5.4 revised 11/22/2023

2) $\int x^3(2+x^4)^5\,dx$

$u = 2+x^4$

$du = 4x^3\,dx$

$\frac{1}{4}\,du = x^3\,dx$ revised 11/22/2023

$= \int (2+x^4)^5 (x^3\,dx)$

$= \int u^5 \left(\frac{1}{4}\,du\right)$

$= \frac{1}{4}\left[\frac{u^6}{6}\right] + C = \frac{1}{24}u^6 + C = \frac{1}{24}(2+x^4)^6 + C$

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4) $\int \frac{dt}{(1-6t)^4} = \int \frac{1}{(1-6t)^4}\,dt$

$u = 1-6t$

$du = -6\,dt$

$\frac{1}{-6}\,du = dt \Rightarrow \frac{-1}{6}\,du = dt$

$= \int \frac{1}{u^4}\left(\frac{-1}{6}\,du\right) = \frac{-1}{6}\int u^{-4}\,du$

$= \frac{-1}{6}\left[\frac{u^{-3}}{-3}\right] + C = \frac{1}{18u^3} + C = \frac{1}{18(1-6t)^3} + C$

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6) $\int \frac{r^2}{\sqrt{r^3+2}}\,dr$

$u = r^3+2$

$du = 3r^2\,dr$

$\frac{1}{3}\,du = r^2\,dr$

$= \int \frac{1}{\sqrt{r^3+2}}(r^2\,dr)$

$= \int \frac{1}{\sqrt{u}}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\int u^{-\frac{1}{2}}\,du$

$= \frac{1}{3}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = \frac{2}{3}\sqrt{u} + C = \frac{2}{3}\sqrt{r^3+2} + C$

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8) $\int x^2(x^3+5)^9\,dx = \int (x^3+5)^9(x^2\,dx)$ $u = x^3+5$ $du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$= \int u^9\left(\frac{1}{3}\,du\right) = \frac{1}{3}\left[\frac{u^{10}}{10}\right] + C$

$= \frac{1}{30}u^{10} + C = \frac{1}{30}(x^3+5)^{10} + C$

10) $\int (3t+2)^{2.4}\,dt$

$u = 3t+2$
$du = 3\,dt$
$\frac{1}{3}du = dt$

$= \int u^{2.4}\left(\frac{1}{3}du\right)$

$= \frac{1}{3}\left[\frac{u^{3.4}}{3.4}\right] + C = \frac{1}{10.2}u^{3.4} + C = \frac{1}{10.2}(3t+2)^{3.4} + C$

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12) $\int w^2\sqrt{6+2w^3}\,dw$

$u = 6+2w^3$
$du = 6w^2\,dw$
$\frac{1}{6}du = w^2\,dw$

$= \int \sqrt{6+2w^3}\,(w^2\,dw)$

$= \int \sqrt{u}\left(\frac{1}{6}du\right) = \frac{1}{6}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{6}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{1}{6}\left(\frac{2}{3}\right)\left(\sqrt{u}\right)^3 + C = \frac{1}{9}\left(\sqrt{6+2w^3}\right)^3 + C$

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14) $\int \frac{t}{\sqrt{2.7t^2+8}}\,dt$

$u = 2.7t^2+8$
$du = 5.4t\,dt$
$\frac{1}{5.4}du = t\,dt$

$= \int \frac{1}{\sqrt{2.7t^2+8}}\,(t\,dt)$

$= \int \frac{1}{\sqrt{u}}\left(\frac{1}{5.4}du\right) = \frac{1}{5.4}\int u^{-\frac{1}{2}}\,du = \frac{1}{5.4}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C$

$= \frac{2}{5.4}\sqrt{u} + C = \frac{2}{5.4}\sqrt{2.7t^2+8} + C$

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16) $\int \frac{x}{(x^2+1)^2}\,dx$

$u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \int \frac{1}{(x^2+1)^2}\,(x\,dx)$

$= \int \frac{1}{u^2}\left(\frac{1}{2}du\right) = \frac{1}{2}\int u^{-2}\,du$

$= \frac{1}{2}\left[\frac{u^{-1}}{-1}\right] + C = \frac{-1}{2u} + C = \frac{-1}{2(x^2+1)} + C$

