4.2

2) Suppose f is continuous function defined on a closed interval $[a, b]$.

a) The Extreme Value Theorem guarantees the existence of an Absolute Maximum and an Absolute minimum value for f.

b) 1) Find the critical numbers of f in the interval (a, b) and compute the values of f at these numbers.

2) Find the values of f at the endpoints of the interval.

3) The largest of the output values from steps 1 and 2 is the Absolute Maximum value; the smallest of these values is the Absolute minimum value.

---

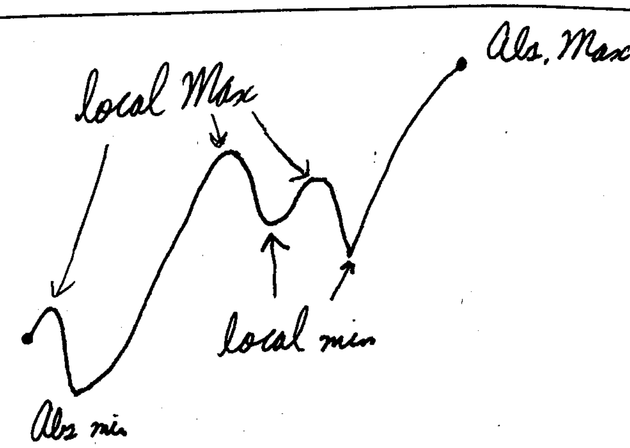
4) Absolute Max: e

Absolute min: t

local Max: a, s

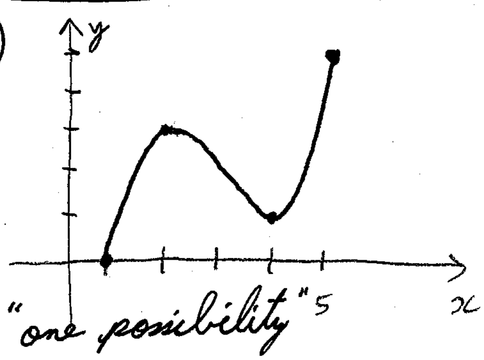
local min: b, c, d, r

6)

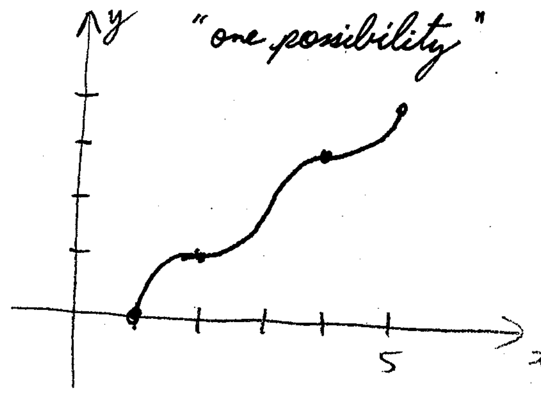


---

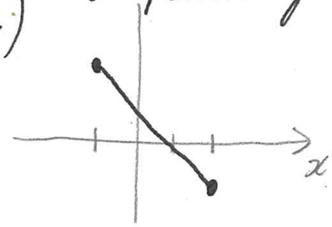
8)



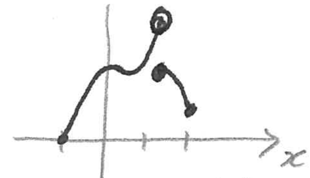
10)



12) a) "one possibility"

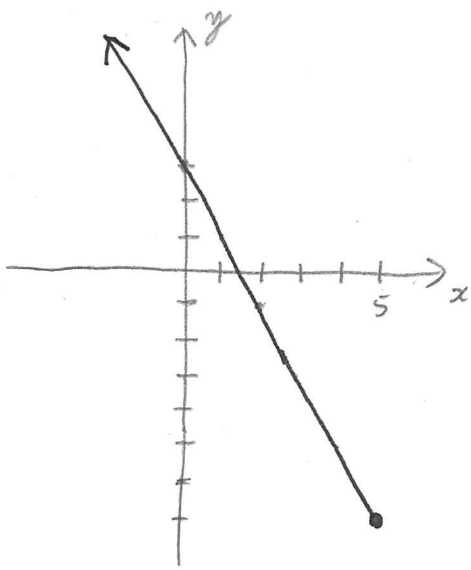


b) in order to draw this the function cannot be continuous



"one possibility"

