section 9.1

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \Longleftrightarrow \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \Longleftrightarrow \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \Longleftrightarrow \tan\theta = \frac{1}{\cot\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\tan\theta}$$

Pythagorean Identities

$$\underline{\cos^2\theta + \sin^2\theta = 1}$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \rightarrow \underline{1 + \tan^2\theta = \sec^2\theta}$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \longrightarrow \underline{\cot^2\theta + 1 = \csc^2\theta}$$

Even-Odd Identities

odd

$\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$

$\cot(-\theta) = -\cot\theta$

even

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

6) $\sin(-x)\cos(-x)\csc(-x)$

$= (-\sin x)(\cos x)(-\csc x)$

$= (-\sin x)(\cos x)\left(\frac{-1}{\sin x}\right) = \underline{\underline{\cos x}}$

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8) $\csc x + \cos x \cot(-x)$

$= \csc x + (\cos x)(-\cot x)$

$= \left(\frac{1}{\sin x}\right) + (\cos x)\left(\frac{-\cos x}{\sin x}\right)$

$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} = \frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \underline{\underline{\sin x}}$

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10) $3\sin^3 t \csc t + \cos^2 t + 2\cos(-t)\cos t$

$= 3\sin^3 t\left(\frac{1}{\sin t}\right) + \cos^2 t + 2(\cos t)\cos t$

$= 3\sin^2 t + \cos^2 t + 2\cos^2 t$

$= 3\sin^2 t + 3\cos^2 t$

$= 3(\sin^2 t + \cos^2 t)$

$= 3(1)$

$= \underline{\underline{3}}$

12) $\dfrac{-\sin(-x)\cos x \sec x \csc x \tan x}{\cot x}$

$= \dfrac{-(-\sin x)\cos x\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)}{\frac{\cos x}{\sin x}}$

$= \dfrac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}} = \left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right) = \dfrac{\sin^2 x}{\cos^2 x} = \underline{\underline{\tan^2 x}}$

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14) $\left(\dfrac{\tan x}{\csc^2 x} + \dfrac{\tan x}{\sec^2 x}\right)\left(\dfrac{1+\tan x}{1+\cot x}\right) - \dfrac{1}{\cos^2 x}$

$= \left(\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\sin^2 x}} + \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}}\right)\left(\dfrac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}}\right) - \dfrac{1}{\cos^2 x}$

$= \left(\dfrac{\sin^3 x}{\cos x} + \dfrac{\sin x\cos^2 x}{\cos x}\right)\left(\dfrac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}}\right) - \dfrac{1}{\cos^2 x}$

$= \left(\dfrac{\sin^3 x + \sin x\cos^2 x}{\cos x}\right)\left(\dfrac{\frac{\cos x+\sin x}{\cos x}}{\frac{\sin x+\cos x}{\sin x}}\right) - \dfrac{1}{\cos^2 x}$

$= \left(\dfrac{\sin x(\sin^2 x+\cos^2 x)}{\cos x}\right)\left(\left(\dfrac{\cos x+\sin x}{\cos x}\right)\left(\dfrac{\sin x}{\sin x+\cos x}\right)\right) - \dfrac{1}{\cos^2 x}$

$= \left(\dfrac{(\sin x)(1)}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right) - \dfrac{1}{\cos^2 x} = \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$

$= \dfrac{\sin^2 x - 1}{\cos^2 x} = \dfrac{-(1-\sin^2 x)}{\cos^2 x} = \dfrac{-(\cos^2 x)}{\cos^2 x} = \underline{\underline{-1}}$

simplify 1st trig expression in terms of 2nd trig expression

16) $\dfrac{\tan x + \cot x}{\csc x}$; $\cos x$

$$= \frac{\tan x}{\csc x} + \frac{\cot x}{\csc x} = \frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\sin x}\right)} + \frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)} = \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin x}{1}\right) + \left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{1}\right)$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} = \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right) = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x} = \underline{\underline{\frac{1}{\cos x}}}$$

