4) the time intervals are 12 second apart

a) using velocities on left

distance : $L_5 = (30 \text{ ft/sec})(12 \text{ sec}) + (28)(12) + (25)(12) + (22)(12) + (24)(12)$

$= 360 + 336 + 300 + 264 + 288 = 1548 \text{ ft}$

b) using velocities on right

distance: $R_5 = (28 \text{ ft/sec})(12) + (25)(12) + (22)(12) + (24)(12) + (27)(12)$

$= 336 + 300 + 264 + 288 + 324 = 1512 \text{ ft}$

c) the estimate of part a may be too high because the motorcycle had only one occasion of maximum velocity of 30 ft/set at the start.
We can safely assume that this motorcycle travelled at least 1500 ft.

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8) using midpoint approximation and intervals of 10 sec.

$M_3 = (\approx 50 \text{ km/hr})(10 \text{ sec}) + (\approx 95 \text{ km/hr})(10 \text{ sec}) + (\approx 115 \text{ km/hr})(10 \text{ sec})$

height at $t=5$ height at $t=15$ height at $t=25$

$= \approx 500 + \approx 950 + \approx 1150 = \approx \underline{\underline{2600 \text{ km}}}$

18) a) $\int_0^2 g(x)\,dx$ this is an area of a triangle with sides 2 units horizontally and 4 units vertically.

$A_L = \int_0^2 g(x)\,dx = \frac{1}{2}(2)(4) = 4 \text{ units}^2$

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b) $\int_2^6 g(x)\,dx$ this is an area of half of a circle with radius 2.

$A_M = \int_2^6 g(x)\,dx = \frac{1}{2}(\pi(2)^2) = \frac{1}{2}(4\pi) = 2\pi \text{ units}^2$

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c) $\int_0^7 g(x)\,dx$ this contains 2 areas of part a and b. The leftover $\int_6^7 g(x)\,dx$ is another triangle with sides 1 unit horizontally and 1 unit vertically.

$A_R = \int_6^7 g(x)\,dx = \frac{1}{2}(1)(1) = \frac{1}{2} \text{ units}^2$

$$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$$

$$= A_L + A_M + A_R$$

$$= 4 \text{ units}^2 + 2\pi \text{ units}^2 + \frac{1}{2} \text{ units}^2$$

$$= \underline{\underline{\left(\frac{9}{2} + 2\pi\right) \text{ units}^2}}$$

20) $\int_0^3 \left(\frac{1}{2}x - 1\right) dx$

$\frac{1}{2}x - 1$ is a linear function and this will be an area of a triangle with $(0, -1)$ and $(3, \frac{1}{2})$; so horizontally 3 units and vertically $\frac{3}{2}$ units.

$$\int_0^3 \left(\frac{1}{2}x - 1\right) dx = \frac{1}{2}(3)\left(\frac{3}{2}\right) = \underline{\underline{\frac{9}{4} \text{ units}^2}}$$

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22) $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

$\sqrt{4-x^2}$ traces the upper arc of a circle from -2 to 2; so this will be an area of half circle of radius 2.

$$\int_{-2}^{2} \sqrt{4-x^2}\, dx = \frac{1}{2}\left(\pi (2)^2\right) = \underline{\underline{2\pi \text{ units}^2}}$$

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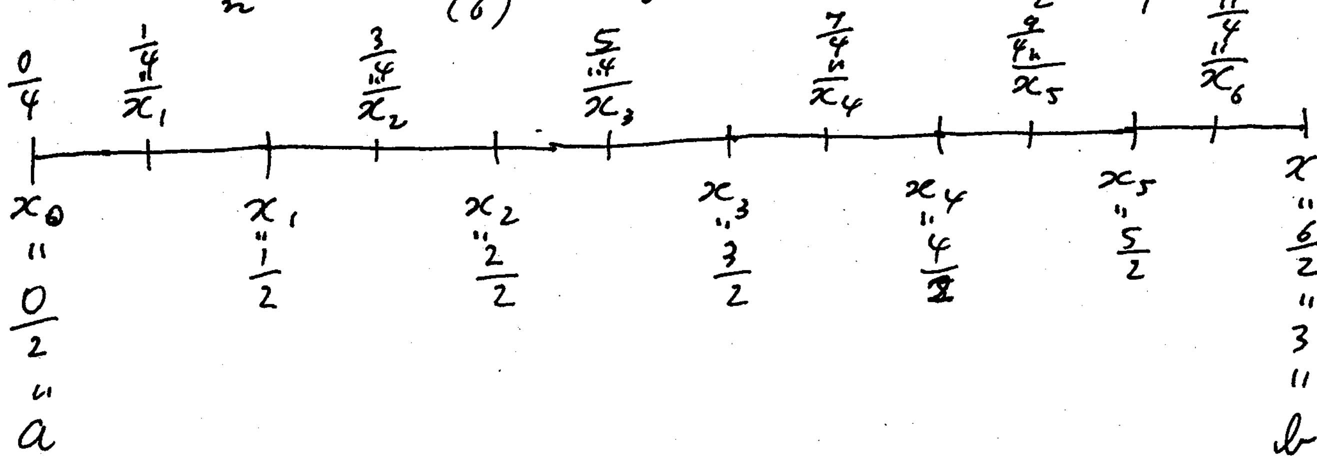
26) $\int_0^3 \ln(x^2+3)\, dx$, $\quad n = 6 \quad$ $f(x) = \ln(x^2+3)$

