4.6

2) let x and y be 2 numbers

$$y - x = 100 \Rightarrow y = 100 + x = x + 100$$

$P = xy$ is minimum

$$P = x(x+100) = x^2 + 100x$$

$$\frac{dP}{dx} = [2x] + 100[1] = 2x + 100$$

$$\frac{d^2P}{dx^2} = 2[1] = 2$$

C.P.:

$$0 = \frac{dP}{dx} = 2x + 100$$

$$0 = 2x + 100$$

$$-100 = 2x$$

$$-50 = x$$

$$\left.\frac{d^2P}{dx^2}\right|_{x=-50} = 2 > 0 \text{ C.U.}$$

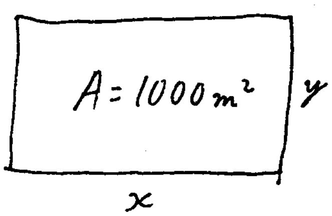
local min

$$y = (-50) + 100 = 50$$

the 2 numbers are $x = -50$ and $y = 50$.

---

6)



$$1000m^2 = A = xy \Rightarrow y = \frac{1000}{x}$$

$$P = 2x + 2y = 2x + 2\left(\frac{1000}{x}\right) = 2x + 2000x^{-1}$$

$$\frac{dP}{dx} = 2[1] + 2000[-1x^{-2}] = 2 - 2000x^{-2} = 2 - \frac{2000}{x^2}$$

$$\frac{d^2P}{dx^2} = [0] - 2000[-2x^{-3}] = \frac{4000}{x^3}$$

C.P.:

$$0 = \frac{dP}{dx} = 2 - \frac{2000}{x^2}$$

$$0 = 2 - \frac{2000}{x^2}$$

$$\frac{2000}{x^2} = 2$$

$$2000 = 2x^2$$

$$0 = 2x^2 - 2000$$

$$0 = 2(x^2 - 1000)$$

$$0 = 2(x + \sqrt{1000})(x - \sqrt{1000})$$

$x + \sqrt{1000} = 0$ | $x - \sqrt{1000} = 0$

$x = -\sqrt{1000}$ | $x = \sqrt{1000}$

test at C.P. $x = -\sqrt{1000}$

$$\left.\frac{d^2P}{dx^2}\right|_{x=-\sqrt{1000}} = \frac{4000}{(-\sqrt{1000})^3} < 0 \text{ C.D.}$$

local Max

discard

test at C.P. $x = \sqrt{1000}$

$$\left.\frac{d^2P}{dx^2}\right|_{x=\sqrt{1000}} = \frac{4000}{(\sqrt{1000})^3} > 0 \text{ C.U.}$$

local min

6) continued...

$x = \sqrt{1000} = \sqrt{(100)(10)} = \sqrt{100}\,\sqrt{10} = 10\sqrt{10}$

$y = \dfrac{1000}{10\sqrt{10}} = \dfrac{100}{\sqrt{10}}$

the dimension of this rectangle is $x = 10\sqrt{10}$ m and $y = \dfrac{100}{\sqrt{10}}$ m.

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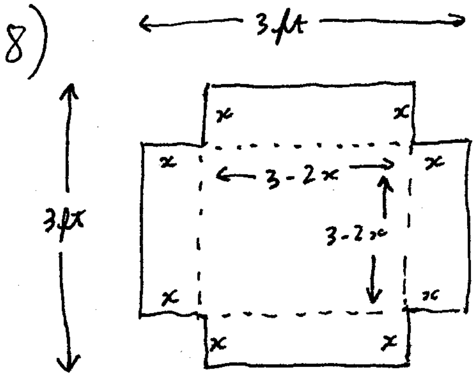
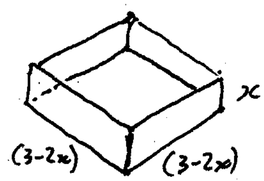
8)





$V = (3-2x)(3-2x)(x)$

$V = (9-12x+4x^2)(x)$

$V = 9x - 12x^2 + 4x^3$

$V = 4x^3 - 12x^2 + 9x$

$\dfrac{dV}{dx} = 4[3x^2] - 12[2x] + 9[1] = 12x^2 - 24x + 9$

$\dfrac{d^2V}{dx^2} = 12[2x] - 24[1] + [0] = 24x - 24$

C.P. :