16) $f(x) = 3 - 2x, \quad x \leq 5$



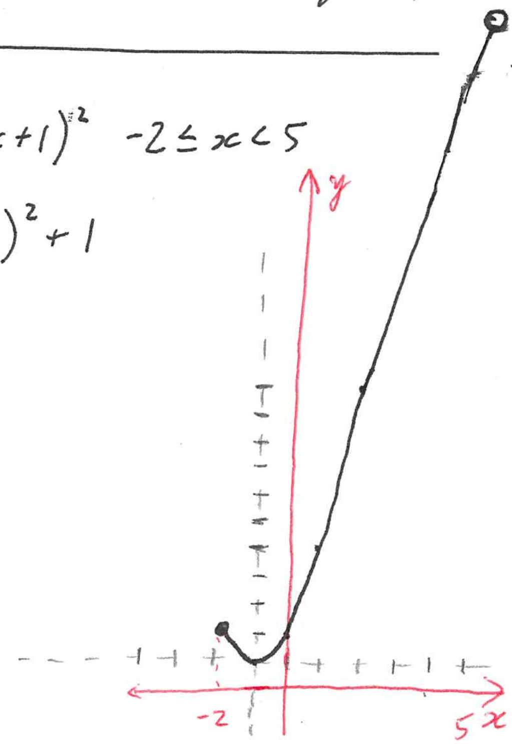
18) $f(x) = 1 + (x+1)^2 \quad -2 \leq x < 5$

$y = (x - (-1))^2 + 1$

H-shift: -1

V-shift: $+1$

like: $y = x^2$



24) $f(x) = x^3 + x^2 - x$

$$\frac{df}{dx} = [3x^2] + [2x] - [1]$$

$$\frac{df}{dx} = 3x^2 + 2x - 1$$

C.P.:

$$0 = \frac{df}{dx} = 3x^2 + 2x - 1$$

$$0 = (x+1)(3x-1)$$

$x + 1 = 0$ | $3x - 1 = 0$

$\underline{\underline{x = -1}}$ | $\underline{\underline{x = \frac{1}{3}}}$

28) $f(x) = x^3 + x^2 + x$

$$\frac{df}{dx} = [3x^2] + [2x] + [1]$$

$$\frac{df}{dx} = 3x^2 + 2x + 1$$

C.P.:

$$0 = \frac{df}{dx} = 3x^2 + 2x + 1$$

not factorable in real number

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(1)}}{2(3)} \Rightarrow \text{not real number}$$

<u>no Critical number</u>

30) $f(x) = x e^{2x}$

$$\frac{df}{dx} = [1](e^{2x}) + (x)[e^{2x}(2)]$$
$$= \{e^{2x} + 2x e^{2x}\}$$
$$\frac{df}{dx} = e^{2x}\{1+2x\}$$

C.P.:

$$0 = \frac{df}{dx} = e^{2x}\{1+2x\}$$

$0 = e^{2x}$ discard | $0 = 1+2x$, $-1 = 2x$, $\frac{-1}{2} = x$

---

32) $h(p) = \dfrac{p-1}{p^2-4}$

$$\frac{dh}{dp} = \frac{[1](p^2-4)-(p-1)[2p]}{(p^2-4)^2}$$
$$\frac{dh}{dp} = \frac{(p^2-4)-(2p^2-2p)}{(p^2-4)^2}$$
$$= \frac{p^2-4-2p^2+2p}{(p^2-4)^2}$$
$$\frac{dh}{dp} = \frac{-p^2+2p-4}{(p^2-4)^2}$$

C.P.:

$$0 = \frac{dh}{dp} = \frac{-p^2+2p-4}{(p^2-4)^2}$$
$$0 = -p^2+2p-4$$
$$p^2-2p+4 = 0$$
$$p = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(4)}}{2(1)} \Rightarrow \text{not real number}$$

not factorable in real number

no Critical number

---

38) $f(x) = x^3 - 6x^2 + 9x + 2 \quad [-1, 4]$

$$\frac{df}{dx} = [3x^2] - 6[2x] + 9[1] + [0]$$
$$= 3x^2 - 12x + 9$$

C.P:

$$0 = \frac{df}{dx} = 3x^2 - 12x + 9$$
$$0 = 3(x^2 - 4x + 3)$$
$$0 = 3(x-1)(x-3)$$

