3.5

$$\frac{d}{du}(\ln u) = \frac{1}{u}(1) = \frac{1}{u}$$

$$\frac{d}{du}(\ln x) = [0] = 0$$

because this was asked to take derivative in respect to u not x

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2) $\sqrt{x} + \sqrt{y} = 4$

$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 4$

a) $\frac{dy}{dx} = ?$

$$\left[\frac{1}{2}x^{-\frac{1}{2}}\right] + \left[\frac{1}{2}y^{-\frac{1}{2}}\left(\frac{dy}{dx}\right)\right] = [0]$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{1}{2\sqrt{y}}\left(\frac{dy}{dx}\right) = \frac{-1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \left(\frac{-1}{2\sqrt{x}}\right)\left(\frac{2\sqrt{y}}{1}\right) = \frac{-\sqrt{y}}{\sqrt{x}}$$

b) $\sqrt{y} = 4 - \sqrt{x}$

$$y = (4-\sqrt{x})^2$$

$$y = (4-x^{\frac{1}{2}})^2$$

$$\frac{dy}{dx} = 2(4-x^{\frac{1}{2}})^1\left(-\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= \frac{2(4-\sqrt{x})}{-2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-(4-\sqrt{x})}{\sqrt{x}}$$

---

c) from part a) and replace $\sqrt{y} = 4-\sqrt{x}$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}} = \frac{-(4-\sqrt{x})}{\sqrt{x}}$$

part b method does not work in general. Only works in special cases.

4) $x^2 - y^2 = 1$

$$[2x] - \left[2y\left(\frac{dy}{dx}\right)\right] = [0]$$

$$2x = 2y\left(\frac{dy}{dx}\right.$$

$$\frac{2x}{2y} = \frac{dy}{dx}$$

$$\underline{\underline{\frac{dy}{dx} = \frac{x}{y}}}$$

product rule

6) $x^2 - 2xy + y^3 = c$

$$x^2 - (2x)(y) + y^3 = c$$

$$[2x] - \left\{[2](y) + (2x)\left[1\left(\frac{dy}{dx}\right)\right]\right\} + \left[3y^2\left(\frac{dy}{dx}\right)\right] = [0]$$

$$2x - 2y - 2x\left(\frac{dy}{dx}\right) + 3y^2\left(\frac{dy}{dx}\right) = 0$$

$$3y^2\left(\frac{dy}{dx}\right) - 2x\left(\frac{dy}{dx}\right) = 2y - 2x$$

$$\left(\frac{dy}{dx}\right)(3y^2 - 2x) = 2y - 2x$$

$$\underline{\underline{\frac{dy}{dx} = \frac{2y - 2x}{3y^2 - 2x}}}$$

product rule

8) $y^5 + x^2y^2 = 1 + ye^{x^2}$

$$\left[5y^4\left(\frac{dy}{dx}\right)\right] + \left\{[2x](y^2) + (x^2)\left[2y\left(\frac{dy}{dx}\right)\right]\right\} = [0] + \left\{\left[1\left(\frac{dy}{dx}\right)\right](e^{x^2}) + (y)\left[e^{x^2}(2x)\right]\right\}$$

$$5y^4\left(\frac{dy}{dx}\right) + 2xy^2 + 2x^2y\left(\frac{dy}{dx}\right) = e^{x^2}\left(\frac{dy}{dx}\right) + 2xye^{x^2}$$

$$5y^4\left(\frac{dy}{dx}\right) + 2x^2y\left(\frac{dy}{dx}\right) - e^{x^2}\left(\frac{dy}{dx}\right) = 2xye^{x^2} - 2xy^2$$

$$\left(\frac{dy}{dx}\right)(5y^4 + 2x^2y - e^{x^2}) = 2xye^{x^2} - 2xy^2$$

$$\underline{\underline{\frac{dy}{dx} = \frac{2xye^{x^2} - 2xy^2}{5y^4 + 2x^2y - e^{x^2}}}}$$

