Correct definition from Stewart Essential Calculus 2e:

(2). Definition of a Definite Integral

If f is a function defined on $[a, b]$, the **definite integral of f from a to b** is the number

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

provided that this limit exists. If it does exist, we say that f is **integrable** on $[a, b]$.

The precise meaning of the limit that defines the integral in Definition 2 is as follows:

$\int_a^b f(x)\,dx = I$ means that for every $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

$$\left| I - \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i \right| < \varepsilon$$

for all partitions P of $[a, b]$ with max $\Delta x_i < \delta$ and for all possible choices of x_i^* in $[x_{i-1}, x_i]$.

Incorrect/imprecise definitions.

Stewart 9e:

2 Definition of a Definite Integral

If f is a function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0(= a), x_1, x_2, \cdots, x_n(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any **sample points** in these subintervals, so x_i^* lies in the i th subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\, \Delta x$$

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that f is **integrable** on $[a, b]$.

Strang:

DEFINITION

If $f(x)$ is a function defined on an interval $[a, b]$, the **definite integral** of *f* from *a* to *b* is given by

$$\int_a^b f(x)\, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\, \Delta x, \qquad (5.8)$$

provided the limit exists. If this limit exists, the function $f(x)$ is said to be integrable on $[a, b]$, or is an **integrable function**.