

Read How to

Solve a system of nonlinear equations by substitution
and

Solve a system of equations by elimination

in online text "Intermediate Algebra 2e" section 11.5

Note: for elimination method, we must have a variable with identical power (exponent). Otherwise elimination will not work easily.

The skills that we use for substitution method in this class will help us more in future courses.

Graphing method is omitted.

$$188) \begin{cases} y = 2x + 2 \\ y = -x^2 + 2 \end{cases}$$

substitution

$$2x + 2 = -x^2 + 2$$

$$x^2 + 2x = 0$$

$$x(x + 2) = 0$$

$$x = 0 \quad \left| \quad \begin{array}{l} x + 2 = 0 \\ x = -2 \end{array} \right.$$

$$x = 0$$

$$y = 2(0) + 2 = 2$$

$$\underline{\underline{x = 0, y = 2}}$$

$$x = -2$$

$$y = 2(-2) + 2 = -2$$

$$\underline{\underline{x = -2, y = -2}}$$

$$190) \begin{cases} x+y=2 \rightarrow x=2-y \\ x=y^2 \end{cases} \text{ substitution}$$

$$2-y=y^2$$

$$0=y^2+y-2$$

$$0=(y+2)(y-1)$$

$$y+2=0$$

$$y=-2$$

$$x=(-2)^2=4$$

$$\underline{x=4, y=-2}$$

$$y-1=0$$

$$y=1$$

$$x=(1)^2$$

$$\underline{x=1, y=1}$$

$$192) \begin{cases} y=\frac{3}{2}x+3 \\ y=-x^2+2 \end{cases} \text{ substitution}$$

$$\frac{3}{2}x+3=-x^2+2$$

$$x^2+\frac{3}{2}x+1=0$$

$$2(x^2+\frac{3}{2}x+1)=2(0)$$

$$2x^2+3x+2=0$$

$$a=2 \quad b=3 \quad c=2$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(2)}}{2(2)} = \frac{-3 \pm \sqrt{9-16}}{4} = \frac{-3 \pm \sqrt{-7}}{4}$$

no real number solution

$$194) \begin{cases} x=-2 \\ x^2+y^2=4 \end{cases} \text{ substitution}$$

substitution

$$(-2)^2+y^2=4$$

$$4+y^2=4$$

$$y^2=0$$

$$y=0$$

$$\underline{x=-2, y=0}$$

$$196) \begin{cases} x=2 \\ (x+2)^2+(y+3)^2=16 \end{cases} \text{ substitution}$$

substitution

$$((2)+2)^2+(y+3)^2=16$$

$$(4)^2+(y+3)^2=16$$

$$16+(y+3)^2=16$$

$$(y+3)^2=0$$

$$y+3=0$$

$$y=-3$$

$$\underline{x=2, y=-3}$$

$$198) \begin{cases} y = -2x + 4 \\ y = \sqrt{x} + 1 \end{cases} \quad \text{substitution}$$

$$-2x + 4 = \sqrt{x} + 1$$

$$0 = 2x + \sqrt{x} - 3 \quad \text{here let } a = \sqrt{x}$$

$$0 = 2(\sqrt{x})^2 + (\sqrt{x}) - 3$$

$$0 = (2(\sqrt{x}) + 3)(\sqrt{x} - 1)$$

$$2(\sqrt{x}) + 3 = 0$$

$$2(\sqrt{x}) = -3$$

$$\sqrt{x} = -\frac{3}{2}$$

$$x = \frac{9}{4}$$

$$y = \sqrt{\frac{9}{4}} + 1 = \frac{3}{2} + 1 \\ = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$$

check

$$\left(\frac{5}{2}\right) \stackrel{?}{=} -2\left(\frac{9}{4}\right) + 4$$

$$\frac{5}{2} \quad \frac{-9}{2} + \frac{8}{2}$$

$$\frac{5}{2} \neq \frac{-1}{2}$$

discard

$$\sqrt{x} - 1 = 0$$

$$\sqrt{x} = 1$$

$$x = 1$$

$$y = \sqrt{1} + 1 = 2$$

check

$$(2) \stackrel{?}{=} -2(1) + 4$$

$$2 = 2$$

$$\underline{\underline{x = 1, y = 2}}$$

$$200) \begin{cases} x^2 + 4y^2 = 4 \\ y = \frac{1}{2}x - 1 \end{cases} \quad \text{substitution}$$

