

201 Sample Final C

No calculators, cell phones, or other electronic devices allowed.

YOU MUST SHOW WORK, OR GIVE EXPLANATIONS, JUSTIFYING ALL ANSWERS.

1. (12 pts) Find the derivative $\frac{dy}{dx}$ and **simplify your answer**.

(a) $y = (x + \frac{1}{x})^9$

(b) $y = \frac{x^3 - 5x^4 + x^8}{e^{3x}}$

(c) $y = 3^x \cos(2x)$

(d) $x^2 + 6xy - 2y^2 = 3$ (use implicit differentiation)

2. (12 pts) Evaluate each integral and **simplify your answer**.

(a) $\int x(\sqrt[3]{x} + \frac{\sec^2(x)}{x} + x^{-3}) dx$

(b) $\int \sin^3(x) \cos(x) dx$

(c) $\int \frac{\sin(\ln x)}{x} dx$

(d) $\int_0^1 \frac{3}{(2x+1)^3} dx$

3. (9 pts) Find each limit (as a real number or $\pm\infty$), or state DNE for does not exist.

(a) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x})$

(b) $\lim_{x \rightarrow \infty} (1 + 2x)^{1/(2 \ln x)}$

(c) $\lim_{x \rightarrow 0} \frac{\sin x}{\tan 4x}$

4. (10 pts) **No credit unless you use the requested methods.**

(a) (6 pts) Let $f(x) = \begin{cases} x^2, & x < 2 \\ 8 - 2x, & x \geq 2 \end{cases}$

i. Use the definition to show $f(x)$ is continuous at $x = 2$.

ii. Is $f(x)$ differentiable at $x = 2$? Explain.

- (b) (4 pts) Using the Fundamental Theorem of Calculus, Part I, find

$F'(x)$ if $F(x) = \int_{\arctan x}^0 \frac{1}{t^3 + 1} dt$.

5. (a) (5 pts) Find $\frac{dx}{dt}$ at $x = 1$ if

$y = x^3 + x^2 - 1, \frac{dy}{dt} = -1$.

- (b) (3 pts) The precise definition of $\lim_{x \rightarrow a} f(x) = +\infty$ states that for every $M > 0$ there must be a $\delta > 0$ such that if $0 < |x - a| < \delta$ then $f(x) > M$. Find the largest value of δ (as a function of M) that you can use to prove $\lim_{x \rightarrow 0} f(x) = \frac{1}{x^2} = +\infty$.

6. (a) (3 pts) State the Mean Value Theorem.
- (b) (5 pts) Suppose $f(x)$ is a differentiable, $f(1) = 2$, and $f'(x) \leq 5$.
What is the largest possible value of $f(4)$?
7. (8 pts) A landscape architect constructs a rectangular garden such that:
- one side uses brick wall costing \$3/ft
 - the other three sides use metal fence costing \$2/ft
 - the area enclosed must be 8 sq ft
- Use calculus to find the costminimizing dimensions.
8. (12 pts) Let $f(x) = \ln x$ for all parts of this problem.
- (a) (3 pts) Find the linear approximation $L(x)$ to $f(x)$ at $x = e^2$.
- (b) (2 pts) Find the differential df at $x = e^2$.
- (c) (3 pts) Use either of the previous answers to estimate the change in f when x increases to $e^2 + 0.2$.
9. (a) (4 pts) Approximate
- $$\int_1^3 \ln x \, dx$$
- using a Riemann sum with $n = 6$ equal length subintervals and left endpoints as the sample points. (You may leave the answer as an unsimplified sum.)
- (b) (5 pts) Find all points on the graph $y = x^{-1}$ where the tangent line is parallel to $y = -4x + 7$.
10. (6 pts) Let $f(x) = \frac{e^x}{x}$.
- (a) Find equations of any vertical asymptotes of the graph of f .
- (b) Does the graph of f have horizontal asymptotes? If so, in which direction(s)?
11. (10 pts) Let $f(x) = -x^3 + 3x^2 - 4$.
- (a) Find the coordinates of all intercepts of its graph and the limits of $f(x)$ as $x \rightarrow \pm\infty$.
(There are exactly two x-intercepts and they are integers that lie in $[-2, 2]$.)
- (b) Find the intervals of increase and the intervals of decrease for $f(x)$ and the coordinates of any local maxima and local minima.
- (c) Find the intervals of concavity and the coordinates of any inflection points.
- (d) Sketch the graph of $f(x)$, including all features from (a) – (c).