$x - 1 = 0$, $x = 1$ | $x - 3 = 0$, $x = 3$

38) continued...

at CP $x=1$

$f(1)=(1)^3-6(1)^2+9(1)+2 = 1-6+9+2 = 6$ Abs Max

at CP $x=3$

$f(3)=(3)^3-6(3)^2+9(3)+2 = 27-54+27+2 = 2$

at endpoint $x=-1$

$f(-1)=(-1)^3-6(-1)^2+9(-1)+2 = -1-6-9+2 = -14$ Abs min

at endpoint $x=4$

$f(4)=(4)^3-6(4)^2+9(4)+2 = 64-96+36+2 = 6$ Abs Max

Abs. Max at $x=1$ and $x=4$ Abs min at $x=-1$

---

40) $f(x)=(x^2-1)^3$ $[-1,2]$

$$\frac{df}{dx} = \left[3(x^2-1)^2(2x)\right] = 6x(x^2-1)^2$$

C.P.:

$$0 = \frac{df}{dx} = 6x(x^2-1)^2$$

$$0 = 6x(x^2-1)^2$$

$$0 = 6x((x+1)(x-1))^2$$

$6x=0$, $x=0$

$((x+1)(x-1))^2=0$

$(x+1)(x-1)=0$

$x+1=0$, $x=-1$ endpoint

$x-1=0$, $x=1$

at CP $x=0$

$f(0)=((0)^2-1)^3=(-1)^3=-1$ Abs min

at CP $x=1$

$f(0)=((1)^2-1)^3=(0)^3=0$

at endpoint $x=-1$

$f(-1)=((-1)^2-1)^3=(0)^3=0$

at endpoint $x=2$

$f(2)=((2)^2-1)^3=(3)^3=27$ Abs Max

Abs Max at $x=2$ Abs min at $x=0$

42) $f(x) = \dfrac{x}{x^2+4}$ $\quad [0, 3]$

$$\frac{df}{dx} = \frac{[1](x^2+4) - (x)[2x]}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

C.P.:

$$0 = \frac{df}{dx} = \frac{4-x^2}{(x^2+4)^2}$$

$$0 = 4 - x^2$$

$$0 = (2+x)(2-x)$$

$2+x=0$	$2-x=0$
$x=-2$	$x=2$
discard	

at CP $x=2$

$$f(2) = \frac{(2)}{(2)^2+4} = \frac{2}{4+4} = \frac{2}{8} = \frac{1}{4} = \frac{1}{4}\left(\frac{13}{13}\right) = \frac{13}{4(13)}$$ Abs Max

at endpoint $x=0$

$$f(0) = \frac{(0)}{(0)^2+4} = \frac{0}{4} = 0$$ Abs min

at endpoint $x=3$

$$f(3) = \frac{(3)}{(3)^2+4} = \frac{3}{9+4} = \frac{3}{13} = \frac{3}{13}\left(\frac{4}{4}\right) = \frac{12}{4(13)}$$

Abs Max at $x=2$ $\qquad$ Abs min at $x=0$

---

44) $f(x) = x - \ln x$ $\quad [\tfrac{1}{2}, 2]$

$$\frac{df}{dx} = [1] - \left[\frac{1}{x}(1)\right] = 1 - \frac{1}{x} = 1\left(\frac{x}{x}\right) - \frac{1}{x} = \frac{x-1}{x}$$

C.P.:

$$0 = \frac{df}{dx} = \frac{x-1}{x}$$

$$0 = x - 1$$

$$1 = x$$

at CP $x=1$

$f(1) = (1) - \ln(1) = 1 - 0 = 1$ Abs min

at endpoint

$f(\tfrac{1}{2}) = (\tfrac{1}{2}) - \ln(\tfrac{1}{2}) \approx 1.19$

at endpoint

$f(2) = (2) - \ln(2) \approx 1.31$ Abs Max

Abs Max at $x=2$ $\qquad$ Abs min at $x=1$