$0 = \dfrac{dV}{dx} = 12x^2 - 24x + 9$

$0 = 3(4x^2 - 8x + 3)$

$0 = 3(2x-1)(2x-3)$

$2x - 1 = 0 \quad | \quad 2x - 3 = 0$

$x = \frac{1}{2} \quad | \quad x = \frac{3}{2}$

test at C.P. $x = \frac{1}{2}$

$\left.\dfrac{d^2V}{dx^2}\right|_{x=\frac{1}{2}} = 24(\tfrac{1}{2}) - 24 = 12 - 24 = -12 < $ C.D.

local Max

test at C.P. $x = \frac{3}{2}$

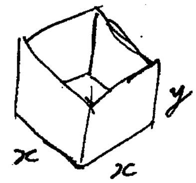
$\left.\dfrac{d^2V}{dx^2}\right|_{x=\frac{3}{2}} = 24(\tfrac{3}{2}) - 24 = 36 - 24 = 12 > $ C.U.

discard local min

$x = \frac{1}{2} \qquad (3 - 2(\tfrac{1}{2})) = 3 - 1 = 2$

the dimension of the box with largest volume is $(3-2x) = 2$ ft $\times$ $(3-2x) = 2$ ft $\times$ $x = \frac{1}{2}$ ft.

10)



$$32000\,cm^3 = V = x^2 y \Rightarrow y = \frac{32000}{x^2}$$

$$S = x^2 + 4xy = x^2 + 4x\left(\frac{32000}{x^2}\right) = x^2 + \frac{4(32000)}{x}$$

$$S = x^2 + 4(32000)x^{-1}$$

$$\frac{dS}{dx} = [2x] + 4(32000)[-1x^{-2}] = 2x - 4(32000)x^{-2} = 2x - \frac{4(32000)}{x^2}$$

$$\frac{d^2S}{dx^2} = 2[1] - 4(32000)[-2x^{-3}] = 2 + \frac{8(32000)}{x^3}$$

C.P.:

$$0 = \frac{dS}{dx} = 2x - \frac{4(32000)}{x^2}$$

$$0 = 2x - \frac{4(32000)}{x^2}$$

$$x^2(0) = \left(2x - \frac{4(32000)}{x^2}\right)x^2$$

$$0 = 2x^3 - 4(32000)$$

$$0 = 2(x^3 - 2(32000))$$

$$0 = 2(x^3 - (2)^6(1000))$$

$$0 = 2(x^3 - (4)^3(10)^3)$$

$$0 = 2(x^3 - (40)^3)$$

$$0 = 2(x-40)\underbrace{(x^2 + 40x + (40)^2)}_{\text{not factorable}}$$

$$x - 40 = 0$$

$$x = 40$$

test at C.P. $x = 40$

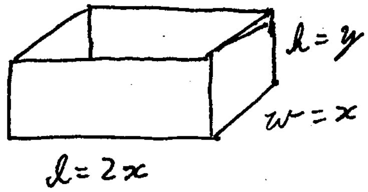
$$\left.\frac{d^2S}{dx^2}\right|_{x=40} = 2 + \frac{8(32000)}{(40)^3} > 0 \text{ C.U.}$$

local min

$$y = \frac{32000}{(40)^2} = \frac{32000}{1600} = \frac{320}{16} = 20$$

$$\boxed{\text{the dimension of the box is } x = 40\text{ cm} \times x = 40\text{ cm} \times y = 20\text{ cm}}$$

12)



$$10\,m^3 = V = (2x)(x)(y) = 2x^2 y$$

$$\Downarrow$$

$$y = \frac{10}{2x^2} \Rightarrow y = \frac{5}{x^2}$$

base: $A_b = (2x)(x) = 2x^2$ costing \$10 per m^2

sides (short): $A_{ss} = 2(x)(y) = 2xy$
(long): $A_{sl} = 2(2x)(y) = 4xy$ $\Big\}$ costing \$6 per m^2

$$C = \$10\,(A_b) + \$6\,(A_{ss} + A_{sl})$$

$$C = 10(2x^2) + 6(2xy + 4xy) = 20x^2 + 36\,xy$$

$$C = 20x^2 + 36x\left(\frac{5}{x^2}\right) = 20x^2 + \frac{5(36)}{x} = 20x^2 + 5(36)\,x^{-1}$$

$$C = 20x^2 + 5(4)(9)\,x^{-1} = 20x^2 + 20(9x^{-1}) = 20\left\{x^2 + 9x^{-1}\right\}$$

$$\frac{dC}{dx} = 20\left\{[2x] + 9[-1x^{-2}]\right\} = 20\left\{2x - 9x^{-2}\right\} = 20\left\{2x - \frac{9}{x^2}\right\}$$

$$\frac{d^2C}{dx^2} = 20\left\{2[1] - 9[-2x^{-3}]\right\} = 20\left\{2 + \frac{18}{x^3}\right\}$$

