section 7.4

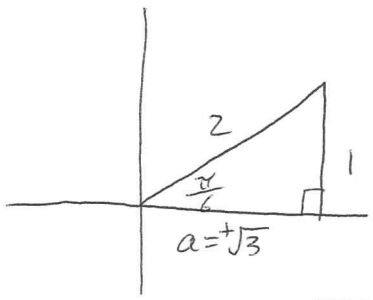
Alternate Forms of the Pythagorean Identity

$\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$

$1 + \tan^2 t = \sec^2 t$

$\frac{\cos^2 t}{\sin^2 t} + \frac{\sin^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}$

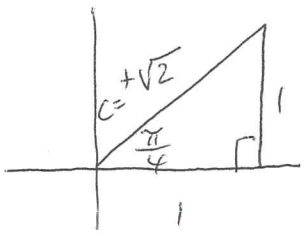
$\cot^2 t + 1 = \csc^2 t$



$a^2 + (1)^2 = (2)^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \pm\sqrt{3}$

6) $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

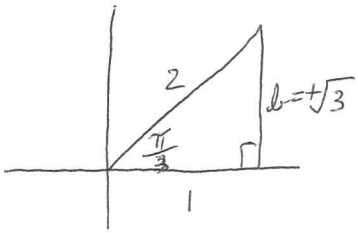
8) $\sin\frac{\pi}{6} = \frac{1}{2} \rightarrow \csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{(\frac{1}{2})} = 2$



$c^2 = (1)^2 + (1)^2$
$c^2 = 1 + 1$
$c^2 = 2$
$c = \pm\sqrt{2}$

10) $\tan\frac{\pi}{4} = \frac{1}{1} = 1$

12) $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \rightarrow \csc\frac{\pi}{4} = \frac{1}{\sin\frac{\pi}{4}} = \frac{1}{(\frac{1}{\sqrt{2}})} = \sqrt{2}$

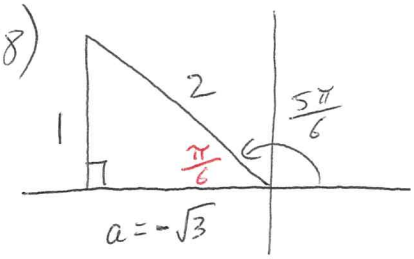


$b^2 + (1)^2 = (2)^2$
$b^2 + 1 = 4$
$b^2 = 3$
$b = \pm\sqrt{3}$

14) $\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$

16) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \rightarrow \csc\frac{\pi}{3} = \frac{1}{\sin\frac{\pi}{3}} = \frac{1}{(\frac{\sqrt{3}}{2})} = \frac{2}{\sqrt{3}}$

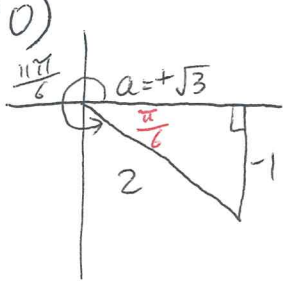
18)



$a^2 + (1)^2 = (2)^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \pm\sqrt{3}$

$\tan\frac{5\pi}{6} = \frac{1}{(-\sqrt{3})} = \frac{-1}{\sqrt{3}}$

20)

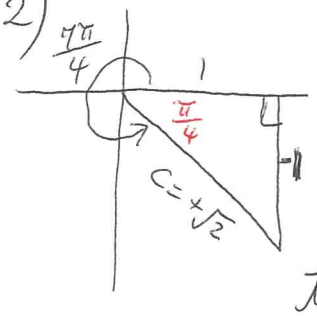


$a^2 + (1)^2 = (2)^2$
$a^2 + 1 = 4$
$a^2 = 3$
$a = \pm\sqrt{3}$

$\csc\frac{11\pi}{6} = \frac{1}{\sin\frac{11\pi}{6}} = \frac{1}{(\frac{-1}{2})} = -2$

7.4 12

22)