10) $\sqrt{x+y} = 1 + x^2y^2$

chain rule   product rule

$(x+y)^{\frac{1}{2}} = 1 + x^2y^2$

$$\left[\frac{1}{2}(x+y)^{-\frac{1}{2}}\left(1+1\left(\frac{dy}{dx}\right)\right)\right] = [0] + \left\{[2x](y^2) + (x^2)\left[2y\left(\frac{dy}{dx}\right)\right]\right\}$$

$$\frac{1+1\left(\frac{dy}{dx}\right)}{2\sqrt{x+y}} = 2xy^2 + 2x^2y\left(\frac{dy}{dx}\right)$$

$$\frac{1}{2\sqrt{x+y}} + \frac{1}{2\sqrt{x+y}}\left(\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\left(\frac{dy}{dx}\right)$$

$$\frac{1}{2\sqrt{x+y}}\left(\frac{dy}{dx}\right) - 2x^2y\left(\frac{dy}{dx}\right) = 2xy^2 - \frac{1}{2\sqrt{x+y}}$$

$$\left(\frac{dy}{dx}\right)\left(\frac{1}{2\sqrt{x+y}} - 2x^2y\right) = 2xy^2 - \frac{1}{2\sqrt{x+y}}$$

$$\underline{\underline{\frac{dy}{dx} = \frac{2xy^2 - \frac{1}{2\sqrt{x+y}}}{\frac{1}{2\sqrt{x+y}} - 2x^2y}}}$$

---

12) $\ln(q+r) = qr$ $\frac{dq}{dr} = ?$

$$\left[\frac{1}{(q+r)}\left(1\left(\frac{dq}{dr}\right)+1\right)\right] = \left\{\left[1\left(\frac{dq}{dr}\right)\right](r) + (q)[1]\right\}$$

$$\frac{1\left(\frac{dq}{dr}\right)+1}{(q+r)} = r\left(\frac{dq}{dr}\right) + q$$

12) continued

$$\frac{1}{q+r}\left(\frac{dq}{dr}\right)+\frac{1}{q+r}=r\left(\frac{dq}{dr}\right)+q$$

$$\frac{1}{q+r}\left(\frac{dq}{dr}\right)-r\left(\frac{dq}{dr}\right)=q-\frac{1}{q+r}$$

$$\left(\frac{dq}{dr}\right)\left(\frac{1}{q+r}-r\right)=q-\frac{1}{q+r}$$

$$\underline{\underline{\frac{dq}{dr}=\frac{q-\frac{1}{q+r}}{\frac{1}{q+r}-r}}}$$

---

14) $e^{2p}-e^{t}=p+t$ $\qquad \left.\frac{dp}{dt}\right|_{p=0,\,t=0}=?$

$$\left[e^{2p}(2)\left(\frac{dp}{dt}\right)\right]-\left[e^{t}(1)\right]=\left[1\left(\frac{dp}{dt}\right)\right]+[1]$$

$$2e^{2p}\left(\frac{dp}{dt}\right)-e^{t}=\left(\frac{dp}{dt}\right)+1$$

$$2e^{2p}\left(\frac{dp}{dt}\right)-\left(\frac{dp}{dt}\right)=e^{t}+1$$

$$\left(\frac{dp}{dt}\right)\left(2e^{2p}-1\right)=e^{t}+1$$

$$\frac{dp}{dt}=\frac{e^{t}+1}{2e^{2p}-1} \qquad \left.\frac{dp}{dt}\right|_{p=0,\,t=0}=\frac{e^{(0)}+1}{2e^{2(0)}+1}=\frac{1+1}{2(1)+1}=\underline{\underline{\frac{2}{3}}}$$

16) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ $\quad (-3\sqrt{3}, 1)$

$$\left[\frac{2}{3}x^{-\frac{1}{3}}\right] + \left[\frac{2}{3}y^{-\frac{1}{3}}\left(\frac{dy}{dx}\right)\right] = [0]$$

$$\frac{2}{3(\sqrt[3]{x})} + \frac{2}{3(\sqrt[3]{y})}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{2}{3(\sqrt[3]{y})}\left(\frac{dy}{dx}\right) = \frac{-2}{3(\sqrt[3]{x})}$$

$$\frac{dy}{dx} = \left(\frac{-2}{3(\sqrt[3]{x})}\right)\left(\frac{3(\sqrt[3]{y})}{2}\right) = \frac{-\sqrt[3]{y}}{\sqrt[3]{x}}$$

$$m = \left.\frac{dy}{dx}\right|_{x=-3\sqrt{3},\, y=1} = \frac{-\sqrt[3]{(1)}}{\sqrt[3]{(-3\sqrt{3})}}$$

$$= \frac{-1}{\sqrt[3]{-(\sqrt{3})^3}} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$y-(1) = \frac{1}{\sqrt{3}}(x-(-3\sqrt{3}))$$

$$y-1 = \frac{1}{\sqrt{3}}x + 3$$

$$\underline{y = \frac{1}{\sqrt{3}} + 4}$$

---

18) $y^2(y^2-4) = x^2(x^2-5)$ $\quad (0, -2)$

$$y^4 - 4y^2 = x^4 - 5x^2$$

$$\left[4y^3\left(\frac{dy}{dx}\right)\right] - 4\left[2y\left(\frac{dy}{dx}\right)\right] = [4x^3] - 5[2x]$$

$$4y^3\left(\frac{dy}{dx}\right) - 8y\left(\frac{dy}{dx}\right) = 4x^3 - 10x$$

$$\left(\frac{dy}{dx}\right)(4y^3 - 8y) = 4x^3 - 10x$$

$$\frac{dy}{dx} = \frac{4x^3-10x}{4y^3-8y}$$

$$m = \left.\frac{dy}{dx}\right|_{x=0,\, y=-2} = \frac{4(0)^3-10(0)}{4(-2)^3-8(-2)} = 0$$

$$y-(-2) = 0(x-(0))$$

$$y+2 = 0$$

$$\underline{y = -2}$$

22) $g(z) = 9\ln z + 12z + 8.1$

$$\frac{dg}{dz} = 9\left[\frac{1}{z}(1)\right] + 12[1] + [0] = \underline{\underline{\frac{9}{z} + 12}}$$

---

24) $y = 3\ln t - t^2$

$$\frac{dy}{dx} = 3\left[\frac{1}{t}(1)\right] - [2t] = \underline{\underline{\frac{3}{t} - 2t}}$$

---

26) $f(x) = \ln(x^2 + 10)$

$$\frac{df}{dx} = \left[\frac{1}{(x^2+10)}(2x)\right] = \underline{\underline{\frac{2x}{x^2+10}}}$$

---

28) $f(x) = \ln \sqrt[5]{x} = \ln\left(x^{\frac{1}{5}}\right) = \frac{1}{5}\ln x$

$$\frac{df}{dx} = \frac{1}{5}\left[\frac{1}{x}(1)\right] = \underline{\underline{\frac{1}{5x}}}$$

using laws of logarithm

$\ln(N^p) \Leftrightarrow p \ln N$

---

30) $f(t) = \dfrac{1 + \ln t}{1 - \ln t}$

$$\frac{df}{dt} = \frac{\left[\frac{1}{t}(1)\right](1 - \ln t) - (1 + \ln t)\left[-\frac{1}{t}(1)\right]}{(1 - \ln t)^2}$$

$$= \frac{\frac{1}{t} - \frac{\ln t}{t} + \frac{1}{t} + \frac{\ln t}{t}}{(1 - \ln t)^2} = \frac{\frac{2}{t}}{(1 - \ln t)^2} = \underline{\underline{\frac{2}{t(1 - \ln t)^2}}}$$