$$x^2 + 4\left(\frac{1}{2}x - 1\right)^2 = 4$$

$$x^2 + 4\left(\frac{1}{4}x^2 - x + 1\right) = 4$$

$$x^2 + x^2 - 4x + 4 = 4$$

$$2x^2 - 4x = 0$$

$$2x(x - 2) = 0$$

$$2x = 0$$

$$x = 0$$

$$y = \frac{1}{2}(0) - 1 = -1$$

check

$$(0)^2 + 4(-1)^2 \stackrel{?}{=} 4$$

$$4 = 4$$

✓

$$\underline{\underline{x = 0, y = -1}}$$

$$x - 2 = 0$$

$$x = 2$$

$$y = \frac{1}{2}(2) - 1$$

$$= 1 - 1 = 0$$

check

$$(2)^2 + 4(0)^2 \stackrel{?}{=} 4$$

$$4 = 4$$

✓

$$\underline{\underline{x = 2, y = 0}}$$

$$202) \begin{cases} 9x^2 + y^2 = 9 \\ y = x + 3 \end{cases} \quad \text{substitution}$$

202) continued

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$$9x^2 + (x+3)^2 = 9$$

$$9x^2 + (x^2 + 6x + 9) = 9$$

$$10x^2 + 6x = 0$$

$$2x(5x+3) = 0$$

$$2x = 0$$

$$x = 0$$

$$y = (0) + 3 = 3$$

check

$$9(0)^2 + (3)^2 \stackrel{?}{=} 9$$

$$9 = 9$$

$$\underline{\underline{x=0, y=3}}$$

$$5x+3=0$$

$$5x = -3$$

$$x = \frac{-3}{5}$$

$$y = \left(\frac{-3}{5}\right) + 3 = \frac{-3}{5} + \frac{15}{5} = \frac{12}{5}$$

check

$$9\left(\frac{-3}{5}\right)^2 + \left(\frac{12}{5}\right)^2 \stackrel{?}{=} 9$$

$$\frac{81}{25} + \frac{144}{25} \quad 9$$

$$\frac{225}{25}$$

$$9 = 9$$

$$\underline{\underline{x = \frac{-3}{5}, y = \frac{12}{5}}}$$

204) $\begin{cases} 4x^2 + y^2 = 4 \\ y = 4 \end{cases}$ substitution

$$4x^2 + (4)^2 = 4$$

$$4x^2 + 16 = 4$$

$$4x^2 = -12$$

$$x^2 = \frac{-12}{4} = -3$$

no real number solution

$$206) \begin{cases} 3x^2 - y = 0 \\ y = 2x - 1 \end{cases}$$

substitution

$$3x^2 - (2x - 1) = 0$$

$$3x^2 - 2x + 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$$3x + 1 = 0$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$y = 2\left(-\frac{1}{3}\right) - 1$$

$$= -\frac{2}{3} - \frac{3}{3} = -\frac{5}{3}$$

check

$$3\left(-\frac{1}{3}\right)^2 - \left(-\frac{5}{3}\right) \stackrel{?}{=} 0$$

$$-\frac{1}{3} + \frac{5}{3}$$

$$\frac{4}{3} \neq 0$$

no solution

$$x - 1 = 0$$

$$x = 1$$

$$y = 2(1) - 1 = 1$$

check

$$3(1)^2 - (1) \stackrel{?}{=} 0$$

$$3 - 1$$

$$2 \neq 0$$

$$208) \begin{cases} y = x^2 + 3 \\ y = x + 3 \end{cases}$$

substitution

$$x^2 + 3 = x + 3$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0$$

$$y = (0) + 3 = 3$$

check

$$(3) \stackrel{?}{=} (0)^2 + 3$$

$$3 \stackrel{?}{=} 3$$

$$x = 0, y = 3$$

$$x - 1 = 0$$

$$x = 1$$

$$y = (1) + 3 = 4$$

check

$$(4) \stackrel{?}{=} (1)^2 + 3$$

$$4 \stackrel{?}{=} 4$$

$$x = 1, y = 4$$

$$210) \begin{cases} x^2 + y^2 = 25 \\ x - y = 1 \rightarrow x = y + 1 \end{cases} \quad \text{substitution} \quad 2(y + 4)(y - 3) = 0$$