$c^2=(1)^2+(1)^2$

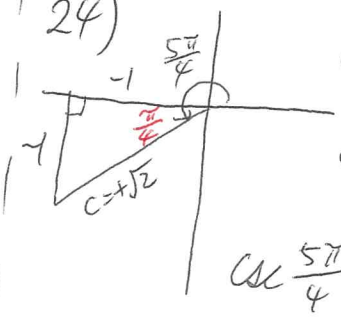
$c^2=1+1$

$c^2=2$

$c=\pm\sqrt{2}$

$\tan\frac{7\pi}{4}=\frac{(-1)}{1}=-1$

24)



$c^2=(1)^2+(1)^2$

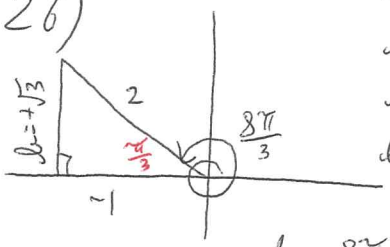
$c^2=1+1$

$c^2=2$

$c=\pm\sqrt{2}$

$\csc\frac{5\pi}{4}=\frac{1}{\sin\frac{5\pi}{4}}=\frac{1}{\left(\frac{-1}{\sqrt{2}}\right)}=-\sqrt{2}$

26)



$b^2+(-1)^2=(2)^2$

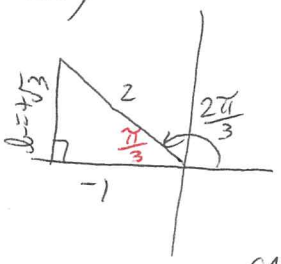
$b^2+1=4$

$b^2=3$

$b=\pm\sqrt{3}$

$\tan\frac{8\pi}{3}=\frac{+\sqrt{3}}{(-1)}=-\sqrt{3}$

28)



$b^2+(-1)^2=(2)^2$

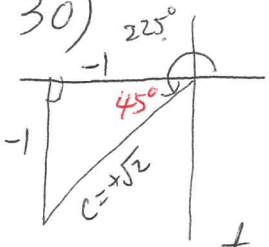
$b^2+1=4$

$b^2=3$

$b=\pm\sqrt{3}$

$\csc\frac{2\pi}{3}=\frac{1}{\sin\frac{2\pi}{3}}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}$

30)



$c^2=(-1)^2+(-1)^2$

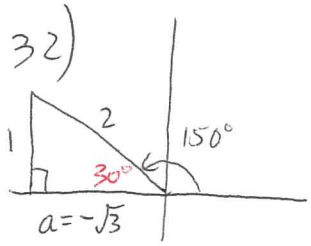
$c^2=1+1$

$c^2=2$

$c=\pm\sqrt{2}$

$\tan(225^\circ)=\frac{(-1)}{(-1)}=1$

32)



$a^2+(1)^2=(2)^2$

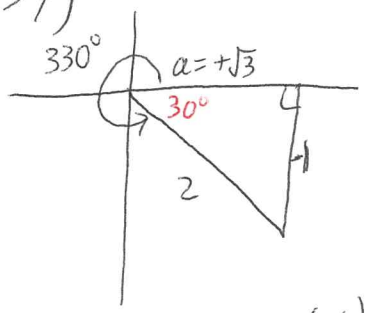
$a^2+1=4$

$a^2=3$

$a=\pm\sqrt{3}$

$\csc(150^\circ)=\frac{1}{\sin(150^\circ)}=\frac{1}{\left(\frac{1}{2}\right)}=2$

34)



$a^2+(-1)^2=(2)^2$

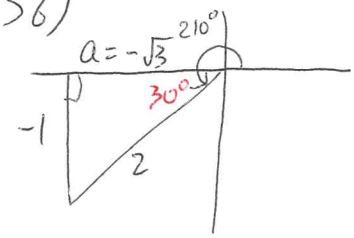
$a^2+1=4$

$a^2=3$

$a=\pm\sqrt{3}$

$\tan(330^\circ)=\frac{(-1)}{\sqrt{3}}=\frac{-1}{\sqrt{3}}$

36)



