201 Sample Final B

No calculators, cell phones, or other electronic devices allowed.

YOU MUST SHOW WORK, OR GIVE EXPLANATIONS, JUSTIFYING ALL ANSWERS.

1. (12 pts) Find the derivative $\dfrac{dy}{dx}$ and **simplify your answer**.
   (a) $y = \sqrt{7 + \sqrt{7x}}$
   (b) $y = (\ln(2x) + \tan x)(\pi - \frac{1}{x^2})$
   (c) $y = x^{\sin x}$
   (d) $xy^3 - x^2 = e^y - 2$ (assume you may use implicit differentiation)

2. (12 pts) Evaluate each integral and **simplify your answer**.
   (a) $\displaystyle\int \frac{2x^4 + x + x^6}{x^4}\,dx$
   (b) $\displaystyle\int \frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}\,dx$
   (c) $\displaystyle\int x3^{x^2}\,dx$
   (d) $\displaystyle\int_0^{\pi/2} (\cos^3(3x) - \cos(3x))\,dx$

3. (9 pts) Find each limit (as a real number or $\pm\infty$), or state DNE for does not exist.
   (a) $\displaystyle\lim_{x\to\infty} \frac{e^x + 2x}{\ln x}$
   (b) $\displaystyle\lim_{x\to 0^+} x^{2x}$
   (c) $\displaystyle\lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$

4. (9 pts) **No credit in either part unless you use the requested methods.**
   (a) (4 pts) Use differentials to estimate the error in computing the volume of a ball if the radius r is measured to be 2 ± 0.1 cm.
   Volume $V = \frac{4}{3}\pi r^3$.
   (b) (5 pts) Use the limit definition of derivative to find $f'(x)$ for $f(x) = \dfrac{1}{x+1}$, and find the tangent line to $y = f(x)$ at $x = 2$.

5. (9 pts) A particle moves in a straight line with velocity $v(t) = t^2 - 5t + 4$, with time $t \geq 0$ measured in seconds and distance in meters.
   (a) For which t is the particle moving left, for which is it moving right?
   (b) Find the acceleration $a(t)$ (include units), and determine when the particle is speeding up and when it is slowing down.
   (c) Find the position $s(t)$ if $s(0) = 3$ meters.

6. (7 pts) A kite is being flown at a constant height of 300 ft; wind blows and carries it horizontally at 25 ft/s. How fast must the string be let out to maintain the height when the kite is 500 ft from the person holding it?

7. (a) (4 pts) Find a and b so that $f(x)$ is continuous:

$$f(x) = \begin{cases} (x-1)^2, & x < 1 \\ ax + b, & 1 \le x \le 4 \\ \sqrt{2x+1}, & x > 4 \end{cases}$$

   (b) (3 pts) State the Mean Value Theorem (MVT).

   (c) (4 pts) Explain why MVT applies to $f(x) = 1/x^2$ on $[2, 4]$, and find all c satisfying the conclusion.

8. (6 pts) Find the absolute maximum and minimum of

$$f(x) = (x^2 + x)^{2/3}$$

on $[-3, 4]$, and the x-values where they occur.

9. (a) (3 pts) State the Intermediate Value Theorem.

   (b) (3 pts) Show that $xe^x = 2$ for some $x \in (0, 1)$.

10. (a) (4 pts) Approximate

$$\int_2^4 x^2\, dx$$

using a Riemann sum with $n = 4$ equal length subintervals and right endpoints as the sample points.
(You may leave the answer as an unsimplified sum.)

   (b) (5 pts) Using the Fundamental Theorem of Calculus, Part I, determine the concavity of

$$F(x) = \int_1^x (\ln t)^2\, dt, \quad x > 1.$$

11. (10 pts) Given

$$f(x) = \frac{x^2}{x^2 - 1}, \qquad f'(x) = \frac{-2x}{(x^2-1)^2}, \qquad f''(x) = \frac{6x^2 + 1}{(x^2-1)^3},$$

   (a) Find the domain of $f(x)$, the coordinates of all intercepts, and the equations of all horizontal and vertical asymptotes of the graph of $f(x)$.

   (b) Find the intervals of increase and the intervals of decrease for $f(x)$ and the coordinates of any local maxima and local minima.

   (c) Find the intervals of concavity and the coordinates of any inflection points.

   (d) Sketch the graph of $f(x)$ including all features from (a)–(c).