18) $\int \frac{e^v}{(2e^v+5)^2}\,dv$

$u = 2e^v + 5$

$du = 2e^v\,dv$

$\frac{1}{2}du = e^v\,dv$

$= \int \frac{1}{(2e^v+5)^2}(e^v\,dv)$

$= \int \frac{1}{u^2}\left(\frac{1}{2}du\right) = \frac{1}{2}\int u^{-2}\,du$

$= \frac{1}{2}\left[\frac{u^{-1}}{-1}\right] + C = \frac{-1}{2u} + C = \underline{\underline{\frac{-1}{2(2e^v+5)} + C}}$

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20) $\int \frac{4}{x(\ln x)^2}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \frac{4}{(\ln x)^2}\left(\frac{1}{x}\,dx\right)$

$= \int \frac{4}{u^2}(du) = 4\int u^{-2}\,du$

$= 4\left[\frac{u^{-1}}{-1}\right] + C = \frac{-4}{u} + C = \underline{\underline{\frac{-4}{\ln x} + C}}$

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22) $\int p^2 e^{p^3+2}\,dp$

$u = p^3 + 2$

$du = 3p^2\,dp$

$\frac{1}{3}du = p^2\,dp$

$= \int e^{p^3+2}(p^2\,dp)$

$= \int e^u\left(\frac{1}{3}du\right)$

$= \frac{1}{3}\left[e^u\right] + C = \underline{\underline{\frac{1}{3}e^{p^3+2} + C}}$

24) $\int \frac{z^2}{2z^3-10}\,dz$

$u = 2z^3 - 10$

$du = 6z^2\,dz$

$\frac{1}{6}du = z^2\,dz$

$= \int \frac{1}{2z^3-10}(z^2 dz)$

$= \int \frac{1}{u}\left(\frac{1}{6}du\right)$

$= \frac{1}{6}\left[\ln|u|\right] + C = \underline{\underline{\frac{1}{6}\ln|2z^3-10| + C}}$

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26) $\int \frac{e^x}{e^x+1}\,dx$

$u = e^x + 1$

$du = e^x\,dx$

$= \int \frac{1}{e^x+1}(e^x dx)$

$= \int \frac{1}{u}(du)$

$= \left[\ln|u|\right] + C = \ln|e^x+1| + C = \underline{\underline{\ln(e^x+1) + C}}$

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28) $\int 3^{x^2+5}\,x\,dx$

$u = x^2 + 5$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int 3^{x^2+5}(x\,dx)$

$= \int 3^u\left(\frac{1}{2}du\right)$

$= \frac{1}{2}\left[\frac{3^u}{\ln(3)}\right] + C \longleftarrow$ $\int a^u\,du = \frac{a^u}{\ln a} + C$ formula used

$= \underline{\underline{\frac{1}{2\ln 3}\,3^{x^2+5} + C}}$

30) $\displaystyle\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx$

$u = 3ax + bx^3$

$du = (3a + 3bx^2)\,dx$

$du = 3(a + bx^2)\,dx$

$\frac{1}{3}du = (a+bx^2)\,dx$

$$= \int \frac{1}{\sqrt{3ax+bx^3}}\left((a+bx^3)\,dx\right)$$

$$= \int \frac{1}{\sqrt{u}}\left(\frac{1}{3}du\right) = \frac{1}{3}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{3}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = \frac{2}{3}\sqrt{u} + C = \underline{\underline{\frac{2}{3}\sqrt{3ax+bx^3} + C}}$$

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34) $\displaystyle\int_0^1 (3t-1)^{50}\,dt$

$\displaystyle\int (3t-1)^{50}\,dt$

$u = 3t-1$

$du = 3\,dt$

$\frac{1}{3}du = dt$

$$= \int u^{50}\left(\frac{1}{3}du\right)$$

$$= \frac{1}{3}\left[\frac{u^{51}}{51}\right] + C = \frac{1}{(3)(51)}(3t-1)^{51} + C$$

$$\int_0^1 (3t-1)^{50}\,dt = \left[\frac{1}{(3)(51)}(3t-1)^{51} + C\right]_0^1$$

$$= \left[\frac{1}{(3)(51)}(3(1)-1)^{51} + C\right] - \left[\frac{1}{(3)(51)}(3(0)-1)^{51} + C\right]$$

$$= \left[\frac{1}{(3)(51)}(2)^{51}\right] - \left[\frac{1}{(3)(51)}(-1)^{51}\right]$$

