left[\frac{p^{\frac{5}{2}}}{\frac{5}{2}}\right] + \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= \frac{2}{5}(z-1)^{\frac{5}{2}} + \frac{2}{3}(z-1)^{\frac{3}{2}} + C = \frac{2}{5}\left(\sqrt{z-1}\right)^5 + \frac{2}{3}\left(\sqrt{z-1}\right)^3 + C$

---

10) $\int (5-3x)^{10}\,dx$

$p = 5-3x$

$dp = -3\,dx$

$\frac{-1}{3}\,dp = dx$

$= \int p^{10}\left(\frac{-1}{3}\,dp\right)$

$= \frac{-1}{3}\left[\frac{p^{11}}{11}\right] + C = \frac{-1}{33}(5-3x)^{11} + C$

---

12) $\int \sin t\sqrt{1+\cos t}\,dt$

$p = 1+\cos t$

$dp = -\sin t\,dt$

$-1\,dp = \sin t\,dt$

$= \int \sqrt{1+\cos t}\,(\sin t\,dt)$

$= \int \sqrt{p}\,(-1\,dp) = \int -p^{\frac{1}{2}}\,dp = -1\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= \frac{-2}{3}(1+\cos t)^{\frac{3}{2}} + C = \frac{-2}{3}\left(\sqrt{1+\cos t}\right)^3 + C$

5.5/3

14) $\int \sec^2 2\theta \, d\theta$

$p = 2\theta$
$dp = 2\, d\theta$
$\frac{1}{2} dp = d\theta$

$= \int \sec^2 p \left(\frac{1}{2} dp\right)$

$= \frac{1}{2}\left[\tan p\right] + C = \underline{\underline{\frac{1}{2}\tan 2\theta + C}}$

---

16) $\int y^2 (4-y^3)^{\frac{2}{3}} dy$

$p = 4 - y^3$
$dp = -3y^2 dy$
$-\frac{1}{3} dp = y^2 dy$

$= \int (4-y^3)^{\frac{2}{3}} (y^2 dy)$

$= \int p^{\frac{2}{3}} \left(\frac{-1}{3} dp\right) = \frac{-1}{3}\left[\frac{p^{\frac{5}{3}}}{\frac{5}{3}}\right] + C = \frac{-1}{5}(4-y^3)^{\frac{5}{3}} + C$

$= \underline{\underline{\frac{-1}{5}\left(\sqrt[3]{4-y^3}\right)^5 + C}}$

---

18) $\int \frac{z^2}{z^3+1} dz$

$p = z^3 + 1$
$dp = 3z^2 dz$
$\frac{1}{3} dp = z^2 dz$

$= \int \frac{1}{z^3+1} (z^2 dz)$

$= \int \frac{1}{p}\left(\frac{1}{3} dp\right) = \frac{1}{3}\left[\ln|p|\right] + C = \underline{\underline{\frac{1}{3}\ln|z^3+1| + C}}$

20) $\int e^{-5r}\,dr$

$p = -5r$

$dp = -5\,dr$

$-\frac{1}{5}dp = dr$

$= \int e^{p}\left(-\frac{1}{5}dp\right)$

$= -\frac{1}{5}\left[e^{p}\right] + C = \frac{-1}{5}e^{-5r} + C = \frac{-1}{5e^{5r}} + C$

---

22) $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

$p = \frac{1}{x} = x^{-1}$

$dp = -1x^{-2}dx = \frac{-1}{x^2}dx$

$-1\,dp = \frac{1}{x^2}dx$

$= \int \sin\left(\frac{1}{x}\right)\left(\frac{1}{x^2}dx\right)$

$= \int \sin p\,(-1\,dp) = -\left[-\cos p\right] + C = \cos\left(\frac{1}{x}\right) + C$

---

24) $\int \frac{t+1}{3t^2+6t-5}\,dt$

$p = 3t^2+6t-5$

$dp = 6t+6\ dt$

$dp = 6(t+1)dt$

$\frac{1}{6}dp = (t+1)dt$

$= \int \frac{1}{3t^2+6t-5}(t+1)dt$

$= \int \frac{1}{p}\left(\frac{1}{6}dp\right) = \frac{1}{6}\left[\ln|p|\right] + C = \frac{1}{6}\ln\left|3t^2+6t-5\right| + C$