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18) $\dfrac{\cos x}{1 + \sin x} + \tan x$; $\cos x$

$$= \left(\frac{\cos x}{1 + \sin x}\right)\left(\frac{1 - \sin x}{1 - \sin x}\right) + \tan x$$

$$= \frac{\cos x\,(1 - \sin x)}{1 - \sin^2 x} + \tan x = \frac{\cos x - (\cos x)(\sin x)}{\cos^2 x} + \tan x$$

$$= \frac{\cos x - \sin x \cos x}{\cos^2 x} + \tan x = \frac{\cos x}{\cos^2 x} - \frac{\sin x \cos x}{\cos^2 x} + \tan x$$

$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x} + \tan x = \frac{1}{\cos x} - \tan x + \tan x = \underline{\underline{\frac{1}{\cos x}}}$$

20) $\dfrac{1}{1-\cos x} - \dfrac{\cos x}{1+\cos x}$; $\csc x = \dfrac{1}{\sin x}$

$= \left(\dfrac{1}{1-\cos x}\right)\left(\dfrac{1+\cos x}{1+\cos x}\right) - \left(\dfrac{\cos x}{1+\cos x}\right)\left(\dfrac{1-\cos x}{1-\cos x}\right)$

$= \dfrac{(1+\cos x)}{1-\cos^2 x} - \dfrac{\cos x(1-\cos x)}{1-\cos^2 x}$

$= \dfrac{(1+\cos x)}{\sin^2 x} - \dfrac{(\cos x - \cos^2 x)}{\sin^2 x} = \dfrac{(1+\cos x)-(\cos x-\cos^2 x)}{\sin^2 x}$

$= \dfrac{1+\cos^2 x}{\sin^2 x} = \dfrac{1+(1-\sin^2 x)}{\sin^2 x} = \dfrac{2-\sin^2 x}{\sin^2 x}$

$= \dfrac{2}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x} = \underline{\underline{2\csc^2 x - 1}}$

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22) $\dfrac{1}{\csc x - \sin x}$; $\sec x = \dfrac{1}{\cos x}$ and $\tan x$

$= \dfrac{1}{\left(\frac{1}{\sin x}\right) - \sin x} = \dfrac{1}{\left(\frac{1}{\sin x}\right) - \left(\frac{\sin x}{1}\right)\left(\frac{\sin x}{\sin x}\right)} = \dfrac{1}{\frac{1-\sin^2 x}{\sin x}}$

$= \dfrac{1}{\frac{\cos^2 x}{\sin x}} = \dfrac{1}{1}\left(\dfrac{\sin x}{\cos^2 x}\right) = \dfrac{\sin x}{\cos^2 x}$

$= \left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{1}{\cos x}\right) = \underline{\underline{(\tan x)(\sec x)}}$

24) $\tan x$; $\sec x = \frac{1}{\cos x}$

* using Pythagorean Identity in Trigonometric form

$$1 + \tan^2 x = \sec^2 x$$
$$\tan^2 x = \sec^2 x - 1$$
$$\underline{\underline{\tan x = \pm\sqrt{\sec^2 x - 1}}}$$

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26) $\sec x$; $\sin x$

$$= \frac{1}{\cos x}$$
$$= \underline{\underline{\frac{1}{\pm\sqrt{1 - \sin^2 x}}}}$$

* $\cos^2 x + \sin^2 x = 1$

$$\cos^2 x = 1 - \sin^2 x$$
$$\cos x = \pm\sqrt{1 - \sin^2 x}$$

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28) $\cot x$; $\csc x = \frac{1}{\sin x}$

* $\cot^2 x + 1 = \csc^2 x$

$$\underline{\underline{\pm\sqrt{\cot^2 x + 1} = \csc x}}$$

verify the identity

30) $\cos x(\tan x - \sec(-x)) = \sin x - 1$

$\cos x\left(\frac{\sin x}{\cos x} - (\sec x)\right)$

$\cos x\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right)$

$\cos x\left(\frac{\sin x}{\cos x}\right) - \cos x\left(\frac{1}{\cos x}\right)$

$\sin x - 1 \overset{\checkmark}{=} \sin x - 1$

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32) $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$\sin^2 x + \cos^2 x + 2\sin x\cos x$

