section 8.3

Inverse Trigonometric Functions

inverse sine function (arcsine) $y = \sin^{-1}(x)$

domain: $[-1, 1]$ range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ QIV and QI

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inverse cosine function (arccosine) $y = \cos^{-1}(x)$

domain: $[-1, 1]$ range: $[0, \pi]$ QI and QII

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inverse tangent function (arctangent) $y = \tan^{-1}(x)$

domain: $(-\infty, \infty)$ range $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ QIV and QI

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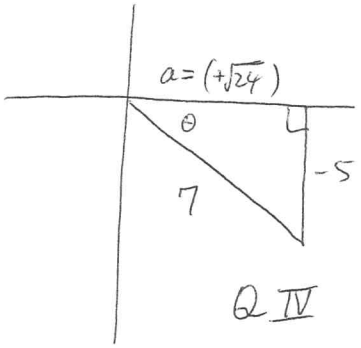
we can use Quadrant restriction of range to draw a correct triangle to make our evaluations easier.

for example:

$\theta = \sin^{-1}\left(\frac{-5}{7}\right)$

$\sin\theta = \frac{-5}{7} < 0$ QIV



$a^2 + (-5)^2 = (7)^2$

$a^2 + 25 = 49$

$a^2 = 24$

$a = \pm\sqrt{24}$

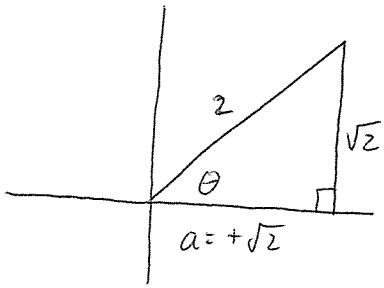
8) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\Downarrow$

$\sin\theta = \frac{\sqrt{2}}{2} > 0 \rightarrow$ Q I

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$



$a^2 + (\sqrt{2})^2 = (2)^2$

$a^2 + 2 = 4$

$a^2 = 2$

$a = \pm\sqrt{2}$

$\theta = \frac{\pi}{4}$

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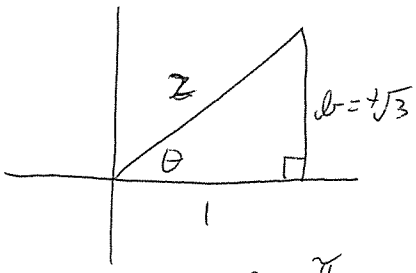
10) $\cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\cos\theta = \frac{1}{2} > 0 \rightarrow$ QI

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$



$b^2 + (1)^2 = (2)^2$

$b^2 + 1 = 4$

$b^2 = 3$

$b = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}$

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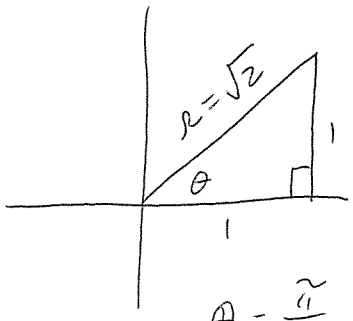
12) $\tan^{-1}(1)$