---

26) $\int \sin x \sin(\cos x)\,dx$

$p = \cos x$

$dp = -\sin x\,dx$

$-1\,dp = \sin x\,dx$

$= \int \sin(\cos x)(\sin x\,dx)$

$= \int \sin p\,(-1\,dp) = -\left[-\cos p\right] + C$

$= \cos(\cos x) + C$

$28)\ \int x\sqrt{x+2}\,dx$

$p = x+2 \rightarrow (p-2) = x$
$dp = dx$

5.5/5

* triple substitution

$= \int (p-2)\sqrt{p}\,dp = \int (p-2)p^{\frac{1}{2}}\,dp$

$= \int \left(p^{\frac{3}{2}} - 2p^{\frac{1}{2}}\right)dp = \left[\frac{p^{\frac{5}{2}}}{\frac{5}{2}}\right] - 2\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C = \underline{\underline{\frac{2}{5}\left(\sqrt{x+2}\right)^5 - \frac{4}{3}\left(\sqrt{x+2}\right)^3 + C}}$

$30)\ \int \frac{dx}{ax+b}\ (a \neq 0)$

$p = ax+b$
$dp = a\,dx$
$\frac{1}{a}dp = dx$

$= \int \frac{1}{ax+b}\,dx$

$= \int \frac{1}{p}\left(\frac{1}{a}dp\right) = \frac{1}{a}\left[\ln|p|\right] + C = \underline{\underline{\frac{1}{a}\ln|ax+b| + C}}$

$32)\ \int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$

$p = \arcsin x = \sin^{-1}x$

$dp = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int e^{\sin^{-1}x}\left(\frac{1}{\sqrt{1-x^2}}\,dx\right)$

$= \int e^p\,dp = \left[e^p\right] + C = \underline{\underline{e^{\sin^{-1}x} + C}}$

$34)\ \int \frac{\sec^2 x}{\tan^2 x}\,dx$

$p = \tan x$
$dp = \sec^2 x\,dx$

$= \int \frac{1}{\tan^2 x}\left(\sec^2 x\,dx\right) = \int \frac{1}{p^2}\,dp = \int p^{-2}\,dp$

$= \left[\frac{p^{-1}}{-1}\right] + C = \frac{-1}{p} + C = \underline{\underline{\frac{-1}{\tan x} + C = -\cot x + C}}$

36) $\int \frac{1}{(x^2+1)\arctan x}\,dx$

$p = \arctan x = \tan^{-1}x$

$dp = \frac{1}{x^2+1}\,dx$

5.5/6

$= \int \frac{1}{\tan^{-1}x}\left(\frac{1}{x^2+1}\,dx\right)$

$= \int \frac{1}{p}\,dp = [\ln|p|] + C = \underline{\underline{\ln|\tan^{-1}x| + C}}$

---

38) $\int \frac{\sin\theta\cos\theta}{1+\sin^2\theta}\,d\theta$

$p = 1+\sin^2\theta$

$dp = 2\sin\theta\cos\theta\,d\theta$

$\frac{1}{2}dp = \sin\theta\cos\theta\,d\theta$

$= \int \frac{1}{1+\sin^2\theta}(\sin\theta\cos\theta\,d\theta)$

$= \int \frac{1}{p}\left(\frac{1}{2}dp\right) = \frac{1}{2}[\ln|p|] + C = \underline{\underline{\frac{1}{2}\ln|1+\sin^2\theta| + C}}$

---

40) $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx$

$p = \frac{\pi}{x} = \pi x^{-1}$

$dp = -\pi x^{-2}dx = \frac{-\pi}{x^2}dx$

$\frac{-1}{\pi}dp = \frac{1}{x^2}dx$

$= \int \cos\left(\frac{\pi}{x}\right)\left(\frac{1}{x^2}dx\right)$

$= \int \cos p\left(\frac{-1}{\pi}dp\right) = \frac{-1}{\pi}[\sin p] + C = \underline{\underline{\frac{-1}{\pi}\sin\left(\frac{\pi}{x}\right) + C}}$