$$\Delta x = \frac{b-a}{n} = \frac{(3)-(0)}{(6)} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4} \qquad \frac{\Delta x}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$



26) continued

$$L_6 = \Delta x\left[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right]$$

$$= \left(\tfrac{1}{2}\right)\left\{\ln\left(\left(\tfrac{0}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{1}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{2}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{3}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{4}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{5}{2}\right)^2+3\right.\right.$$

$$R_6 = \Delta x\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)\right]$$

$$= \left(\tfrac{1}{2}\right)\left\{\ln\left(\left(\tfrac{1}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{2}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{3}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{4}{2}\right)^2+3\right) + \ln\left(\left(\tfrac{5}{3}\right)^2+3\right) + \ln\left(\left(\tfrac{6}{2}\right)^2+3\right)\right]$$

$$M_6 = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4) + f(\bar{x}_5) + f(\bar{x}_6)\right]$$

$$= \left(\tfrac{1}{2}\right)\left\{\ln\left(\left(\tfrac{1}{4}\right)^2+3\right) + \ln\left(\left(\tfrac{3}{4}\right)^2+3\right) + \ln\left(\left(\tfrac{5}{4}\right)^2+3\right) + \ln\left(\left(\tfrac{7}{4}\right)^2+3\right) + \ln\left(\left(\tfrac{9}{4}\right)^2+3\right) + \ln\left(\left(\tfrac{11}{4}\right)^2+3\right)\right]$$

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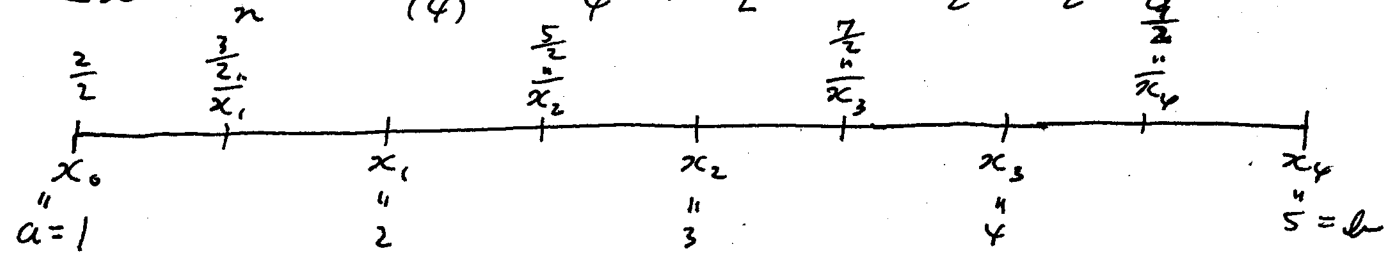
28) $\int_1^5 x^2 e^{-x}\,dx$, $n=4$ $\qquad f(x) = x^2 e^{-x} = \dfrac{x^2}{e^x}$

$$\Delta x = \frac{b-a}{n} = \frac{(5)-(1)}{(4)} = \frac{4}{4} = 1 = \frac{2}{2} \qquad \frac{\Delta x}{2} = \frac{1}{2}$$



$$L_4 = \Delta x\left[f(x_0) + f(x_1) + f(x_2) + f(x_3)\right]$$

$$= (1)\left[\frac{(1)^2}{e^{(1)}} + \frac{(2)^2}{e^{(2)}} + \frac{(3)^2}{e^{(3)}} + \frac{(4)^2}{e^{(4)}}\right]$$

$$R_4 = \Delta x\left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right] = (1)\left[\frac{(2)^2}{e^{(2)}} + \frac{(3)^2}{e^{(3)}} + \frac{(4)^2}{e^{(4)}} + \frac{(5)^2}{e^{(5)}}\right]$$

$$M_4 = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4)\right]$$

$$= (1)\left[\frac{\left(\frac{3}{2}\right)^2}{e^{\left(\frac{3}{2}\right)}} + \frac{\left(\frac{5}{2}\right)^2}{e^{\left(\frac{5}{2}\right)}} + \frac{\left(\frac{7}{2}\right)^2}{e^{\left(\frac{7}{2}\right)}} + \frac{\left(\frac{9}{2}\right)^2}{e^{\left(\frac{9}{2}\right)}}\right]$$