$(\sin^2 x + \cos^2 x) + 2\sin x\cos x$

$1 + 2\sin x\cos x \overset{\checkmark}{=} 1 + 2\sin x\cos x$

prove or disprove the identity

34) $\dfrac{1}{1+\cos x} - \dfrac{1}{1-\cos(-x)} \overset{?}{=} -2\cot x \csc x$

$\dfrac{1}{1+\cos x} - \dfrac{1}{1-\cos x}$ $\qquad -2\left(\dfrac{\cos x}{\sin x}\right)\left(\dfrac{1}{\sin x}\right)$

$\left(\dfrac{1}{1+\cos x}\right)\left(\dfrac{1-\cos x}{1-\cos x}\right) - \left(\dfrac{1}{1-\cos x}\right)\left(\dfrac{1+\cos x}{1+\cos x}\right)$ $\qquad \dfrac{-2\cos x}{\sin^2 x}$

$\dfrac{(1-\cos x)}{1-\cos^2 x} - \dfrac{(1+\cos x)}{1-\cos^2 x}$

$\dfrac{(1-\cos x)}{\sin^2 x} - \dfrac{(1+\cos x)}{\sin^2 x}$

$\dfrac{(1-\cos x)-(1+\cos x)}{\sin^2 x}$

$\dfrac{1-\cos x-1-\cos x}{\sin^2 x}$

$\dfrac{-2\cos x}{\sin^2 x} = \dfrac{-2\cos x}{\sin^2 x}$

proved

36) $\left(\dfrac{\sec^2(-x)-\tan^2 x}{\tan x}\right)\left(\dfrac{2+2\tan x}{2+2\cot x}\right)-2\sin^2 x \overset{?}{=} \cos 2x$

$\left(\dfrac{\sec^2 x-\tan^2 x}{\tan x}\right)\left(\dfrac{2(1+\tan x)}{2(1+\cot x)}\right)-2\sin^2 x$ $\qquad 1-2\sin^2 x$

$\left(\dfrac{(1+\tan^2 x)-\tan^2 x}{\tan x}\right)\left(\dfrac{1+\tan x}{1+\cot x}\right)-2\sin^2 x$

$\left(\dfrac{1}{\tan x}\right)\left(\dfrac{1+\dfrac{\sin x}{\cos x}}{1+\dfrac{\cos x}{\sin x}}\right)-2\sin^2 x$

$\left(\dfrac{1}{\dfrac{\sin x}{\cos x}}\right)\left(\dfrac{\dfrac{\cos x+\sin x}{\cos x}}{\dfrac{\sin x+\cos x}{\sin x}}\right)-2\sin^2 x$

$\left(\dfrac{\cos x}{\sin x}\right)\left(\left(\dfrac{\cos x+\sin x}{\cos x}\right)\left(\dfrac{\sin x}{\sin x+\cos x}\right)\right)-2\sin^2 x$

in section 9.3 double angle formulas
is introduced
$\cos(2\theta)=1-\sin^2\theta$

$1-2\sin^2 x = 1-2\sin^2 x$

proved

40) $$\frac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} \overset{?}{=} \sin^2\theta$$

$$\frac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1-\left(\frac{\sin^2\theta}{\cos^2\theta}\right)} = \frac{\cos^2\theta - \sin^2\theta}{1\left(\frac{\cos^2\theta}{\cos^2\theta}\right) - \frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}} = \left(\frac{\cos^2\theta - \sin^2\theta}{1}\right)\left(\frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\right) = \underline{\underline{\cos^2\theta}}$$

false

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42) $$\frac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta} \overset{?}{=} \sec^2\theta$$

$$\frac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta} = \frac{\left(\frac{1}{\cos\theta}\right) + \left(\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{\cos\theta}{\sin\theta}\right) + \left(\frac{\cos\theta}{1}\right)} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \left(\frac{\cos\theta}{1}\right)\left(\frac{\sin\theta}{\sin\theta}\right)}$$

$$= \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta\cos\theta}{\sin\theta}} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{\cos\theta(1+\sin\theta)}{\sin\theta}} = \left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta}{\cos\theta(1+\sin\theta)}\right)$$

$$= \frac{\sin\theta}{\cos^2\theta} = (\sin\theta)\left(\frac{1}{\cos^2\theta}\right) = \underline{\underline{\sin\theta\sec^2\theta}}$$

false