section 6.7 (Real-World Applications 28-39)

for this section only read about Modeling Exponential Growth and Decay (beginning part)

read Characteristics of the Exponential function

$y = A_0 e^{kt}$ [gray box (1st one)]

$y(t) = A_0 e^{kt}$ note $y(0) = A_0$

28) prescribes 125 mg $\rightarrow A_0 = 125$

decays about 30% each hour $\rightarrow$ amount remaining is 70% each hour

$\rightarrow y(1) = 70\%(125) = 0.7(125)$

$0.7(125) = y(1) = 125 e^{k(1)}$

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$0.7 = e^{k}$ $\qquad y(t) = 125 e^{(\ln(0.7))t}$

$\Downarrow$

$\ln(0.7) = k$

for half-life: $y(t) = 0.5 A_0 = 0.5(125)$ and find t

$0.5(125) = 125 e^{(\ln(0.7))t}$

$0.5 = e^{(\ln(0.7))t}$

$\Downarrow$

$\ln(0.5) = (\ln(0.7))t \rightarrow t = \dfrac{\ln(0.5)}{\ln(0.7)}$ hours

exact answer

do exercise 29 and also use for 30

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32) $A_0 = 0.5\,g$ $\qquad y(t) = A_0 e^{kt}$

decay rate 1.15% per day $\rightarrow$ remains 98.85% per day

$\rightarrow y(1) = 98.85\% A_0 = 0.9885(0.5)$

$0.9885(0.5) = y(1) = 0.5e^{k(1)}$

$0.9885(0.5) = 0.5e^{k(1)}$

$0.9885 = e^{k}$

$\Downarrow$

$\ln(0.9885) = k$

$y(t) = 0.5\,e^{(\ln(0.9885))t}$

$y(60) = 0.5\,e^{(\ln(0.9885))(60)}$

$= 0.5\,e^{60\ln(0.9885)}$

$= 0.5\,e^{\ln(0.9885)^{60}}$

$y(60) = \underline{\underline{0.5(0.9885)^{60}\,g}}$ exact answer

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34) half-life is 1590 years $\rightarrow y(1590) = 0.5A_0$

$0.5A_0 = y(1590) = A_0 e^{k(1590)}$

$0.5A_0 = A_0 e^{1590k}$

$0.5 = e^{1590k}$

$\Downarrow$

$\ln(0.5) = 1590k$

$\frac{\ln(0.5)}{1590} = k$

exact answer

$\underline{\underline{k = \left(\frac{\ln(0.5)}{1590}\right) \times 100\%}}$

36) 60% of carbon-14 present $\rightarrow y(t) = 0.6A_0$

half-life is 5730 years $\rightarrow y(5730) = 0.5A_0$

$$0.5A_0 = y(5730) = A_0 e^{k(5730)}$$

$$0.5A_0 = A_0 e^{k(5730)}$$

$$0.5 = e^{5730k}$$

$$\Downarrow$$

$$\ln(0.5) = 5730k$$

$$\frac{\ln(0.5)}{5730} = k$$

$$y(t) = A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$

$$0.6A_0 = A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$

$$0.6 = e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$

$$\Downarrow$$

$$\ln(0.6) = \left(\frac{\ln(0.5)}{5730}\right)t$$

$$\frac{5730 \cdot \ln(0.6)}{\ln(0.5)} = t$$

Exact answer $\quad t = \frac{5730 \ln(0.6)}{\ln(0.5)}$ years

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38) 360 bacteria after 5 minutes $\rightarrow y(5) = 360$

1000 bacteria after 20 minutes $\rightarrow y(20) = 1000$

$$(360) = y(5) = A_0 e^{k(5)}$$

$$360 = A_0 e^{5k}$$

$$\frac{360}{e^{5k}} = A_0$$

$$(1000) = y(20) = A_0 e^{k(20)}$$

$$1000 = A_0 e^{20k}$$

$$1000 = \left(\frac{360}{e^{5k}}\right) e^{20k}$$

$$\frac{1000}{360} = \frac{e^{20k}}{e^{5k}} = e^{(20k-5k)} = e^{15k}$$

$$\Downarrow$$

$$\ln\left(\frac{1000}{360}\right) = 15k \rightarrow k = \frac{\ln\left(\frac{1000}{360}\right)}{15}$$

38) continued...

$$A_0 = \frac{360}{e^{5\left(\frac{\ln\left(\frac{1000}{360}\right)}{15}\right)}} = \frac{360}{e^{\frac{1}{3}\ln\left(\frac{1000}{360}\right)}}$$

$$= \frac{360}{e^{\ln\left(\frac{1000}{360}\right)^{\frac{1}{3}}}} = \frac{360}{\left(\frac{1000}{360}\right)^{\frac{1}{3}}} = \frac{360}{\sqrt[3]{\frac{1000}{360}}}$$

$$= \frac{360}{\left(\frac{\sqrt[3]{1000}}{\sqrt[3]{360}}\right)} = \frac{360}{1}\left(\frac{\sqrt[3]{360}}{\sqrt[3]{1000}}\right)$$

$$= \frac{360\left(\sqrt[3]{360}\right)}{\sqrt[3]{1000}} = \frac{360\left(\sqrt[3]{360}\right)}{10}$$

$= 36\left(\sqrt[3]{360}\right)$ bacteria

exact answer.