201 Sample C su26

No calculators, cell phones, or other electronic devices allowed.

YOU MUST SHOW WORK, OR GIVE EXPLANATIONS, JUSTIFYING ALL ANSWERS.

1. (12 pts) Find the derivative $\dfrac{dy}{dx}$ and **simplify your answer**.

   (a) $y = (x + \dfrac{1}{x})^9$

   (b) $y = \dfrac{x^3 - 5x^4 + x^8}{e^{3x}}$

   (c) $y = 3^x \cos(2x)$

   (d) $x^2 + 6xy - 2y^2 = 3$ (use implicit differentiation)

2. (12 pts) Evaluate each integral and **simplify your answer**.

   (a) $\displaystyle\int x(\sqrt[3]{x} + \frac{\sec^2(x)}{x} + x^{-3})\, dx$

   (b) $\displaystyle\int \sin^3(x)\cos(x)\, dx$

   (c) $\displaystyle\int \frac{\sin(\ln x)}{x}\, dx$

   (d) $\displaystyle\int_0^1 \frac{3}{(2x+1)^3}\, dx$

3. (9 pts) Find each limit (as a real number or $\pm\infty$), or state DNE for does not exist.

   (a) $\lim\limits_{x\to\infty} (\sqrt{x^2+3x} - \sqrt{x^2-2x})$

   (b) $\lim\limits_{x\to\infty} (1+2x)^{1/(2\ln x)}$

   (c) $\lim\limits_{x\to 0} \dfrac{\sin x}{\tan 4x}$

4. (10 pts) **No credit unless you use the requested methods.**

   (a) (6 pts) Let $f(x) = \begin{cases} x^2, & x < 2 \\ 8 - 2x, & x \geq 2 \end{cases}$

   i. Use the definition to show $f(x)$ is continuous at $x = 2$.

   ii. Is $f(x)$ differentiable at $x = 2$? Explain.

   (b) (4 pts) Using the Fundamental Theorem of Calculus, Part I, find $F'(x)$ if $F(x) = \displaystyle\int_{\arctan x}^{0} \frac{1}{t^3+1}\, dt$.

5. (a) (5 pts) Find $\dfrac{dx}{dt}$ at $x = 1$ if $y = x^3 + x^2 - 1$, $\dfrac{dy}{dt} = -1$.

   (b) (3 pts) The precise definition of $\lim\limits_{x\to a} f(x) = +\infty$ states that for every $M > 0$ there must be a $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) > M$. Find the largest value of δ (as a function of M) that you can use to prove $\lim\limits_{x\to 0} f(x) = \dfrac{1}{x^2} = +\infty$.

6. (a) (3 pts) State the Mean Value Theorem.

   (b) (5 pts) Suppose $f(x)$ is a differentiable , $f(1) = 2$, and $f'(x) \leq 5$. What is the largest possible value of $f(4)$?

7. (8 pts) A landscape architect constructs a rectangular garden such that:
   - one side uses brick wall costing \$3/ft
   - the other three sides use metal fence costing \$2/ft
   - the area enclosed must be 8 sq ft

   Use calculus to find the costminimizing dimensions.

8. (12 pts) Let $f(x) = \ln x$ for all parts of this problem.

   (a) (3 pts) Find the linear approximation $L(x)$ to $f(x)$ at $x = e^2$.

   (b) (2 pts) Find the differential df at $x = e^2$.

   (c) (3 pts) Use either of the previous answers to estimate the change in f when x increases to $e^2 + 0.2$.

9. (a) (4 pts) Approximate

$$\int_1^3 \ln x \, dx$$

   using a Riemann sum with $n = 6$ equal length subintervals and left endpoints as the sample points. (You may leave the answer as an unsimplified sum.)

   (b) (5 pts) Find all points on the graph $y = x^{-1}$ where the tangent line is parallel to $y = -4x + 7$.

10. (6 pts) Let $f(x) = \dfrac{e^x}{x}$.

    (a) Find equations of any vertical asymptotes of the graph of f.

    (b) Does the graph of f have horizontal asymptotes? If so, in which direction(s)?

11. (10 pts) Let $f(x) = -x^3 + 3x^2 - 4$.

    (a) Find the coordinates of all intercepts of its graph and the limits of $f(x)$ as $x \to \pm\infty$. (There are exactly two x-intercepts and they are integers that lie in $[-2, 2]$.)

    (b) Find the intervals of increase and the intervals of decrease for $f(x)$ and the coordinates of any local maxima and local minima.

    (c) Find the intervals of concavity and the coordinates of any inflection points.

    (d) Sketch the graph of $f(x)$, including all features from (a) – (c).