Department of Mathematics,CCNY

Homework 6- Math 201

Chapter: 3.9, 3.10, 4, & 5

Student's Last Name, First Name: _____

Instruction: You must write very clearly and neaty, and show all your work to receive credit.

1. Let $f(x) = \dfrac{1}{\sqrt{x-10}}$.
   (a) Use the limit definition of the derivative to find $f'(x)$.
   (b) Use linear approximation or differential to estimate $f(14.2)$.

2. A ladder 10 feet long is leaning against a vertical wall with its other end on the ground. The bottom of the ladder is sliding away from the wall at a rate of 7 feet per second. How fast is the Top of the ladder moving down from the wall when the bottom of the ladder is 6 feet from the ground? You must include the correct units with your answer.

3. Two cars starts moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

4. A water tank has the shape of inverted circular cone with base radius 2 meters and height 4 meters. If water is bbeing pumped into the tank at a rate of 3 cubic meters per minute. Find the rate at which the water level is rising when the water is 5 meters deep.

5. Find the area of the largest shaded rectangle that can be inscribed in a semicircle of radius 6. See Figure 1 below. **Hint:** Graph of the semicircle is $y = \sqrt{36 - x^2}$; and a vertex (x, y) of the rectangle can be expressed as $\left(x, \sqrt{36 - x^2}\right)$

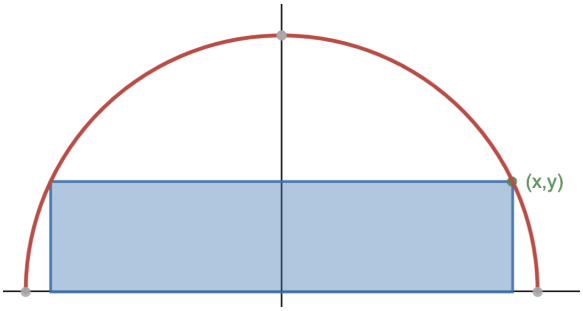

Figure 1: Graph of $y = \sqrt{36 - x^2}$

6. A rectangular box with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs 10 \$/m^2. Material for the sides costs 6 \$/m^2. Find the dimension that minimizes the cost of materials for such box.

7. If 1200 cm^2 of material is available to make a box with a square base, find the dimension of the box that minimize the amount of material used.

8. Find the point on the curve $y = 2 + \sqrt{x+3}$ that s nearest to the point (4,2).

9. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \dfrac{x^2}{x^2-1}$. Find all signifiant features (Domain and rang of f, the intercepts, end behavior limits, asymptotes, interval of increase or decrease, inflection if any and interval of concavity) then use these information to sketch graph of f.

10. Repeat question (9.) above with $f(x) = \dfrac{1}{3}\,x^3 + \dfrac{3}{2}\,x^2 - 4x - 1$.

11. Let $F(x) = \displaystyle\int_{32}^{x^5} \sqrt{1+t^5}\; dt$.
    (a) Using the Fundamental Theorem of Calculus, Part I, find $F'(x)$.
    (b) Find $(F^{-1})'(0)$.

12. Give the definition for $\displaystyle\int_a^b f(x)\; dx$ in terms of Riemann sums. Make sure to include the condition about f to be integrable.
    (a) Use your definition with 5 subinterval and left end points to estimate the integral for $f(x) = \sqrt{x + \sec(x)}$ on $[0/\pi/3]$
    (b) Use your definition with 5 subinterval and both right and left end points to estimate the integral for $f(x) = \sqrt{x+1}$ on $[1, 11]$. Is the integral overestimated? Underestimated? Explain your reason.