section 9.2

Sum and Difference formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

omit Sum and Difference formulas for Tangent

4) $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} = \underline{\underline{\frac{1-\sqrt{3}}{2\sqrt{2}}}}$

or $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \underline{\underline{\frac{\sqrt{2}-\sqrt{6}}{4}}}$

6) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

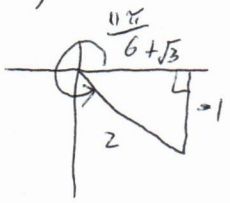
$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \underline{\underline{\frac{\sqrt{3}+1}{2\sqrt{2}}}}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \underline{\underline{\frac{\sqrt{6}+\sqrt{2}}{4}}}$

10) $\sin\left(x+\frac{11\pi}{6}\right) = \sin x \cos\left(\frac{11\pi}{6}\right) + \cos x \sin\left(\frac{11\pi}{6}\right)$



$= (\sin x)\left(\frac{\sqrt{3}}{2}\right) + (\cos x)\left(\frac{-1}{2}\right)$

$= \frac{\sqrt{3}\sin x}{2} - \frac{\cos x}{2} = \underline{\underline{\frac{\sqrt{3}\sin x - \cos x}{2}}}$

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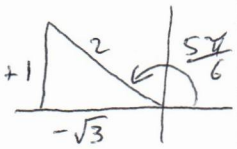
12) $\cos\left(x-\frac{5\pi}{6}\right) = \cos x \cos\left(\frac{5\pi}{6}\right) + \sin x \sin\left(\frac{5\pi}{6}\right)$



$= (\cos x)\left(\frac{-\sqrt{3}}{2}\right) + (\sin x)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{3}\cos x}{2} + \frac{\sin x}{2} = \underline{\underline{\frac{-\sqrt{3}\cos x + \sin x}{2}}}$

$= \underline{\underline{\frac{\sin x - \sqrt{3}\cos x}{2}}}$

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14) $\csc\left(\frac{\pi}{2}-t\right) = ?$

$\sin\left(\frac{\pi}{2}-t\right) = \sin\left(\frac{\pi}{2}\right)\cos t - \cos\left(\frac{\pi}{2}\right)\sin t$

$= (1)(\cos t) - (0)(\sin t) = \cos t$

$\csc\left(\frac{\pi}{2}-t\right) = \frac{1}{\sin\left(\frac{\pi}{2}-t\right)} = \frac{1}{\cos t} = \underline{\underline{\sec t}}$

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16) $\cot\left(\frac{\pi}{2}-x\right) = ?$

$\sin\left(\frac{\pi}{2}-x\right) = \sin\left(\frac{\pi}{2}\right)\cos x - \cos\left(\frac{\pi}{2}\right)\sin x$

$= (1)(\cos x) - (0)\sin x = \cos x$

$\cos\left(\frac{\pi}{2}-x\right) = \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x$

$= (0)(\cos x) + (1)(\sin x) = \sin x$

$\cot\left(\frac{\pi}{2}-x\right)$

$= \frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

$= \frac{\sin x}{\cos x} = \underline{\underline{\tan x}}$

18) $\sin(2x)\cos(5x) - \sin(5x)\cos(2x)$

$= \sin(2x)\cos(5x) - \cos(2x)\sin(5x)$

$= \sin((2x)-(5x)) = \underline{\sin(-3x)}$

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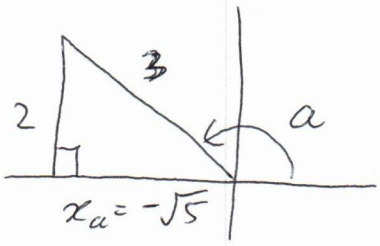
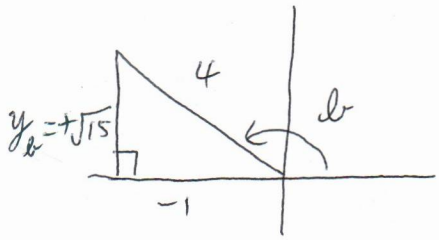
20) $\sin a = \frac{2}{3}$ and $\cos b = \frac{-1}{4}$ in $\left[\frac{\pi}{2}, \pi\right)$





