5.2 revised 11/13/2024

2) $f(x) = 1 - x^3 + 12x^5$

$$\int (1 - x^3 + 12x^5)\,dx = [x] - \left[\frac{x^4}{4}\right] + 12\left[\frac{x^6}{6}\right] + C$$

$$= x - \frac{1}{4}x^4 + 2x^6 + C$$

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4) $g(u) = 2u + \sqrt[3]{u}$

$$\int (2u + \sqrt[3]{u})\,du = \int (2u + u^{\frac{1}{3}})\,du$$

$$= 2\left[\frac{u^2}{2}\right] + \left[\frac{u^{\frac{4}{3}}}{\frac{4}{3}}\right] + C = u^2 + \frac{3}{4}u^{\frac{4}{3}} + C$$

$$= u^2 + \frac{3}{4}\left(\sqrt[3]{u}\right)^4 + C$$

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6) $p(t) = 9.2t^2 - \frac{4}{t}$

$$\int \left(9.2t^2 - \frac{4}{t}\right)dt = 9.2\left[\frac{t^3}{3}\right] - 4\left[\ln|t|\right] + C$$

$$= \frac{9.2}{3}t^3 - 4\ln|t| + C$$

10) $f(x) = \frac{x^2+x+1}{x}$

$$\int \left(\frac{x^2+x+1}{x}\right) dx = \int \left(\frac{x^2}{x} + \frac{x}{x} + \frac{1}{x}\right) dx$$

$$= \int \left(x + 1 + \frac{1}{x}\right) dx = \left[\frac{x^2}{2}\right] + [x] + [\ln|x|] + C$$

$$= \frac{1}{2}x^2 + x + \ln|x| + C$$

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14) $g'(t) = 4.1e^{-0.3t}$

for this we need to use substitution technique (section 5.4).

$$g(t) = \int g'(t)\,dt$$

$$= \int 4.1e^{-0.3t}\,dt$$

$u = -0.3t$

$du = -0.3\,dt$

$\frac{1}{-0.3}\,du = dt$

$$= \int 4.1e^{u}\left(\frac{1}{-0.3}\,du\right)$$

$$= \frac{4.1}{-0.3}e^{u} + C$$

$$= \frac{-4.1}{0.3}e^{-0.3t} + C$$

16) $f'(x) = 2x - \frac{3}{x^4}$, $x > 0$, $f(1) = 3$

$$f(x) = \int f'(x)\,dx = \int \left(2x - \frac{3}{x^4}\right)dx = \int \left(2x - 3x^{-4}\right)dx$$

$$= 2\left[\frac{x^2}{2}\right] - 3\left[\frac{x^{-3}}{-3}\right] + C = x^2 + x^{-3} + C = x^2 + \frac{1}{x^3} + C$$

$$(3) = (1)^2 + \frac{1}{(1)^3} + C$$

$$(3) = 2 + C \quad \Rightarrow f(x) = x^2 + \frac{1}{x^3} + (1)$$

$$1 = C \qquad = \underline{\underline{x^2 + \frac{1}{x^3} + 1}}$$

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18) $f''(x) = 4 - 6x - 40x^3$, $f(0) = 2$, $f'(0) = 1$

$$f'(x) = \int f''(x)\,dx = \int (4 - 6x - 40x^3)\,dx$$

$$= 4[x] - 6\left[\frac{x^2}{2}\right] - 40\left[\frac{x^4}{4}\right] + C = 4x - 3x^2 - 10x^4 + C$$

$$(1) = 4(0) - 3(0)^2 - 10(0)^4 + C$$

$$\Rightarrow f'(x) = 4x - 3x^2 - 10x^4 + (1)$$

$$1 = C$$

$$f(x) = \int f'(x)\,dx = \int (4x - 3x^2 - 10x^4 + 1)\,dx$$

$$= 4\left[\frac{x^2}{2}\right] - 3\left[\frac{x^3}{3}\right] - 10\left[\frac{x^5}{5}\right] + [x] + D = 2x^2 - x^3 - 2x^5 + x + D$$

$$(2) = 2(0)^2 - (0)^3 - 2(0)^5 + (0) + D$$

$$2 = D \qquad \Rightarrow \underline{\underline{f(x) = 2x^2 - x^3 - 2x^5 + x + (2)}}$$