$$(y + 1)^2 + y^2 = 25$$

$$(y^2 + 2y + 1) + y^2 = 25$$

$$2y^2 + 2y - 24 = 0$$

$$2(y^2 + y - 12) = 0$$

$$y + 4 = 0$$

$$y = -4$$

$$x = (-4) + 1 = -3$$

check

$$(-3)^2 + (-4)^2 \stackrel{?}{=} 25$$

$$9 + 16$$

$$25 \stackrel{?}{=} 25$$

$$x = -3, y = -4$$

$$y - 3 = 0$$

$$y = 3$$

$$x = (3) + 1 = 4$$

check

$$(4)^2 + (3)^2 \stackrel{?}{=} 25$$

$$16 + 9$$

$$25 \stackrel{?}{=} 25$$

$$x = 4, y = 3$$

$$212) \begin{cases} x^2 + y^2 = 16 \longrightarrow x^2 + y^2 = 16 \\ x^2 - 2y = 8 \xrightarrow{-1} -x^2 + 2y = -8 \end{cases}$$

$$y^2 + 2y = 8$$

elimination

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$$y^2 + 2y - 8 = 0$$

$$(y+4)(y-2) = 0$$

$$y+4=0$$

$$y = -4$$

$$x^2 - 2(-4) = 8$$

$$x^2 + 8 = 8$$

$$x^2 = 0$$

$$x = 0$$

check

$$(0)^2 + (-4)^2 \stackrel{?}{=} 16$$

$$16 \stackrel{\checkmark}{=} 16$$

$$\underline{x=0, y=-4}$$

$$y-2=0$$

$$y = 2$$

$$x^2 - 2(2) = 8$$

$$x^2 - 4 = 8$$

$$x^2 = 12$$

$$x = \pm\sqrt{12}$$

check

$$(\pm\sqrt{12})^2 + (2)^2 \stackrel{?}{=} 16$$

$$12 + 4 \checkmark$$

$$16 = 16$$

$$\underline{x = +\sqrt{12}, y=2} \text{ and } \underline{x = -\sqrt{12}, y=2}$$

$$214) \begin{cases} x^2 + y^2 = 4 \longrightarrow x^2 + y^2 = 4 \\ x^2 + 2y = 1 \xrightarrow{-1} -x^2 - 2y = -1 \end{cases}$$

$$y^2 - 2y = 3$$

elimination

$$y^2 - 2y - 3 = 0$$

$$(y+1)(y-3) = 0$$

214) continued

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$y+1=0$	check	$y-3=0$
$y=-1$	$(\pm\sqrt{3})^2 + (-1)^2 \stackrel{?}{=} 4$	$y=3$
$x^2 + 2(-1) = 1$	$3+1$	$x^2 + 2(3) = 1$
$x^2 - 2 = 1$	$4 \stackrel{\checkmark}{=} 4$	$x^2 + 6 = 1$
$x^2 = 3$	$x = +\sqrt{3}, y = -1$	$x^2 = -5$
$x = \pm\sqrt{3}$	and	no real number solution
	$x = -\sqrt{3}, y = -1$	

216) $\begin{cases} x^2 + y^2 = 9 \\ x^2 - y = 3 \end{cases} \xrightarrow{-1} \begin{cases} x^2 + y^2 = 9 \\ -x^2 + y = -3 \end{cases}$ *elimination*

$y^2 + y = 6$

$$y^2 + y - 6 = 0$$

$$(y+3)(y-2) = 0$$

$$\begin{aligned} y+3 &= 0 \\ y &= -3 \end{aligned}$$

$$x^2 - (-3) = 3$$

$$x^2 + 3 = 3$$

$$x^2 = 0$$

$$x = 0$$

check

$$(0)^2 + (-3)^2 \stackrel{?}{=} 9$$

$$9 \stackrel{\checkmark}{=} 9$$

$$\underline{x=0, y=-3}$$

$$y-2=0$$

$$y=2$$

$$x^2 - (2) = 3$$

$$x^2 - 2 = 3$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

check

$$(\pm\sqrt{5})^2 + (2)^2 \stackrel{?}{=} 9$$

$$5+4$$

$$9 \stackrel{\checkmark}{=} 9$$

$$\underline{x = +\sqrt{5}, y = 2}$$

and

$$\underline{x = -\sqrt{5}, y = 2}$$

$$218) \begin{cases} x^2 + y^2 = 25 \\ 2x^2 - 3y^2 = 5 \end{cases} \xrightarrow{3} \begin{cases} 3x^2 + 3y^2 = 75 \\ 2x^2 - 3y^2 = 5 \end{cases} \xrightarrow{\text{elimination}} \underline{5x^2 = 80}$$

