4.7

6) $C(q) = 10000 + 340q - 0.3q^2 + 0.0001q^3$

Average Cost function

$$A(q) = \frac{C(q)}{q} = \frac{10000 + 340q - 0.3q^2 + 0.0001q^3}{q}$$

$$= \frac{10000}{q} + 340 - 0.3q + 0.0001q^2$$

$$= 1000q^{-1} + 340 - 0.3q + 0.0001q^2$$

Marginal Cost function

$$C'(q) = \frac{dC}{dq} = [0] + 340[1] - 0.3[2q] + 0.0001[3q^2]$$

$$= 340 - 0.6q + 0.0003q^2$$

---

a) production level of 1000 units

Cost: $C(1000) = 10000 + 340(1000) - 0.3(1000)^2 + 0.0001(1000)^3$

$$= 15000 = \$15000$$

Average Cost: $A(1000) = \frac{C(1000)}{1000} = \frac{\$15000}{1000} = \$15/\text{unit}$

Marginal Cost:

$$C'(1000) = \left.\frac{dC}{dq}\right|_{q=1000} = 340 - 0.6(1000) + 0.0003(1000)^2$$

$$= 40 = \$40/\text{unit}$$

6) continued...

b) we need to find critical number of Average Cost function and verify that this value is a local minimum.

$$\frac{dA}{dq} = 10000\left[-1q^{-2}\right] + [0] - 0.3[1] + 0.0001[2q]$$

$$= -10000\,q^{-2} - 0.3 + 0.0002q$$

$$= \frac{-10000}{q^2} - 0.3 + 0.0002q$$

$$\frac{d^2A}{dq^2} = -10000\left[-2q^{-3}\right] - [0] + 0.0002[1]$$

$$= \frac{20000}{q^3} + 0.0002$$

C.P.:

$$0 = \frac{dA}{dq} = \frac{-10000}{q^2} - 0.3 + 0.0002q$$

this is not a simple algebra problem. It is a complicated cubic equation where factor by grouping does not work.

Using Maple computer software, we get $q \approx 1521$ as a real number solution

test at C.P. $q \approx 1521$

$$\left.\frac{d^2A}{dq^2}\right|_{q=1521} = \frac{20000}{(1521)^3} + 0.0002 > 0 \quad \text{C.U.} \quad \text{local min}$$

6) continued (2nd page)...

c) $A(1521) = \frac{C(1521)}{1521} \approx \frac{10000+340(1521)-0.3(1521)^2+0.0001(1521)}{1521}$

$\approx \frac{184288.5}{1521} \approx \$121/\text{unit}$

---

16) \$10 each, sales ave. 20 per day

price increase by \$1 (−1) lost 2 sales per day (−2)

let q be sales per day.

this exercise is similar to Example 3 (sec 4.7)

a) $D(q) = \underset{\text{cost for (20 per day)}}{\$10} - \underset{\text{price increase}}{(-1)}\underset{\text{lost sales}}{\left(\frac{1}{-2}\right)}(q - \underset{\text{fixed sales of 20 linked to cost in front of \$10}}{20})$

$D(q) = 10 - \frac{1}{2}(q-20) = 10 - \frac{1}{2}q + 10$

$$\underline{\underline{D(q) = 20 - \frac{1}{2}q}}$$

b) each necklace cost \$6. $\Rightarrow C(q) = \$6(q) = 6q$

$R(q) = q(D(q)) = q\left(20 - \frac{1}{2}q\right) = 20q - \frac{1}{2}q^2$

$P(q) = R(q) - C(q) = \left(20q - \frac{1}{2}q^2\right) - (6q) = 14q - \frac{1}{2}q^2$

16) continued...

$$\frac{dP}{dq} = 14[1] - \frac{1}{2}[2q] = 14 - q$$

$$\frac{d^2P}{dq^2} = [0] - [1] = -1$$

C.P.:

$$0 = \frac{dP}{dq} = 14 - q$$

$$0 = 14 - q$$

$$q = 14$$

test at C.P. $q = 14$

$$\left.\frac{d^2P}{dq^2}\right|_{q=14} = -1 < 0 \quad \text{C.D. local Max}$$

$$D(14) = 20 - \frac{1}{2}(14) = 13$$

Terry should sell 14 necklace per day charging \$13 each.

---

20) $p\,q^{0.85} = 81000$

a) $p = \$56000$

$$(56000)\,q^{0.85} = 81000$$

$$q^{0.85} = \frac{81000}{56000}$$

$$q^{0.85} = \frac{81}{56}$$

$$q = \sqrt[0.85]{\frac{81}{56}}$$

$$q \approx 1.54$$

20) continued...

$$E(q) = -\frac{\frac{p}{q}}{\frac{dp}{dq}}$$

b) $pq^{0.85} = 81000$

$$p = \frac{81000}{q^{0.85}}$$

$$p = 81000\,q^{-0.85}$$

$$\frac{dp}{dq} = 81000\left[-0.85\,q^{-1.85}\right]$$

$$= \frac{-0.85(81000)}{q^{1.85}}$$

$$\frac{dp}{dq} = \frac{-85(810)}{q^{1.85}}$$

$$E(1.54) = -\left.\frac{\frac{p}{q}}{\frac{dp}{dq}}\right|_{q=1.54}$$

$$= -\frac{\frac{56000}{1.54}}{\frac{-85(810)}{(1.54)^{1.85}}}$$

$$E(1.54) \approx 1.17 > 1$$

the demand is greater than the relative change in price and it is elastic.