20) $a(t) = 5 + 4t - 2t^2$, $v(0) = 3\ m/sec$, $s(0) = 10\ m$

$$v(t) = \int a(t)\,dt = \int (5 + 4t - 2t^2)\,dt = 5[t] + 4\left[\frac{t^2}{2}\right] - 2\left[\frac{t^3}{3}\right] + C$$
$$= 5t + 2t^2 - \frac{2}{3}t^3 + C$$
$$(3) = 5(0) + 2(0)^2 - \frac{2}{3}(0)^3 + C$$
$$3 = C \Rightarrow v(t) = 5t + 2t^2 - \frac{2}{3}t^3 + (3)$$

$$s(t) = \int v(t)\,dt = \int \left(5t + 2t^2 - \frac{2}{3}t^3 + 3\right)dt$$
$$= 5\left[\frac{t^2}{2}\right] + 2\left[\frac{t^3}{3}\right] - \frac{2}{3}\left[\frac{t^4}{4}\right] + 3[t] + D$$
$$= \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + D$$
$$(10) = \frac{5}{2}(0)^2 + \frac{2}{3}(0)^3 - \frac{1}{6}(0)^4 + 3(0) + D$$
$$10 = D \Rightarrow \underline{\underline{s(t) = \frac{5}{2}t^2 + \frac{2}{3}t^3 - \frac{1}{6}t^4 + 3t + (10)}}$$

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22) $$\int_5^9 (t^2 + 1)\,dt = \left[\frac{t^3}{3} + t + C\right]_5^9$$
$$= \left[\frac{(9)^3}{3} + (9) + C\right] - \left[\frac{(5)^3}{3} + (5) + C\right]$$
$$= \left[\frac{729}{3} + 9\right] - \left[\frac{125}{3} + 5\right] = \frac{729}{3} + 9 - \frac{125}{3} - 5$$
$$= \frac{729}{3} + 4 - \frac{125}{3} = \frac{729}{3} + \frac{12}{3} - \frac{125}{3} = \frac{741 - 125}{3} = \underline{\underline{\frac{616}{3}}}$$

24) $\int_1^3 (1+2x-4x^3)\,dx = \left[x + x^2 - x^4 + C\right]_1^3$

$= \left[(3)+(3)^2-(3)^4+C\right] - \left[(1)+(1)^2-(1)^4+C\right]$

$= \left[3+9-81\right] - \left[1+1-1\right] = 12 - 82 = \underline{\underline{-70}}$

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26) $\int_{-2}^0 (u^5 - u^3 + u^2)\,du = \left[\frac{u^6}{6} - \frac{u^4}{4} + \frac{u^3}{3} + C\right]_{-2}^0$

$= \left[\frac{(0)^6}{6} - \frac{(0)^4}{4} + \frac{(0)^3}{3} + C\right] - \left[\frac{(-2)^6}{6} - \frac{(-2)^4}{4} + \frac{(-2)^3}{3} + C\right]$

$= \left[0 - 0 + 0\right] - \left[\frac{2(32)}{6} - 4 - \frac{8}{3}\right] = -\left[\frac{32}{3} - \frac{12}{3} - \frac{8}{3}\right] = \frac{-12}{3} = \underline{\underline{-4}}$

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28) $\int_1^8 \sqrt[3]{x}\,dx = \int_1^8 x^{\frac{1}{3}}\,dx = \left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C\right]_1^8 = \left[\frac{3}{4}(\sqrt[3]{x})^4 + C\right]_1^8$

$= \left[\frac{3}{4}\left(\sqrt[3]{(8)}\right)^4 + C\right] - \left[\frac{3}{4}\left(\sqrt[3]{(1)}\right)^4 + C\right] = \left[\frac{3}{4}(16)\right] - \left[\frac{3}{4}\right]$

$= \frac{48}{4} - \frac{3}{4} = \underline{\underline{\frac{45}{4}}}$

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32) $\int_{-2}^2 (3u+1)^2\,du = \int_{-2}^2 (9u^2 + 6u + 1)\,du = \left[3u^3 + 3u^2 + u + C\right]_{-2}^2$

$= \left[3(2)^3 + 3(2)^2 + (2) + C\right] - \left[3(-2)^3 + 3(-2)^2 + (-2) + C\right]$

$= \left[24 + 12 + 2\right] - \left[-24 + 12 - 2\right]$

$= 24 + 12 + 2 + 24 - 12 + 2 = \underline{\underline{52}}$

34) $\int_{-2}^{-1}\left(4y^3+\frac{2}{y^3}\right)dy = \int_{-2}^{-1}\left(4y^3+2y^{-3}\right)dy$