$a^2+(-1)^2=(2)^2$

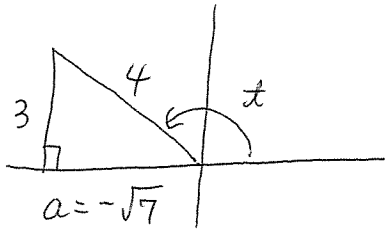
$a^2+1=4$

$a^2=3$

$a=\pm\sqrt{3}$

$\csc(210^\circ)=\frac{1}{\sin(210^\circ)}=\frac{1}{\left(\frac{-1}{2}\right)}=-2$

38) $\sin t = \frac{3}{4}$, t in Q II



$a^2 + (3)^2 = (4)^2$

$a^2 + 9 = 16$

$a^2 = 7$

$a = \pm\sqrt{7}$

$\cos t = \frac{-\sqrt{7}}{4}$

$\tan t = \frac{3}{-\sqrt{7}} = \frac{-3}{\sqrt{7}}$

$\csc t = \frac{1}{\sin t} = \frac{1}{(\frac{3}{4})} = \frac{4}{3}$

$\sec t = \frac{1}{\cos t} = \frac{1}{(\frac{-\sqrt{7}}{4})} = \frac{-4}{\sqrt{7}}$

$\cot t = \frac{1}{\tan t} = \frac{1}{(\frac{4}{3})} = \frac{3}{4}$

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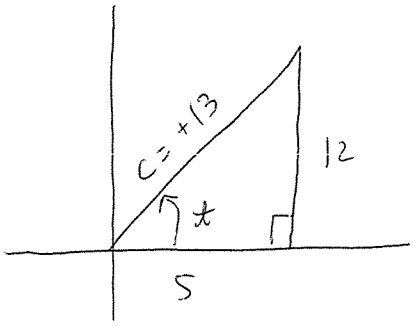
40) $\tan t = \frac{12}{5}$, $0 \le t < \frac{\pi}{2}$

$\Downarrow$

Q I



$c^2 = (5)^2 + (12)^2$

$c^2 = 25 + 144$

$c^2 = 169$

$c = \pm\sqrt{169} = \pm 13$

$\tan t = \frac{12}{5} \rightarrow \cot t = \frac{1}{\tan t} = \frac{1}{(\frac{12}{5})} = \frac{5}{12}$

$\sin t = \frac{12}{13} \rightarrow \csc t = \frac{1}{\sin t} = \frac{1}{(\frac{12}{13})} = \frac{13}{12}$

$\cos t = \frac{5}{13} \rightarrow \sec t = \frac{1}{\cos t} = \frac{1}{(\frac{5}{13})} = \frac{13}{5}$

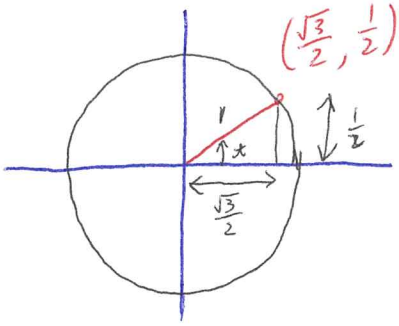
42) $\sin 40^\circ \approx 0.643$ and $\cos 40^\circ \approx 0.766$

$\csc 40^\circ = \frac{1}{\sin 40^\circ} = \frac{1}{0.643}$ $\quad$ $\sec 40^\circ = \frac{1}{\cos 40^\circ} = \frac{1}{0.766}$

$\tan 40^\circ = \frac{\sin 40^\circ}{\cos 40^\circ} = \frac{0.643}{0.766}$ $\quad$ $\cot 40^\circ = \frac{1}{\tan 40^\circ} = \frac{1}{\left(\frac{0.643}{0.766}\right)} = \frac{0.766}{0.643}$

skip 43-48 | exact answers for 52-61 (no calculator)

50)

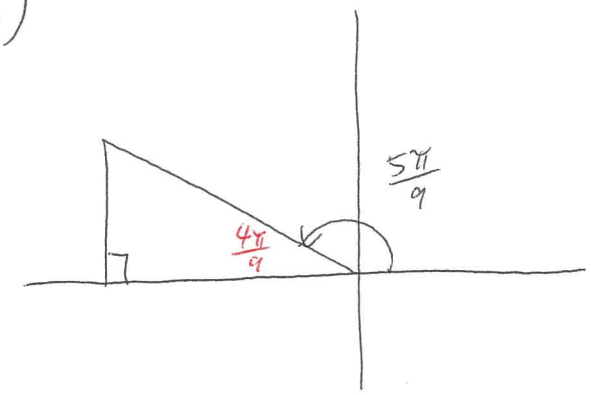


$\sin t = \frac{\left(\frac{1}{2}\right)}{(1)} = \frac{1}{2}$ $\quad$ $\csc t = \frac{1}{\sin t} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

