21200 {01} section 3.8

6-a) P: population in millions, t: time in years

1950	$t=0$	$P(0)=83 \Rightarrow P_0$	$\boxed{P(t)=P_0 e^{rt}}$
1960	$t=10$	$P(10)=100$	

$$P(t)=(83)e^{rt}$$

$$100=P(10)=83e^{r(10)}$$

$$100=83e^{10r}$$

$$\frac{100}{83}=e^{10r}$$

$$\Downarrow$$

$$\ln\left(\frac{100}{83}\right)=10r$$

$$r=\frac{\ln\left(\frac{100}{83}\right)}{10}$$

$$P(t)=83e^{\left(\frac{\ln\left(\frac{100}{83}\right)}{10}\right)t}$$

$1980 \rightarrow P(30)=?$

$$P(30)=83e^{\left(\frac{\ln\left(\frac{100}{83}\right)}{10}\right)(30)}$$

$$=83e^{3\ln\left(\frac{100}{83}\right)}$$

$$=83e^{\ln\left(\frac{100}{83}\right)^3}$$

$$=83\left(\frac{100}{83}\right)^3=\frac{(100)^3}{(83)^2} \text{ millions}$$

$P(30)=145.158949$ millions {calc.}

actual: $1980 \rightarrow 150$ millions

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10) $P(t)=P_0e^{rt}$

let P_0 be original mass, t: time in days

decayed to 64.3% of its original amount after 300 days $\longrightarrow P(300)=0.643P_0$

we can use the same formula used in example above

10) continued...

$$0.643P_0 = P(300) = P_0 e^{r(300)}$$

$$0.643P_0 = P_0 e^{300r}$$

$$0.643 = e^{300r}$$

$$\Downarrow$$

$$\ln(0.643) = 300r$$

$$r = \frac{\ln(0.643)}{300}$$

$$P(t) = P_0 e^{\left(\frac{\ln(0.643)}{300}\right)t}$$

a) $P(t) = 0.5P_0$, $t = ?$

$$0.5P_0 = P(t) = P_0 e^{\left(\frac{\ln(0.643)}{300}\right)t}$$

$$0.5 = e^{\left(\frac{\ln(0.643)}{300}\right)t}$$

$$\Downarrow$$

$$\ln(0.5) = \left(\frac{\ln(0.643)}{300}\right)t$$

$$t = \frac{300\ln(0.5)}{\ln(0.643)} \text{ days}$$

$t = 470.8767758$ days {calc.}

b) $P(t) = \frac{P_0}{3}$, $t = ?$

$$\frac{P_0}{3} = P(t) = P_0 e^{\left(\frac{\ln(0.643)}{300}\right)t}$$

$$\frac{1}{3} = e^{\left(\frac{\ln(0.643)}{300}\right)t}$$

$$\Downarrow$$

$$\ln\left(\frac{1}{3}\right) = \left(\frac{\ln(0.643)}{300}\right)t$$

$$t = \frac{300\ln\left(\frac{1}{3}\right)}{\ln(0.643)} \text{ days}$$

$t = 746.3220321$ days {calc.}

3.8/3

16) T : temperature (Celcius) , T_0 : ambient temperature
t : time in hours $T_0 = 20°C$

Newton's Law of Cooling

$$\boxed{\frac{dT}{dt} = k(T - T_0)} \qquad \frac{dT}{dt} = k(T - 20)$$

$$\frac{1}{T-20}\, dT = k\, dt$$

$$\int \frac{1}{T-20}\, dT = \int k\, dt$$

$$\ln|T-20| = kt + c_1$$

$$\Downarrow$$

$$T - 20 = e^{(kt + c_1)}$$

$$T - 20 = (e^{kt})(e^{c_1})$$

$$T - 20 = C_2 e^{kt}$$

$$T = 20 + C_2 e^{kt}$$

$$T(t) = 20 + C_2 e^{kt}$$

now apply the informations of temperatures of the corpse at 2 different time to find C_2 and k

16) continued...

1:30 PM be $t=0$

$T(0) = 32.5\,°C$

$32.5 = T(0) = 20 + C_2 e^{k(0)}$

$32.5 = 20 + C_2$

$12.5 = C_2$

$T(t) = 20 + 12.5 e^{kt}$

2:30 PM $\rightarrow t = 1$

$T(1) = 30.3\,°C$

use the information above and result from left to find k

$30.3 = T(1) = 20 + 12.5 e^{k(1)}$

$30.3 = 20 + 12.5 e^{k}$

$10.3 = 12.5 e^{k}$

$\frac{10.3}{12.5} = e^{k}$

$\Downarrow$

$\ln\left(\frac{10.3}{12.5}\right) = k$

$T(t) = 20 + 12.5 e^{\left(\ln\left(\frac{10.3}{12.5}\right)\right)t}$

now find t when normal body temperature is 37°C.

$T(t) = 37°C$, $t = ?$

$37 = T(t) = 20 + 12.5 e^{\left(\ln\left(\frac{10.3}{12.5}\right)\right)t}$

$37 = 20 + 12.5 e^{\left(\ln\left(\frac{10.3}{12.5}\right)\right)t}$

$17 = 12.5 e^{\left(\ln\left(\frac{10.3}{12.5}\right)\right)t}$

$\frac{17}{12.5} = e^{\left(\ln\left(\frac{10.3}{12.5}\right)\right)t}$

$\Downarrow$

$\ln\left(\frac{17}{12.5}\right) = \left(\ln\left(\frac{10.3}{12.5}\right)\right)t$

$t = \frac{\ln\left(\frac{17}{12.5}\right)}{\ln\left(\frac{10.3}{12.5}\right)}$ hours "negative"

$t = -1.588372541$ hours before 1:30PM

$t \approx -(1$ hr 35 min$)$

Murder took place around 12:00PM "before 11:55 AM".