---

42) $\int \frac{2^t}{2^t+3}\,dt$

$p = 2^t$

$dp = 2^t(\ln 2)\,dt$

$\frac{1}{\ln 2}dp = 2^t\,dt$

$= \int \frac{1}{2^t+3}(2^t\,dt)$

$= \int \frac{1}{p}\left(\frac{1}{\ln 2}dp\right) = \frac{1}{\ln 2}[\ln|p|] + C = \underline{\underline{\frac{1}{\ln 2}\ln|2^t+3| + C}}$

44) $\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

$p = \tan t$
$dp = \sec^2 t\, dt$

$= \int \frac{\sec^2 t\, dt}{\sqrt{1+\tan t}} = \int \frac{1}{\sqrt{1+\tan t}}(\sec^2 t\, dt) = \int \frac{1}{\sqrt{p}}\, dp = \int p^{-\frac{1}{2}} dp$

$= \left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = \underline{\underline{2\sqrt{1+\tan t} + C}}$

---

46) $\int \frac{\sin x}{1+\cos^2 x}\, dx$

$p = \cos x$
$dp = -\sin x\, dx$
$-1\, dp = \sin x\, dx$

$= \int \frac{1}{1+\cos^2 x}(\sin x\, dx)$

$= \int \frac{1}{1+p^2}(-1\, dp) = -1\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C = \underline{\underline{-\tan^{-1}(\cos x) + C}}$

---

48) $\int \frac{\cos(\ln t)}{t}\, dt$

$p = \ln t$
$dp = \frac{1}{t} dt$

$= \int \cos(\ln t)\left(\frac{1}{t} dt\right)$

$= \int \cos p\, dp = [\sin p] + C = \underline{\underline{\sin(\ln t) + C}}$

---

50) $\int \frac{x}{1+x^4}\, dx = \int \frac{x}{1+(x^2)^2}\, dx$

$p = x^2$
$dp = 2x\, dx$
$\frac{1}{2} dp = x\, dx$

$= \int \frac{1}{1+(x^2)^2}(x\, dx) = \int \frac{1}{1+p^2}\left(\frac{1}{2} dp\right)$

$= \frac{1}{2}\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C = \underline{\underline{\frac{1}{2}\tan^{-1}(x^2) + C}}$

5.5/8

52) $\int x^2\sqrt{2+x}\,dx$

$p = 2+x \rightarrow (p-2) = x$

$dp = dx$

* triple substitution

$= \int (p-2)^2\sqrt{p}\,dp$

$= \int (p^2-4p+4)p^{\frac{1}{2}}\,dp = \int \left(p^{\frac{5}{2}} - 4p^{\frac{3}{2}} + 4p^{\frac{1}{2}}\right)dp$

$= \left[\frac{p^{\frac{7}{2}}}{\frac{7}{2}}\right] - 4\left[\frac{p^{\frac{5}{2}}}{\frac{5}{2}}\right] + 4\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{2}{7}(2+x)^{\frac{7}{2}} - \frac{8}{5}(2+x)^{\frac{5}{2}} + \frac{8}{3}(2+x)^{\frac{3}{2}} + C$

$= \frac{2}{7}\left(\sqrt{2+x}\right)^7 - \frac{8}{5}\left(\sqrt{2+x}\right)^5 + \frac{8}{3}\left(\sqrt{2+x}\right)^3 + C$

---

54) $\int x^3\sqrt{x^2+1}\,dx$

$p = x^2+1 \rightarrow (p-1) = x^2$

$dp = 2x\,dx$

$\frac{1}{2}dp = x\,dx$

* triple substitution

$= \int x^2\sqrt{x^2+1}\,(x\,dx)$

$= \int (p-1)\sqrt{p}\left(\frac{1}{2}dp\right)$

$= \int \frac{1}{2}(p-1)p^{\frac{1}{2}}\,dp = \int \frac{1}{2}\left(p^{\frac{3}{2}} - p^{\frac{1}{2}}\right)dp = \frac{1}{2}\left\{\left[\frac{p^{\frac{5}{2}}}{\frac{5}{2}}\right] - \left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right]\right\} + C$