$\theta = \tan^{-1}(1)$

$\Downarrow$

$\tan\theta = 1 = \frac{1}{1} > 0 \rightarrow$ QI

$\tan^{-1}(1) = \frac{\pi}{4}$



$r^2 = (1)^2 + (1)^2$

$r^2 = 1 + 1$

$r^2 = 2$

$r = \pm\sqrt{2}$

$\theta = \frac{\pi}{4}$

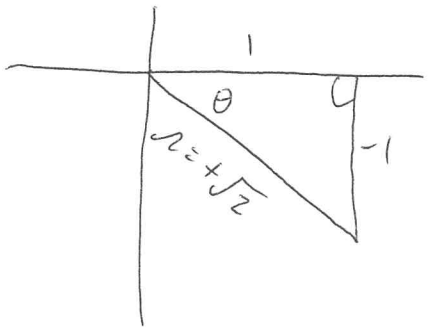
14) $\tan^{-1}(-1)$

$\theta = \tan^{-1}(-1)$

$\Downarrow$

$\tan\theta = -1 = \frac{-1}{1} < 0 \rightarrow$ QIV

$\tan^{-1}(-1) = \frac{-\pi}{4}$



$r^2 = (1)^2 + (-1)^2$

$r^2 = 1 + 1$

$r^2 = 2$

$r = \pm\sqrt{2}$

$\theta = \frac{-\pi}{4}$

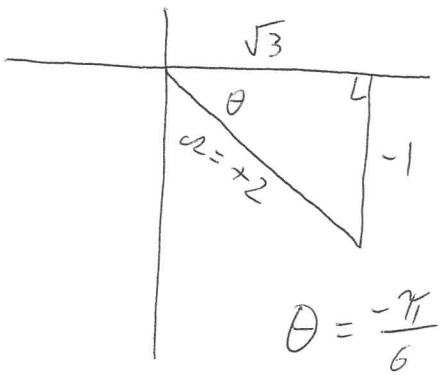
16) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$\Downarrow$

$\tan\theta = \frac{-1}{\sqrt{3}} < 0 \rightarrow$ QIV

$\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$



$r^2 = (\sqrt{3})^2 + (-1)^2$

$r^2 = 3 + 1$

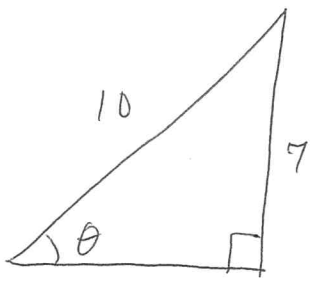
$r^2 = 4$

$r = \pm 2$

$\theta = \frac{-\pi}{6}$

skip 18-21; 22, 23 exact answer (no calculator)

22)



$\sin\theta = \frac{7}{10}$

$\Downarrow$

$\theta = \sin^{-1}\left(\frac{7}{10}\right)$

24) $\sin^{-1}(\cos(\pi)) = \sin^{-1}(-1) = \underline{\underline{\frac{-\pi}{2}}}$

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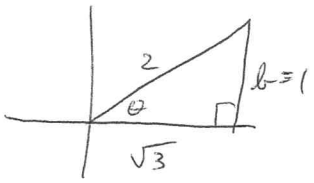
26) $\cos^{-1}(\sin\frac{\pi}{3}) = \cos^{-1}(\frac{\sqrt{3}}{2}) = \underline{\underline{\frac{\pi}{6}}}$

$\theta = \cos^{-1}(\frac{\sqrt{3}}{2})$

$\Downarrow$

$\cos\theta = \frac{\sqrt{3}}{2} > 0 \rightarrow$ QI



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28) $\sin^{-1}(\cos(\frac{-\pi}{2})) = \sin^{-1}(0) = \underline{\underline{0}}$

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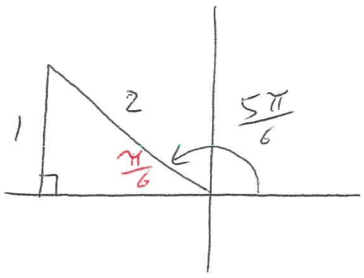
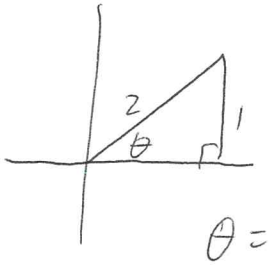
30) $\sin^{-1}(\sin\frac{5\pi}{6}) = \sin^{-1}(\frac{1}{2}) = \underline{\underline{\frac{\pi}{6}}}$



$\theta = \sin^{-1}(\frac{1}{2})$

$\Downarrow$

$\sin\theta = \frac{1}{2} > 0 \rightarrow$ QI



$\theta = \frac{\pi}{6}$

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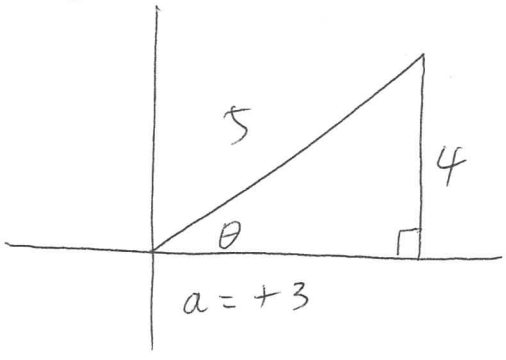
32) $\cos(\sin^{-1}(\frac{4}{5})) = \underline{\cos(\theta)} = \frac{3}{5}$