$\cos t = \frac{\left(\frac{\sqrt{3}}{2}\right)}{(1)} = \frac{\sqrt{3}}{2}$ $\quad$ $\sec t = \frac{1}{\cos t} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$

$\tan t = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}}$ $\quad$ $\cot t = \frac{1}{\tan t} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3}$

52) QII

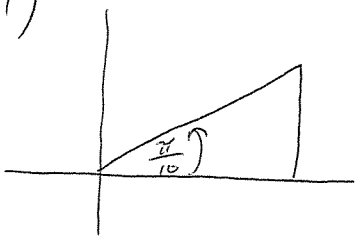


$t^{RA} = \pi - \frac{5\pi}{9} = \frac{9\pi}{9} - \frac{5\pi}{9} = \frac{4\pi}{9}$

$\sin\left(\frac{5\pi}{9}\right) = +\sin\left(\frac{4\pi}{9}\right)$

$\csc\left(\frac{5\pi}{9}\right) = \frac{1}{\sin\left(\frac{5\pi}{9}\right)} = \frac{1}{\sin\left(\frac{4\pi}{9}\right)}$

54) QI



$t' = t = \frac{\pi}{10}$ $\cos\left(\frac{\pi}{10}\right)$

$\sec\left(\frac{\pi}{10}\right) = \frac{1}{\cos\left(\frac{\pi}{10}\right)}$

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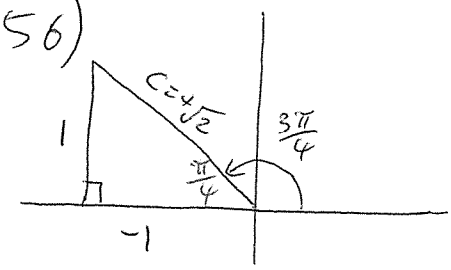
56) QII



$t' = \pi - \frac{3\pi}{4} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$

$c^2 = (-1)^2 + (1)^2$

$c^2 = 1 + 1$

$c^2 = 2$

$c = \pm\sqrt{2}$

$\cos\left(\frac{3\pi}{4}\right) = \frac{(-1)}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\sec\left(\frac{3\pi}{4}\right) = \frac{1}{\cos\left(\frac{3\pi}{4}\right)} = \frac{1}{\left(\frac{-1}{\sqrt{2}}\right)} = -\sqrt{2}$

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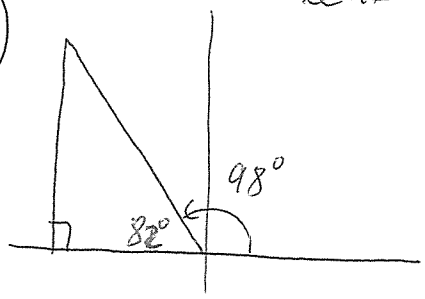
58) QII



$t' = 180° - 98° = 82°$

$\tan(98°) = -\tan(82°)$

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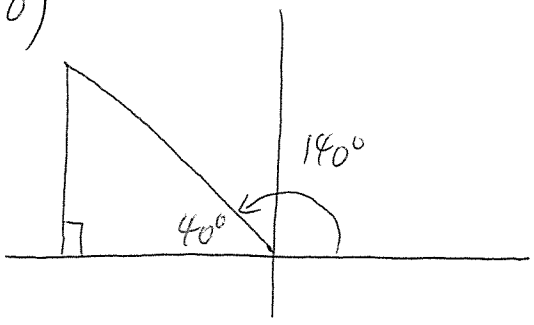
60) QII



$t' = 180° - 140° = 40°$

$\cot(140°) = \frac{1}{\tan(140°)} = \frac{1}{-\tan(40°)} = \frac{-1}{\tan(40°)}$