$= \frac{1}{2}\left\{\frac{2}{5}(x^2+1)^{\frac{5}{2}} - \frac{2}{3}(x^2+1)^{\frac{3}{2}}\right\} + C = \frac{1}{5}\left(\sqrt{x^2+1}\right)^5 - \frac{1}{3}\left(\sqrt{x^2+1}\right)^3 + C$

---

60) $\int_0^1 (3t-1)^{50}\,dt$

$p = 3t$

$dp = 3\,dt$

$\frac{1}{3}dp = dt$

$\int (3t-1)^{50}\,dt = \int p^{50}\left(\frac{1}{3}dt\right)$

$= \frac{1}{3}\left[\frac{p^{51}}{51}\right] + C = \frac{1}{153}(3t-1)^{51} + C$

$\int_0^1 (3t-1)^{50}\,dt = \left[\frac{1}{153}(3t-1)^{51} + C\right]_0^1 = \left[\frac{1}{153}(3(1)-1)^{51} + C\right] - \left[\frac{1}{153}(3(0)-1)^{51} + C\right]$

$= \left[\frac{1}{153}(2)^{51}\right] - \left[\frac{1}{153}(-1)^{51}\right] = \frac{1}{153}\left\{(2)^{51} + 1\right\} = \frac{(2)^{51}+1}{153}$

62) $\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \csc^2\left(\frac{1}{2}t\right)dt$

$p = \frac{1}{2}t$

$dp = \frac{1}{2}dt$

$2dp = dt$

$\int \csc^2\left(\frac{1}{2}t\right)dt$

$= \int \csc^2 p\ (2dp) = 2[-\cot p] + C = -2\cot\left(\frac{1}{2}t\right) + C$

$\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \csc^2\left(\frac{1}{2}t\right)dt = \left[-2\cot\left(\frac{1}{2}t\right)+C\right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} = \left[-2\cot\left(\frac{1}{2}\left(\frac{2\pi}{3}\right)\right)+C\right] - \left[-2\cot\left(\frac{1}{2}\left(\frac{\pi}{3}\right)\right)+C\right]$

$= \left[-2\cot\left(\frac{\pi}{3}\right)\right] - \left[-2\cot\left(\frac{\pi}{6}\right)\right] = \left[-2\left(\frac{1}{\sqrt{3}}\right)\right] - \left[-2\left(\frac{\sqrt{3}}{1}\right)\right]$

$= \frac{-2}{\sqrt{3}} + 2\sqrt{3} = \frac{-2}{\sqrt{3}} + 2\sqrt{3}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{6-2}{\sqrt{3}} = \underline{\underline{\frac{4}{\sqrt{3}}}}$

64) $\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx$

$p = 2 + \sqrt{x} = 2 + x^{\frac{1}{2}}$

$dp = \frac{1}{2}x^{-\frac{1}{2}}dx$

$dp = \frac{1}{2\sqrt{x}}dx$

$2dp = \frac{1}{\sqrt{x}}dx$

$\int \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx = \int \sqrt{2+\sqrt{x}}\left(\frac{1}{\sqrt{x}}dx\right)$

$= \int \sqrt{p}\,(2dp) = \int 2p^{\frac{1}{2}}dp = 2\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{4}{3}\left(2+\sqrt{x}\right)^{\frac{3}{2}} + C$

$= \frac{4}{3}\left(\sqrt{2+\sqrt{x}}\right)^3 + C$

$\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}\,dx = \left[\frac{4}{3}\left(\sqrt{2+\sqrt{x}}\right)^3 + C\right]_1^4 = \left[\frac{4}{3}\left(\sqrt{2+\sqrt{(4)}}\right)^3 + C\right] - \left[\frac{4}{3}\left(\sqrt{2+\sqrt{(1)}}\right)^3 + C\right]$

$= \left[\frac{4}{3}\left(\sqrt{2+2}\right)^3\right] - \left[\frac{4}{3}\left(\sqrt{2+1}\right)^3\right] = \left[\frac{4}{3}(2)^3\right] - \left[\frac{4}{3}\left(\sqrt{3}\right)^3\right]$