$x_a^2 + (2)^2 = (3)^2$

$x_a^2 + 4 = 9$

$x_a^2 = 5$

$x_a = \pm\sqrt{5}$

$y_b^2 + (-1)^2 = (4)^2$

$y_b^2 + 1 = 16$

$y_b^2 = 15$

$y_b = \pm\sqrt{15}$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$= \left(\frac{2}{3}\right)\left(\frac{-1}{4}\right) + \left(\frac{-\sqrt{5}}{3}\right)\left(\frac{\sqrt{15}}{4}\right)$

$= \frac{-2}{12} - \frac{\sqrt{75}}{12} = \underline{\frac{-2-\sqrt{75}}{12}}$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$= \left(\frac{-\sqrt{5}}{3}\right)\left(\frac{-1}{4}\right) + \left(\frac{2}{3}\right)\left(\frac{\sqrt{15}}{4}\right)$

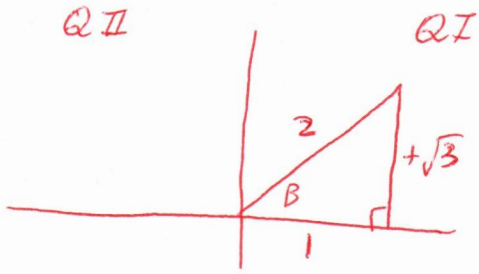
$= \frac{\sqrt{5}}{12} + \frac{2\sqrt{15}}{12} = \underline{\frac{\sqrt{5}+2\sqrt{15}}{12}}$

22) $\sin\left(\cos^{-1}(0) - \cos^{-1}\left(\frac{1}{2}\right)\right)$

$A = \cos^{-1}(0)$ $\quad B = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos A = 0$ $\quad \cos B = \frac{1}{2}$

$A = \frac{\pi}{2}$ $\quad B = \frac{\pi}{3}$



$$\sin\left(\cos^{-1}(0) - \cos^{-1}\left(\frac{1}{2}\right)\right) = \sin\left(\left(\frac{\pi}{2}\right) - \left(\frac{\pi}{3}\right)\right) = \sin\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{6}\right) = \underline{\underline{\frac{1}{2}}}$$

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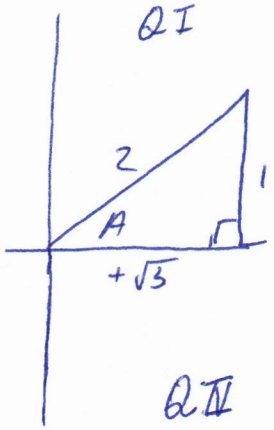
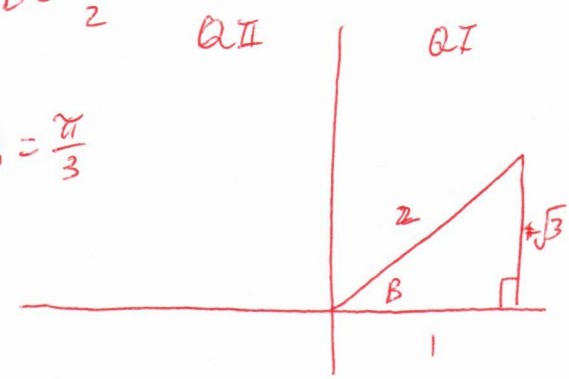
24) $\tan\left(\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right)$

$A = \sin^{-1}\left(\frac{1}{2}\right)$ $\quad B = \cos^{-1}\left(\frac{1}{2}\right)$

$\sin A = \frac{1}{2}$ $\quad \cos B = \frac{1}{2}$

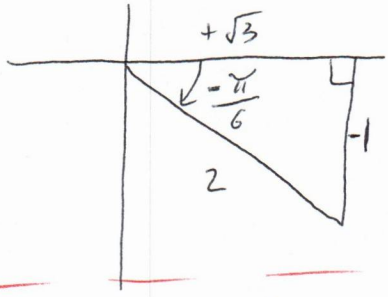
$A = \frac{\pi}{6}$ $\quad B = \frac{\pi}{3}$





$$\tan\left(\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right) = \tan\left(\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{3}\right)\right) = \tan\left(\frac{\pi}{6} - \frac{2\pi}{6}\right)$$

$$= \tan\left(\frac{-\pi}{6}\right) = \underline{\underline{\frac{-1}{\sqrt{3}}}}$$



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