visualize this

$\theta = \sin^{-1}(\frac{4}{5})$

$\Downarrow$

$\sin\theta = \frac{4}{5} > 0 \rightarrow$ QI



$a^2 + (4)^2 = (5)^2$

$a^2 + 16 = 25$

$a^2 = 9$

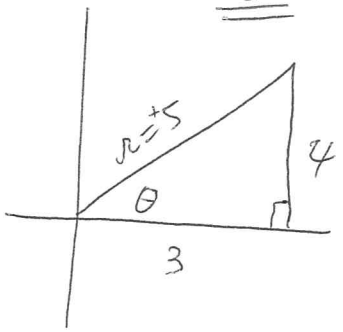
$a = \pm 3$

34) $\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right) = \sin(\theta) = \underline{\underline{\frac{4}{5}}}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\Downarrow$

$\tan\theta = \frac{4}{3} > 0 \rightarrow QI$



$r^2 = (3)^2 + (4)^2$

$r^2 = 9 + 16$

$r^2 = 25$

$r = \pm 5$

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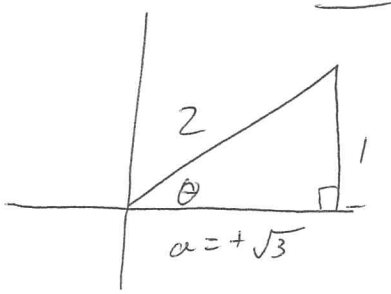
36) $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \cos(\theta) = \underline{\underline{\frac{\sqrt{3}}{2}}}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\Downarrow$

$\sin\theta = \frac{1}{2} > 0 \rightarrow QI$



$a^2 + (1)^2 = (2)^2$

$a^2 + 1 = 4$

$a^2 = 3$

$a = \pm\sqrt{3}$

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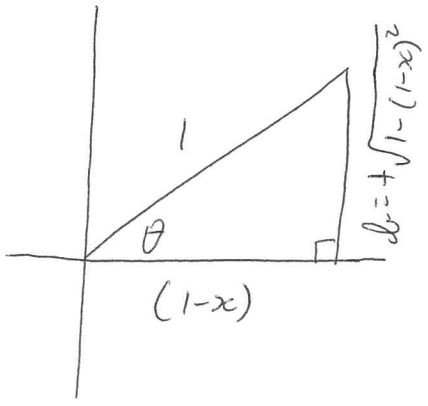
38) $\sin\left(\cos^{-1}(1-x)\right) = \sin(\theta) = \frac{\sqrt{1-(1-x)^2}}{1} = \underline{\underline{\sqrt{1-(1-x)^2}}}$

$\theta = \cos^{-1}(1-x)$

$\Downarrow$

$\cos\theta = (1-x) = \frac{(1-x)}{1}$

and use QI



$b^2 + (1-x)^2 = (1)^2$

$b^2 = (1)^2 - (1-x)^2$

$b = \pm\sqrt{(1)^2 - (1-x)^2}$

$b = +\sqrt{1-(1-x)^2}$

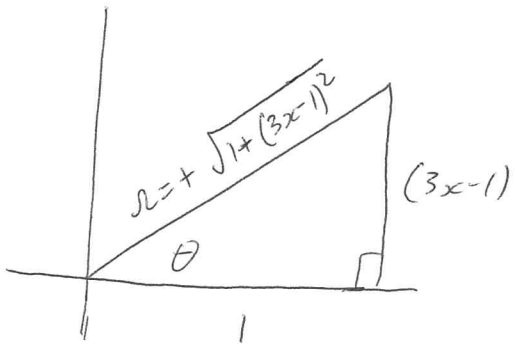
40) $\cos\left(\tan^{-1}(3x-1)\right) = \cos(\theta) = \dfrac{1}{\sqrt{1+(3x-1)^2}}$

$\theta = \tan^{-1}(3x-1)$

$\Downarrow$

$\tan\theta = (3x-1) = \dfrac{(3x-1)}{1}$

and use QI



$r^2 = (1)^2 + (3x-1)^2$

$r = \pm\sqrt{(1)^2+(3x-1)^2}$