$= \left[\frac{32}{3}\right] - \left[\frac{4}{3}\left(3\sqrt{3}\right)\right] = \underline{\underline{\frac{32-12\sqrt{3}}{3}}}$

66) $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$

$p = e^x$

$dp = e^x\,dx$

$\boxed{5.5/10}$

$$\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{1}{1+(e^x)^2}\,(e^x\,dx) = \int \frac{1}{1+p^2}\,dp = \left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right] + C$$

$$= \tan^{-1}(e^x) + C$$

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \left[\tan^{-1}(e^x) + C\right]_0^1 = \left[\tan^{-1}(e^{(1)}) + C\right] - \left[\tan^{-1}(e^{(0)}) + C\right]$$

$$= \left[\tan^{-1}(e)\right] - \left[\tan^{-1}(1)\right] = \underline{\underline{\tan^{-1}(e) - \frac{\pi}{4}}}$$

68) $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx$

$p = \sin x$

$dp = \cos x\,dx$

$$\int \cos x \sin(\sin x)\,dx = \int \sin(\sin x)\,(\cos x\,dx) = \int \sin p\,dp$$

$$= \left[-\cos p\right] + C = -\cos(\sin x) + C$$

$$\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx = \left[-\cos(\sin x) + C\right]_0^{\frac{\pi}{2}}$$

$$= \left[-\cos\left(\sin\left(\frac{\pi}{2}\right)\right) + C\right] - \left[-\cos(\sin(0)) + C\right]$$

$$= \left[-\cos(1)\right] - \left[-\cos(0)\right] = \left[-\cos(1)\right] - \left[-(1)\right]$$

$$= \underline{\underline{1 - \cos(1)}}$$

70) $\int_0^a x\sqrt{a^2-x^2}\,dx$ $\qquad$ $p=a^2-x^2$, $dp=-2x\,dx$, $\frac{-1}{2}dp = x\,dx$ $\qquad$ 5.5/11

$$\int x\sqrt{a^2-x^2}\,dx = \int \sqrt{a^2-x^2}\,(x\,dx)$$

$$= \int \sqrt{p}\left(\frac{-1}{2}dp\right) = \int \frac{-1}{2}p^{\frac{1}{2}}dp = \frac{-1}{2}\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right]+C = \frac{-1}{3}\left(a^2-x^2\right)^{\frac{3}{2}}+C$$

$$= \frac{-1}{3}\left(\sqrt{a^2-x^2}\right)^3+C$$

$$\int_0^a x\sqrt{a^2-x^2}\,dx = \left[\frac{-1}{3}\left(\sqrt{a^2-x^2}\right)^3+C\right]_0^a = \left[\frac{-1}{3}\left(\sqrt{a^2-(a)^2}\right)^3+C\right]-\left[\frac{-1}{3}\left(\sqrt{a^2-(0)^2}\right)^3+C\right]$$

$$= \left[\frac{-1}{3}\left(\sqrt{0}\right)^3\right]-\left[\frac{-1}{3}\left(\sqrt{a^2}\right)^3\right] = [0]-\left[\frac{-1}{3}a^3\right] = \underline{\underline{\frac{1}{3}a^3}}$$

72) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^4 \sin x\,dx$ this should be in section 7.1 (Integration by parts)

74) $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$ $\qquad$ $p=1+2x \longrightarrow p-1=2x$, $\frac{p-1}{2}=x$; $dp=2\,dx$, $\frac{1}{2}dp=dx$