C.P.:

$$0 = \frac{dC}{dx} = 20\left\{2x - \frac{9}{x^2}\right\}$$

$$0 = 20\left\{2x - \frac{9}{x^2}\right\}$$

$$0 = 2x - \frac{9}{x^2}$$

$$0 = 2x^3 - 9$$

$$0 = \left(\sqrt[3]{2}\,x - \sqrt[3]{9}\right)\left(\left(\sqrt[3]{2}\,x\right)^2 + \left(\sqrt[3]{2}\,x\right)\left(\sqrt[3]{9}\right) + \left(\sqrt[3]{9}\right)^2\right)$$

$$\sqrt[3]{2}\,x - \sqrt[3]{9} = 0$$

$$\sqrt[3]{2}\,x = \sqrt[3]{9}$$

$$x = \frac{\sqrt[3]{9}}{\sqrt[3]{2}}$$

$$x = \sqrt[3]{\frac{9}{2}}$$

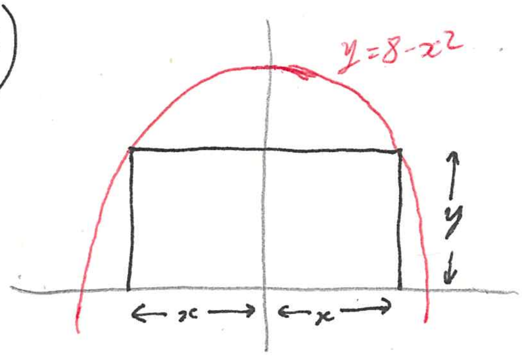
test at C.P. $x = \sqrt[3]{\frac{9}{2}}$

$$\left.\frac{d^2C}{dx^2}\right|_{x=\sqrt[3]{\frac{9}{2}}} = 20\left\{2 + \frac{18}{\left(\sqrt[3]{\frac{9}{2}}\right)^3}\right\} > 0 \text{ C.U.}$$

local min

$$\boxed{\begin{array}{c}\text{the dimension of the box}\\ \text{with minimal cost is}\\ 2x = 2\left(\sqrt[3]{\tfrac{9}{2}}\right)m \times x = \sqrt[3]{\tfrac{9}{2}}\ m\\ \times\ y = \dfrac{5}{\left(\sqrt[3]{\frac{9}{2}}\right)^2}\ m\end{array}}$$

14)



$l = 2x$

$w = y$

$A = (2x)(y) = (2x)(8-x^2)$

$A = 16x - 2x^3$

C.P.:

$0 = \frac{dA}{dx} = 16 - 6x^2$

$0 = 2(8 - 3x^2)$

$0 = 2(\sqrt{8} + \sqrt{3}x)(\sqrt{8} - \sqrt{3}x)$

$\sqrt{8} + \sqrt{3}x = 0$	$\sqrt{8} - \sqrt{3}x = 0$
$\sqrt{3}x = -\sqrt{8}$	$\sqrt{8} = \sqrt{3}x$
$x = \frac{-\sqrt{8}}{\sqrt{3}}$ discard	$\frac{\sqrt{8}}{\sqrt{3}} = x$ or $x = \sqrt{\frac{8}{3}}$

$\frac{dA}{dx} = 16[1] - 2[3x^2] = 16 - 6x^2$

$\frac{d^2A}{dx^2} = [0] - 6[2x] = -12x$

test at C.P, $x = \sqrt{\frac{8}{3}}$

$\left.\frac{d^2A}{dx^2}\right|_{x=\sqrt{\frac{8}{3}}} = -12\left(\sqrt{\frac{8}{3}}\right) < 0$ C.D.

local Max

$y = 8 - \left(\sqrt{\frac{8}{3}}\right)^2 = 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3} = \frac{16}{3}$

$2x = 2\left(\sqrt{\frac{8}{3}}\right) = 2\sqrt{\frac{8}{3}}$

the dimension of the rectangle is $2x = 2\sqrt{\frac{8}{3}}$ by $y = \frac{16}{3}$.