11.5 [9]

$$x^2 = 16$$

$$x = \pm 4$$

$$(\pm 4)^2 + y^2 = 25$$

$$16 + y^2 = 25$$

$$y^2 = 9$$

$$y = \pm 3$$

check

$$2(\pm 4)^2 - 3(\pm 3)^2 \stackrel{?}{=} 5$$

$$2(16) - 3(9)$$

$$32 - 27$$

$$5 \stackrel{\checkmark}{=} 5$$

$$\underline{x = +4, y = +4}$$

$$\underline{x = +4, y = -4}$$

$$\underline{x = -4, y = 4}$$

$$\underline{x = -4, y = -4}$$

$$220) \begin{cases} x^2 + y^2 = 13 \\ x^2 - y^2 = 5 \end{cases} \xrightarrow{\text{elimination}}$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

$$(\pm 3)^2 + y^2 = 13$$

$$9 + y^2 = 13$$

$$y^2 = 4$$

$$y = \pm 2$$

check

$$(\pm 3)^2 - (\pm 2)^2 \stackrel{?}{=} 5$$

$$9 - 4$$

$$5 \stackrel{\checkmark}{=} 5$$

$$\underline{x = +3, y = +2}$$

$$\underline{x = +3, y = -2}$$

$$\underline{x = -3, y = +2}$$

$$\underline{x = -3, y = -2}$$

$$222) \begin{cases} 4x^2 + 9y^2 = 36 \\ 2x^2 - 9y^2 = 18 \end{cases} \quad \text{elimination}$$

$$6x^2 = 54$$

$$x^2 = 9$$

$$x = \pm 3$$

$$4(\pm 3)^2 + 9y^2 = 36$$

$$4(9) + 9y^2 = 36$$

$$36 + 9y^2 = 36$$

$$9y^2 = 0$$

$$y^2 = 0$$

$$y = 0$$

check

$$2(\pm 3)^2 - 9(0)^2 \stackrel{?}{=} 18$$

$$2(9)$$

$$18 \stackrel{\checkmark}{=} 18$$

$$\underline{x = +3, y = 0}$$

$$\underline{x = -3, y = 0}$$

$$224) \begin{cases} 4x^2 - y^2 = 4 \\ 4x^2 + y^2 = 4 \end{cases} \quad \text{elimination}$$

$$8x^2 = 8$$

$$x^2 = 1$$

$$x = \pm 1$$

$$4(\pm 1)^2 + y^2 = 4$$

$$4 + y^2 = 4$$

$$y^2 = 0$$

$$y = 0$$

check

$$4(\pm 1)^2 - (0)^2 \stackrel{?}{=} 4$$

$$4 \stackrel{\checkmark}{=} 4$$

$$\underline{x = +1, y = 0}$$

$$\underline{x = -1, y = 0}$$

$$226) \begin{cases} 2x^2 + y^2 = 11 \rightarrow 2x^2 + y^2 = 11 \\ x^2 + 3y^2 = 28 \xrightarrow{-2} -2x^2 - 6y^2 = -56 \end{cases}$$

elimination

$$-5y^2 = -45$$

$$y^2 = 9$$

$$y = \pm 3$$

$$x^2 + 3(\pm 3)^2 = 28$$

$$x^2 + 3(9) = 28$$

$$x^2 + 27 = 28$$

$$x^2 = 1$$

$$x = \pm 1$$

check

$$2(\pm 1)^2 + (\pm 3)^2 \stackrel{?}{=} 11$$

$$2 + 9$$

$$11 \stackrel{\checkmark}{=} 11$$

$$\underline{x = +1, y = +1}$$

$$\underline{x = +1, y = -1}$$

$$\underline{x = -1, y = +1}$$

$$\underline{x = -1, y = -1}$$