$$= \left[\frac{(2)^{51}}{(3)(51)}\right] - \left[\frac{-1}{(3)(51)}\right]$$

$$= \underline{\underline{\frac{(2)^{51}+1}{(3)(51)}}}$$

← this is an extremely large value

36) $\int_0^2 t^2\sqrt{8-t^3}\,dt$

$$\int t^2\sqrt{8-t^3}\,dt$$

$u = 8-t^3$

$du = -3t^2\,dt$

$-\frac{1}{3}du = t^2\,dt$

$$= \int \sqrt{8-t^3}\,(t^2\,dt)$$

$$= \int \sqrt{u}\left(-\tfrac{1}{3}du\right) = -\tfrac{1}{3}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{3}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C = \frac{-2}{9}\left(\sqrt{u}\right)^3 + C = \frac{-2}{9}\left(\sqrt{8-t^3}\right)^3 + C$$

$$\int_0^2 t^2\sqrt{8-t^3}\,dt = \left[\frac{-2}{9}\left(\sqrt{8-t^3}\right)^3 + C\right]_0^2$$

revised 11/22/2023

$$= \left[\frac{-2}{9}\left(\sqrt{8-(2)^3}\right)^3 + C\right] - \left[\frac{-2}{9}\left(\sqrt{8-(0)^3}\right)^3 + C\right]$$

$$= \left[\frac{-2}{9}\left(\sqrt{8-8}\right)^3\right] - \left[\frac{-2}{9}\left(\sqrt{8}\right)^3\right] = \left[\frac{-2}{9}(0)^3\right] - \left[\frac{-2}{9}\left(2\sqrt{2}\right)^3\right]$$

$$= [0] - \left[\frac{-2}{9}(8)(\sqrt{2})^3\right] = -\left[\frac{-16}{9}(\sqrt{2})(\sqrt{2})(\sqrt{2})\right] = \frac{16}{9}(2)\sqrt{2} = \underline{\underline{\frac{32\sqrt{2}}{9}}}$$

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40) $\int_0^1 \frac{x^3}{3x^4+1}\,dx$

$$\int \frac{x^3}{3x^4+1}\,dx$$

$u = 3x^4+1$

$du = 9x^3\,dx$

$\frac{1}{9}du = x^3\,dx$

$$= \int \frac{1}{3x^4+1}\left(x^3\,dx\right)$$

$$= \int \frac{1}{u}\left(\frac{1}{9}du\right)$$

$$= \frac{1}{9}\left[\ln|u|\right] + C = \frac{1}{9}\ln\left|3x^4+1\right| + C$$

40) continued...

$$\int_0^1 \frac{x^3}{3x^4+1}\,dx = \left[\frac{1}{9}\ln|3x^4+1|+C\right]_0^1$$

$$= \left[\frac{1}{9}\ln|3(1)^4+1|+C\right] - \left[\frac{1}{9}\ln|3(0)^4+1|+C\right]$$

$$= \left[\frac{1}{9}\ln|4|\right] - \left[\frac{1}{9}\ln|1|\right] = \left[\frac{1}{9}\ln 4\right] - \left[\frac{1}{9}(0)\right] = \underline{\underline{\frac{1}{9}\ln 4}}$$

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38) $\displaystyle\int_0^2 \frac{x}{\sqrt{2x^2+1}}\,dx$

$$\int \frac{x}{\sqrt{2x^2+1}}\,dx$$

$u = 2x^2+1$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$$= \int \frac{1}{\sqrt{2x^2+1}}(x\,dx)$$

$$= \int \frac{1}{\sqrt{u}}\left(\frac{1}{4}du\right) = \frac{1}{4}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{4}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C = \frac{2}{4}\sqrt{u} + C = \frac{1}{2}\sqrt{2x^2+1} + C$$

$$\int_0^2 \frac{x}{\sqrt{2x^2+1}}\,dx = \left[\frac{1}{2}\sqrt{2x^2+1}+C\right]_0^2$$

$$= \left[\frac{1}{2}\sqrt{2(2)^2+1}+C\right] - \left[\frac{1}{2}\sqrt{2(0)^2+1}+C\right]$$

$$= \left[\frac{1}{2}\sqrt{9}\right] - \left[\frac{1}{2}\sqrt{1}\right]$$

$$= \left[\frac{1}{2}(3)\right] - \left[\frac{1}{2}(1)\right] = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = \underline{\underline{1}}$$