* triple substitution

$$\int \frac{x}{\sqrt{1+2x}}\,dx$$

$$= \int \frac{\left(\frac{p-1}{2}\right)}{\sqrt{p}}\left(\frac{1}{2}dp\right) = \int \frac{1}{4}\left(\frac{p-1}{p^{\frac{1}{2}}}\right)dp = \int \frac{1}{4}\left(p^{\frac{1}{2}}-p^{-\frac{1}{2}}\right)dp$$

$$= \frac{1}{4}\left\{\left[\frac{p^{\frac{3}{2}}}{\frac{3}{2}}\right]-\left[\frac{p^{\frac{1}{2}}}{\frac{1}{2}}\right]\right\}+C = \frac{1}{4}\left\{\frac{2}{3}(1+2x)^{\frac{3}{2}}-2(1+2x)^{\frac{1}{2}}\right\}+C$$

$$= \frac{1}{6}\left(\sqrt{1+2x}\right)^3-\frac{1}{2}\sqrt{1+2x}+C$$

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx = \left[\frac{1}{6}\left(\sqrt{1+2x}\right)^3-\frac{1}{2}\sqrt{1+2x}+C\right]_0^4 = \left[\frac{1}{6}\left(\sqrt{1+2(4)}\right)^3-\frac{1}{2}\sqrt{1+2(4)}+C\right]-\left[\frac{1}{6}\left(\sqrt{1+2(0)}\right)^3-\frac{1}{2}\sqrt{1+2(0)}+C\right]$$

$$= \left[\frac{1}{6}(\sqrt{9})^3-\frac{1}{2}(\sqrt{9})\right]-\left[\frac{1}{6}(\sqrt{1})^3-\frac{1}{2}(\sqrt{1})\right] = \left[\frac{27}{6}-\frac{3}{2}\right]-\left[\frac{1}{6}-\frac{1}{2}\right] = \left[\frac{27}{6}-\frac{9}{6}\right]-\left[\frac{1}{6}-\frac{3}{6}\right]$$

$$= \left[\frac{18}{6}\right]-\left[\frac{-2}{6}\right] = \frac{20}{6} = \underline{\underline{\frac{10}{3}}}$$

5.5/12

76) $\int_0^2 (x-1)e^{(x-1)^2}\,dx$

$p=(x-1)^2$

$dp = 2(x-1)\,dx$

$\frac{1}{2}dp = (x-1)dx$

$$\int (x-1)e^{(x-1)^2}dx$$

$$= \int e^{(x-1)^2}(x-1)dx = \int e^{p}\left(\tfrac{1}{2}dp\right) = \tfrac{1}{2}\left[e^{p}\right]+C = \tfrac{1}{2}e^{(x-1)^2}+C$$

$$\int_0^2 (x-1)e^{(x-1)^2}dx = \left[\tfrac{1}{2}e^{(x-1)^2}+C\right]_0^2 = \left[\tfrac{1}{2}e^{((2)-1)^2}+C\right]-\left[\tfrac{1}{2}e^{((0)-1)^2}+C\right]$$

$$= \left[\tfrac{1}{2}e^{(1)^2}\right]-\left[\tfrac{1}{2}e^{(-1)^2}\right] = \left[\tfrac{1}{2}e\right]-\left[\tfrac{1}{2}e\right] = \underline{\underline{0}}$$

---

78) $\int_1^4 \frac{1}{(x+1)\sqrt{x}}\,dx$

$p = \sqrt{x} = x^{\frac{1}{2}}$

$dp = \frac{1}{2}x^{-\frac{1}{2}}dx$

$dp = \frac{1}{2\sqrt{x}}dx$

$2dp = \frac{1}{\sqrt{x}}dx$

$$\int \frac{1}{(x+1)\sqrt{x}}dx = \int \frac{1}{((\sqrt{x})^2+1^2)}\left(\frac{1}{\sqrt{x}}dx\right)$$

$$= \int \frac{1}{p^2+1^2}(2dp) = 2\left[\frac{1}{1}\tan^{-1}\left(\frac{p}{1}\right)\right]+C = 2\tan^{-1}(\sqrt{x})+C$$

$$\int_1^4 \frac{1}{(x+1)\sqrt{x}}dx = \left[2\tan^{-1}(\sqrt{x})+C\right]_1^4 = \left[2\tan^{-1}(\sqrt{(4)})+C\right]-\left[2\tan^{-1}(\sqrt{(1)})+C\right]$$

$$= \left[2\tan^{-1}(2)\right]-\left[2\tan^{-1}(1)\right] = 2\tan^{-1}(2)-2\left(\frac{\pi}{4}\right)$$

$$= \underline{\underline{2\tan^{-1}(2)-\frac{\pi}{2}}}$$