201 Sample A su26

No calculators, cell phones, or other electronic devices allowed.

YOU MUST SHOW WORK, OR GIVE EXPLANATIONS, JUSTIFYING ALL ANSWERS.

1. (9 pts) Find the derivative $\dfrac{dy}{dx}$ and **simplify your answer**.

   (a) $y = (\arctan(x) + x^{-3} + \sqrt{x})^{25}$

   (b) $y = (x^2 + 5)\ln x$

   (c) $y = x^{x+2}$

2. (12 pts) Evaluate each integral and **simplify your answer**.

   (a) $\displaystyle\int x(\sqrt{x} + \frac{1}{x} + x^5)\ dx$

   (b) $\displaystyle\int x\sin(x^2)\ dx$

   (c) $\displaystyle\int \frac{2x^3}{1+x^4}\ dx$

   (d) $\displaystyle\int_1^e \frac{\ln(x)}{x}\ dx$

3. (9 pts) Find each limit (as a real number or $\pm\infty$), or state DNE for does not exist.

   (a) $\displaystyle\lim_{x\to\infty} \frac{3x^3 + 6x^2}{4x^3 - 11}$

   (b) $\displaystyle\lim_{x\to 0} \frac{x}{\tan x}$

   (c) $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x^2} - \frac{1}{x^3}\right)$

4. (a) (4 pts) Assuming $xe^y + \sin(xy) + y = 0$ defines y as a differentiable function of x, find $\frac{dy}{dx}(x, y)$ and find the tangent line at $(0, 0)$.

   (b) (3 pts) If $f(1) = -3$ and $f'(1) = 10$ find the indicated derivative of the inverse function:

   $$(f^{-1})'(-3)$$

   .

   (c) (3 pts) The precise definition of limit states that $\lim_{x\to a} f(x) = L$, where L is a real number, if for every arbitrary number $\epsilon > 0$, there exists a number $\delta > 0$ such that $0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$. Find the largest value of δ (as a function of ϵ) that you can use to prove $\lim_{x\to 1}(5 - 2x) = 3$.

5. (12 pts) **No credit in any part unless you use the requested methods.**

   (a) For $\displaystyle\int_0^1 \sqrt{x}\ dx$:

      i. (4 pts) Approximate this integral using a Riemann sum with $n = 4$ equal length subintervals and right endpoints as the sample points.
      (You may leave the answer as an unsimplified sum.)

      ii. (4 pts) Using the definition, express the value of this integral as a limit of Riemann sums with n equal subintervals and right endpoints as the sample points.
      (You need **not** evaluate the limit or find the numerical value of the integral.)

   (b) (4 pts) Use linear approximation or differentials to estimate $\sqrt{8.9}$.

6. (7 pts) A 5-ft-tall person walks toward a wall at a rate of 2 ft/s. A spotlight is on the ground 40 ft from the wall. How fast does the height of the person's shadow on the wall change when the person is 10 ft from the wall?

7. (6 pts) Let

$$f(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{1}{x}, & 0 < x \le 1 \\ 2, & 1 < x < 3 \\ 4, & x = 3 \\ 2, & x > 3 \end{cases}$$

Use the definition to show why $f(x)$ is not continuous at $x = 0$, $x = 1$, and $x = 3$, and identify the type of each discontinuty.

8. (a) (3 pts) State the Intermediate Value Theorem.

   (b) (3 pts) Prove that

$$f(x) = 2x + 4x^3 + 3$$

   has a root in the interval $(-1, 1)$.

   (c) Let

$$F(x) = \int_0^{x^3} e^{-t^2}\, dt.$$

   i. (3 pts) Using the Fundamental Theorem of Calculus, Part I, find $F'(x)$.

   ii. (2 pts) Is the function $F(x)$ increasing or decreasing? Explain.

9. (6pts) Find $f(x)$ if $f''(x) = 2x + 3\sin x, \quad f(0) = 2, \quad f'(0) = 4.$

10. (8 pts) A rectangular box with an open top has vertical sides, a square bottom, and a volume of 32 cubic meters. Use calculus to find the dimensions of the box with the least possible surface area.

11. (10 pts) You are given

$$f(x) = \frac{2x^2}{(x-4)^2}, \qquad f'(x) = \frac{-16x}{(x-4)^3}, \qquad f''(x) = \frac{32(x+2)}{(x-4)^4}.$$

   (a) Find the domain of $f(x)$, the coordinates of all intercepts, and the equations of all horizontal and vertical asymptotes of the graph of $f(x)$.

   (b) Find the intervals of increase and the intervals of decrease for $f(x)$ and the coordinates of any local maxima and local minima.

   (c) Find the intervals of concavity and the coordinates of any inflection points.

   (d) Sketch the graph of $f(x)$ including all features from (a)(c).