32) $F(y) = y \ln(1+e^y)$

$$\frac{dF}{dy} = [1](\ln(1+e^y)) + (y)\left[\frac{1}{(1+e^y)}(e^y(1))\right]$$

$$= \underline{\underline{\ln(1+e^y) + \frac{ye^y}{1+e^y}}}$$

---

34) $G(u) = \ln\sqrt{\frac{3u+2}{3u-2}} = \ln\left(\frac{\sqrt{3u+2}}{\sqrt{3u-2}}\right)$

$$= +\ln(\sqrt{3u+2}) - \ln(\sqrt{3u-2}) = \ln(3u+2)^{\frac{1}{2}} - \ln(3u-2)^{\frac{1}{2}}$$

$$= \frac{1}{2}\ln(3u+2) - \frac{1}{2}\ln(3u-2)$$

$$\frac{dG}{du} = \frac{1}{2}\left[\frac{1}{(3u+2)}(3)\right] - \frac{1}{2}\left[\frac{1}{(3u-2)}(3)\right] = \underline{\underline{\frac{3}{2(3u+2)} - \frac{3}{2(3u-2)}}}$$

---

36) $y = [\ln(3x-2)]^2$

$$\frac{dy}{dx} = 2[\ln(3x-2)]^1\left(\frac{1}{(3x-2)}(3)\right) = \underline{\underline{\frac{6\ln(3x-2)}{3x-2}}}$$

---

38) $y = [\ln(1+e^x)]^2$

$$\frac{dy}{dx} = 2[\ln(1+e^x)]^1\left(\frac{1}{(1+e^x)}(e^x(1))\right) = \underline{\underline{\frac{2e^x\ln(1+e^x)}{1+e^x}}}$$

40) $f(x) = \dfrac{\ln x}{x^2}$ find $f'(x) = \dfrac{df}{dx}$ and $f''(x) = \dfrac{d^2 f}{dx^2}$

$$\frac{df}{dx} = \frac{\left[\frac{1}{x}(1)\right](x^2) - (\ln x)[2x]}{(x^2)^2} = \frac{x - 2x\ln x}{x^4} = \frac{x(1-2\ln x)}{x^4}$$

$$= \underline{\underline{\frac{1-2\ln x}{x^3}}}$$

$$\frac{d^2 f}{dx^2} = \frac{\left[-2\left(\frac{1}{x}(1)\right)\right](x^3) - (1-2\ln x)\left[3x^2\right]}{(x^3)^2} = \frac{-2x^2 - 3x^2 + 6x^2\ln x}{x^6}$$

$$= \frac{6x^2\ln x - 5x^2}{x^6} = \frac{x^2(6\ln x - 5)}{x^6} = \underline{\underline{\frac{6\ln x - 5}{x^4}}}$$

---

52) $y = \ln(x + 2y)$ $(1, 0)$

$$\left[1\left(\frac{dy}{dx}\right)\right] = \left[\frac{1}{(x+2y)}\left(1 + 2\left(\frac{dy}{dx}\right)\right)\right]$$

$$1\left(\frac{dy}{dx}\right) = \frac{1 + 2\left(\frac{dy}{dx}\right)}{x+2y}$$

$$1\left(\frac{dy}{dx}\right) = \frac{1}{x+2y} + \frac{2}{x+2y}\left(\frac{dy}{dx}\right)$$

$$1\left(\frac{dy}{dx}\right) - \frac{2}{x+2y}\left(\frac{dy}{dx}\right) = \frac{1}{x+2y}$$

$$\left(\frac{dy}{dx}\right)\left(1 - \frac{2}{x+2y}\right) = \frac{1}{x+2y}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x+2y}}{1 - \frac{2}{x+2y}}$$

$$m = \left.\frac{dy}{dx}\right|_{x=1,\, y=0}$$

$$= \frac{\frac{1}{(1)+2(0)}}{1 - \frac{2}{(1)+2(0)}}$$

$$= \frac{\frac{1}{1}}{1 - \frac{2}{1}} = \frac{1}{1-2}$$

$$= \frac{1}{-1} = \underline{\underline{-1}}$$