

Please PRINT your name and ID# on the cover of your exam booklet and indicate if you are handing-in more than one booklet. Write clearly and cross-out work not to be graded.

ALL ANSWERS GO IN THE EXAM BOOK. **NO CALCULATORS ALLOWED.**

1. Find the derivative,  $\frac{dy}{dx}$ , of each of the following functions  $y(x)$  and **simplify** your answers: (20 pts.)

(a)  $y = \arcsin(\sqrt{x})$  (i.e.  $y = \sin^{-1}(\sqrt{x})$ )

(b)  $y = x^{\cosh x}$

2. Find the following limits: (15 pts.)

(a)

$$(i) \lim_{x \rightarrow \infty} \frac{e^x}{\ln x} \quad (ii) \lim_{x \rightarrow \infty} \frac{\ln x}{e^x} \quad (15 \text{ pts.})$$

(b)  $\lim_{x \rightarrow 0^+} (\sin x)(\ln x)$

3. Find the volume of the solid formed by rotating about the **x-axis** the region R bounded by  $y = \frac{1}{\sqrt{1+x^2}}$ ,  $x = 1$ , the x-axis, and the y-axis. (20 pts.)

**DO TWO of the following three problems:**

4. Find the work done in moving a particle along the x-axis from  $x = 0$  to  $x = \pi/3$  subject to a force  $F(x) = 2^{\tan x} \sec^2 x$ . (15 pts.)

5. In each case, solve for  $x$ : (15 pts.)

(a)  $\log_5(a^x) = b$

(b)  $\sinh^{-1}(x) = 1$  (in terms of exponentials)

(c)  $y = \frac{4x-1}{2x+3}$

6. If  $P(t)$  represents population at time  $t$ , a more realistic model with constant relative birth, death, and emigration rates  $a$ ,  $b$ , and  $m$ , respectively, gives the differential equation (15 pts.)

$$\frac{dP}{dt} = kP - m, \quad k = a - b.$$

Assume  $m > 0$ , and  $a > b$ , so  $k > 0$ .

(a) Find the solution of this equation that satisfies  $P(0) = P_0$ . (Hint: separate the variables by dividing both sides of the equation by  $kP - m$  and integrating as in the case when  $m = 0$ , which gave simple exponential growth.)

P.S. If you can do  $\int \frac{dx}{5x-1}$ , you can do this!

(b) Check that your answer gives pure exponential growth if  $m = 0$ .

(c) Does the population always grow? Explain. (Hint: are there conditions under which the solution is constant?)