Section 6.5

Law/Rule for Logarithms (for $b>0$ and $b\neq 1$)

Product: $\log_b(MN) = \log_b(M) + \log_b(N)$

$\ln(MN) = \ln(M) + \ln(N)$

Quotient: $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

$\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$

Power: $\log_b(M^n) = n\log_b(M)$

$\ln(M^n) = n\ln(M)$

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note: $\log_b(1) = 0$ because $1 = b^0$

$\ln(1) = 0$ because $1 = e^0$

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change of base formula: $\log_b(M) = \frac{\ln(M)}{\ln(b)}$

4) $\ln(3ab \cdot 5c) = \ln(3ab) + \ln(5c)$

$= \underline{\ln(3) + \ln(a) + \ln(b) + \ln(5) + \ln(c)}$

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6) $\log_4\left(\frac{\frac{x}{3}}{w}\right) = \log_4\left(\frac{x}{3}\right) - \log_4(w)$

$= \underline{\log_4(x) - \log_4(3) - \log_4(w)}$

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8) $\log_2(y^x) = \underline{x \log_2(y)}$

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10) $\log_3(2) + \log_3(a) + \log_3(11) + \log_3(b)$

$= \log_3((2)(a)(11)(b)) = \underline{\log_3(22ab)}$

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12) $\ln(a) - \ln(d) - \ln(c) = \ln\left(\frac{a}{(d)(c)}\right) = \underline{\ln\left(\frac{a}{cd}\right)}$

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14) $\frac{1}{3}\ln(8) = \ln(8^{\frac{1}{3}}) = \ln(\sqrt[3]{8}) = \underline{\ln(2)}$

16) $\ln\left(\frac{a^{-2}}{b^{-4}c^5}\right) = \ln\left(\frac{b^4}{a^2c^5}\right)$

$= +\ln(b^4) - \ln(a^2) - \ln(c^5)$

$= +4\ln(b) - 2\ln(a) - 5\ln(c)$

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18) $\ln\left(y\sqrt{\frac{y}{1-y}}\right) = \ln(y) + \ln\left(\sqrt{\frac{y}{1-y}}\right)$

$= \ln|y| + \ln\left(\frac{\sqrt{y}}{\sqrt{1-y}}\right) =$

$= \ln(y) + \{+\ln(\sqrt{y}) - \ln(\sqrt{1-y})\}$

$= \ln(y) + \ln(y^{\frac{1}{2}}) - \ln((1-y)^{\frac{1}{2}})$

$= \ln(y) + \frac{1}{2}\ln(y) - \frac{1}{2}\ln(1-y)$

$= \frac{3}{2}\ln(y) - \frac{1}{2}\ln(1-y)$

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20) $\log(2x^4) + \log(3x^5) = \log((2x^4)(3x^5))$

$= \log(6x^9)$

22) $2\log(x)+3\log(x+1)=\log(x^2)+\log((x+1)^3)$

$=\log\left((x^2)(x+1)^3\right)$

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24) $4\log_7(c)+\frac{\log_7(a)}{3}+\frac{\log_7(b)}{3}$

$=4\log_7(c)+\frac{1}{3}\log_7(a)+\frac{1}{3}\log_7(b)$

$=\log_7(c^4)+\log_7(a^{\frac{1}{3}})+\log_7(b^{\frac{1}{3}})$

$=\log_7\left((c^4)(a^{\frac{1}{3}})(b^{\frac{1}{3}})\right)=\log_7\left((c^4)(\sqrt[3]{a})(\sqrt[3]{b})\right)$

$=\log_7\left((c^4)(\sqrt[3]{ab})\right)$

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26) $\log_{14}(55{,}875)=\frac{\log(55{,}875)}{\log(14)}$

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28) $\log_5(6)=a$ $\log_5(11)=b$

$\log_6(55)=\frac{\log_5(55)}{\log_5(6)}=\frac{\log_5((5)(11))}{\log_5(6)}=\frac{\log_5(5)+\log_5(11)}{\log_5(6)}$

$=\frac{1+b}{a}$