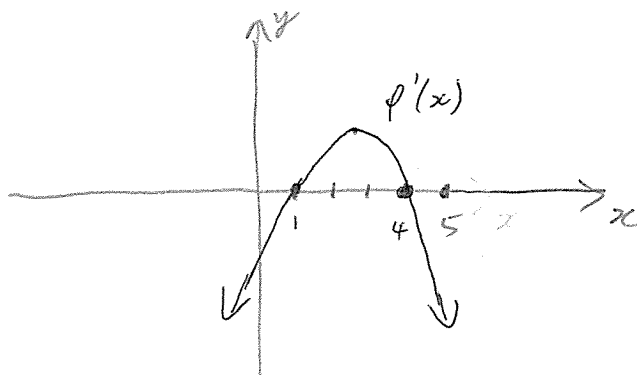


2.4

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$$2) \quad a) f'(0) \approx -1.5 \quad b) f'(1) = 0 \quad c) f'(2) \approx 0.8$$

$$d) f'(3) \approx 0.75 \quad e) f'(4) = 0 \quad f) f'(5) \approx -0.5$$



$$18) \quad g(x) = 3x + 5$$

$$g(x+h) = 3(x+h) + 5 = 3x + 3h + 5$$

$$g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{(3x + 3h + 5) - (3x + 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h} = \lim_{h \rightarrow 0} 3 = \underline{\underline{3}}$$

$$20) \quad f(x) = 1.5x^2 - x + 3.7$$

$$f(x+h) = 1.5(x+h)^2 - (x+h) + 3.7 = 1.5(x^2 + 2xh + h^2) - x - h + 3.7$$

$$= 1.5x^2 + 3xh + 1.5h^2 - x - h + 3.7$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(1.5x^2 + 3xh + 1.5h^2 - x - h + 3.7) - (1.5x^2 - x + 3.7)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3xh + 1.5h^2 - h}{h} = \lim_{h \rightarrow 0} \frac{h(3x + 1.5h - 1)}{h} = \lim_{h \rightarrow 0} (3x + 1.5h - 1)$$

$$= 3x + 1.5(0) - 1 = \underline{\underline{3x - 1}}$$

$$22) y = 2x^3 + 7x^2 = y(x)$$

$$\begin{aligned} y(x+h) &= 2(x+h)^3 + 7(x+h)^2 = 2(x+h)(x^2 + 2xh + h^2) + 7(x^2 + 2xh + h^2) \\ &= 2(x^3 + 3x^2h + 3xh^2 + h^3) + 7x^2 + 14xh + 7h^2 \\ &= 2x^3 + 6x^2h + 6xh^2 + 2h^3 + 7x^2 + 14xh + 7h^2 \end{aligned}$$

$$\begin{aligned} y'(x) &= \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \rightarrow 0} \frac{(2x^3 + 6x^2h + 6xh^2 + 2h^3 + 7x^2 + 14xh + 7h^2) - (2x^3 + 7x^2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3 + 14xh + 7h^2}{h} = \lim_{h \rightarrow 0} \frac{h(6x^2 + 6xh + 2h^2 + 14x + 7h)}{h} \\ &= \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2 + 14x + 7h) = 6x^2 + 6x(0) + 2(0)^2 + 14x + 7(0) = \underline{\underline{6x^2 + 14x}} \end{aligned}$$

$$24) N(t) = \frac{1}{t^2} \quad N(t+h) = \frac{1}{(t+h)^2}$$

$$\begin{aligned} N'(t) &= \lim_{h \rightarrow 0} \frac{N(t+h) - N(t)}{h} = \lim_{h \rightarrow 0} \frac{\left(\frac{1}{(t+h)^2}\right) - \left(\frac{1}{t^2}\right)}{h} \quad \text{GLCD} = (t^2(t+h)^2) \\ &= \lim_{h \rightarrow 0} \left(\frac{\frac{1}{(t+h)^2} - \frac{1}{t^2}}{h} \right) \left(\frac{t^2(t+h)^2}{t^2(t+h)^2} \right) = \lim_{h \rightarrow 0} \frac{1(t^2) - 1(t+h)^2}{h t^2 (t+h)^2} \\ &= \lim_{h \rightarrow 0} \frac{t^2 - (t^2 + 2th + h^2)}{h t^2 (t+h)^2} = \lim_{h \rightarrow 0} \frac{-2th - h^2}{h t^2 (t+h)^2} = \\ &= \lim_{h \rightarrow 0} \frac{h(-2t - h)}{h t^2 (t+h)^2} = \lim_{h \rightarrow 0} \frac{-2t - h}{t^2 (t+h)^2} \\ &= \frac{-2t - (0)}{t^2 (t+(0))^2} = \frac{-2t}{t^2 (t)^2} = \frac{-2t}{t^4} = \underline{\underline{\frac{-2}{t^3}}} \end{aligned}$$