$= \left[y^4+2\left[\frac{y^{-2}}{-2}\right]+C\right]_{-2}^{-1} = \left[y^4-\frac{1}{y^2}+C\right]_{-2}^{-1}$

$= \left[(-1)^4-\frac{1}{(-1)^2}+C\right]-\left[(-2)^4-\frac{1}{(-2)^2}+C\right]$

$= [1-1]-\left[16-\frac{1}{4}\right] = [0]-\left[\frac{64}{4}-\frac{1}{4}\right] = \underline{\underline{\frac{-63}{4}}}$

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36) $\int_1^9 \frac{3x-2}{\sqrt{x}}dx = \int_1^9\left(\frac{3x}{\sqrt{x}}-\frac{2}{\sqrt{x}}\right)dx = \int_1^9\left(3\sqrt{x}-\frac{2}{\sqrt{x}}\right)dx$

$= \int_1^9\left(3x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)dx = \left[3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]-2\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]+C\right]_1^9$

$= \left[2\left(\sqrt{x}\right)^3-4\sqrt{x}+C\right]_1^9 = \left[2\left(\sqrt{(9)}\right)^3-4\sqrt{(9)}+C\right]-\left[2\left(\sqrt{(1)}\right)^3-4\sqrt{(1)}+C\right]$

$= [54-12]-[2-4] = [42]-[-2] = \underline{\underline{44}}$

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38) $\int_1^2\left(x+\frac{1}{x}\right)dx = \left[\frac{x^2}{2}+\ln|x|+C\right]_1^2$

$= \left[\frac{(2)^2}{2}+\ln|(2)|+C\right]-\left[\frac{(1)^2}{2}+\ln|(1)|+C\right]$

$= [2+\ln(2)]-\left[\frac{1}{2}+\ln(1)\right]$

$= [2+\ln(2)]-\left[\frac{1}{2}+0\right] = \underline{\underline{\frac{3}{2}+\ln(2)}}$

42) $y = \frac{3}{x} + 2x^3$, $2 \le x \le 3$

$$A = \int_2^3 \left(\frac{3}{x} + 2x^3\right) dx = \left[3\ln|x| + \frac{1}{2}x^4 + C\right]_2^3$$

$$= \left[3\ln|(3)| + \frac{1}{2}(3)^4 + C\right] - \left[3\ln|(2)| + \frac{1}{2}(2)^4 + C\right]$$

$$= \left[3\ln(3) + \frac{81}{2}\right] - \left[3\ln(2) + \frac{16}{2}\right]$$

$$= \left[\ln(3^3) + \frac{81}{2}\right] - \left[\ln(2^3) + \frac{16}{2}\right]$$

$$= \left[\ln(27) + \frac{81}{2}\right] - \left[\ln(8) + \frac{16}{2}\right] = \frac{65}{2} + \ln(27) - \ln(8)$$

$$= \underline{\underline{\frac{65}{2} + \ln\left(\frac{27}{8}\right)}}$$

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52) $\int (8x - 3)\,dx = 8\left[\frac{x^2}{2}\right] - 3[x] + C = \underline{\underline{4x^2 - 3x + C}}$

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54) $\int (9.3 - 1.5v^2)\,dv = 9.3[v] - 1.5\left[\frac{v^3}{3}\right] + C = \underline{\underline{9.3v - 0.5v^3 + C}}$

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56) $\int (x + e^x)\,dx = \left[\frac{x^2}{2}\right] + \left[e^x\right] + C = \underline{\underline{\frac{1}{2}x^2 + e^x + C}}$

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60) $\int \left(\frac{3}{s} - \frac{2}{s^2}\right) ds = \int \left(\frac{3}{s} - 2s^{-2}\right) ds$

$= 3\left[\ln|s|\right] - 2\left[\frac{s^{-1}}{-1}\right] + C = \underline{\underline{3\ln|s| + \frac{2}{s} + C}}$

62) $\int x(1+2x^4)\,dx = \int (x+2x^5)\,dx$

$= \left[\frac{x^2}{2}\right] + 2\left[\frac{x^6}{6}\right] + C = \underline{\underline{\frac{1}{2}x^2 + \frac{1}{3}x^6 + C}}$