26) f(x) = (3+x)/(1-3x)

1-3x=0
1=3x
1/3=x

domain: (-inf, 1/3) U (1/3, inf)

f(x+h) = (3+(x+h))/(1-3(x+h)) = (3+x+h)/(1-3x-3h)

GLCD = (1-3x)(1-3x-3h)

f'(x) = lim_{h->0} (f(x+h)-f(x))/h = lim_{h->0} ((3+x+h)/(1-3x-3h) - (3+x)/(1-3x))/h
= lim_{h->0} ((3+x+h)/(1-3x-3h) - (3+x)/(1-3x)) / (h/1) * (1/((1-3x)(1-3x-3h))) = lim_{h->0} ((3+x+h)(1-3x) - (3+x)(1-3x-3h)) / (h(1-3x)(1-3x-3h))
= lim_{h->0} (3+3x+h-9x-3x^2-3xh) - (3-9x-9h+x-3x^2-3xh) / (h(1-3x)(1-3x-3h)) = lim_{h->0} (h-9h) / (h(1-3x)(1-3x-3h))
= lim_{h->0} h(1-9) / (h(1-3x)(1-3x-3h)) = lim_{h->0} (1-9) / ((1-3x)(1-3x-3h)) = (1-9) / ((1-3x)(1-3x-3(0)))
= 8 / ((1-3x)(1-3x)) = 8 / (1-3x)^2
domain: (-inf, 1/3) U (1/3, inf)

28) f(x) = x + sqrt(x) x >= 0
0 <= x domain: [0, inf)

f(x+h) = (x+h) + sqrt(x+h) = x+h+sqrt(x+h)

f'(x) = lim_{h->0} (f(x+h)-f(x))/h = lim_{h->0} ((x+h+sqrt(x+h)) - (x+sqrt(x)))/h
= lim_{h->0} (h + sqrt(x+h) - sqrt(x)) / h = lim_{h->0} { h/h + (sqrt(x+h) - sqrt(x))/h }
= lim_{h->0} { 1 + ((sqrt(x+h) - sqrt(x)) / h) * ((sqrt(x+h) + sqrt(x)) / (sqrt(x+h) + sqrt(x))) }

28) continued

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$$\begin{aligned} &= \lim_{h \rightarrow 0} \left\{ 1 + \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} \right\} = \lim_{h \rightarrow 0} \left\{ 1 + \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \right\} \\ &= \lim_{h \rightarrow 0} \left\{ 1 + \frac{1}{\sqrt{x+h} + \sqrt{x}} \right\} = 1 + \frac{1}{\sqrt{x+(0)} + \sqrt{x}} = 1 + \frac{1}{\sqrt{x} + \sqrt{x}} = \underline{\underline{1 + \frac{1}{2\sqrt{x}}}} \end{aligned}$$

$x > 0$
 $0 < x$ "because even root appears in denominator" domain: $(0, \infty)$

30) $g(x) = \frac{1}{\sqrt{x}}$ $x > 0$
 $0 < x$ domain: $(0, \infty)$

$$g(x+h) = \frac{1}{\sqrt{x+h}} = \frac{1}{\sqrt{x+h}}$$

$$g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{\left(\frac{1}{\sqrt{x+h}}\right) - \left(\frac{1}{\sqrt{x}}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \right) \left(\frac{\sqrt{x} \sqrt{x+h}}{\sqrt{x} \sqrt{x+h}} \right) = \lim_{h \rightarrow 0} \frac{(\sqrt{x}) - (\sqrt{x+h})}{h(\sqrt{x} \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \left(\frac{\sqrt{x} - \sqrt{x+h}}{h \sqrt{x} \sqrt{x+h}} \right) \left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \right) = \lim_{h \rightarrow 0} \frac{(x) - (x+h)}{h \sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h \sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})} = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x} \sqrt{x+h} (\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x} \sqrt{x+(0)} (\sqrt{x} + \sqrt{x+(0)})} = \frac{-1}{\sqrt{x} \sqrt{x} (\sqrt{x} + \sqrt{x})} = \frac{-1}{\sqrt{x} \sqrt{x} (2\sqrt{x})}$$

$$= \underline{\underline{\frac{-1}{2(\sqrt{x})^3}}} \quad \text{domain: } (0, \infty)$$

38) a) Increasing: $[0, 1)$, $(3, 5)$
decreasing: $(1, 3)$, $(5, 6]$

b) local max or local min (point of extrema/critical point),
 $x = 1, x = 3, x = 5$

