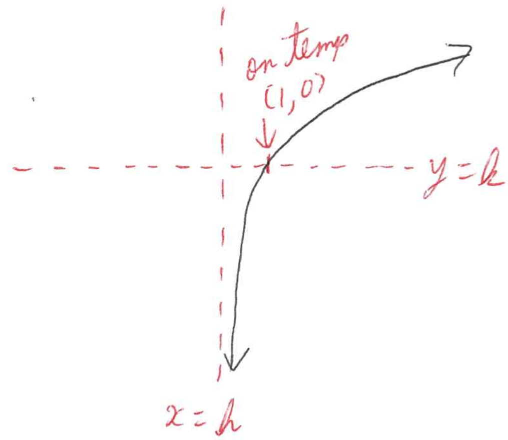
section 6.4

general form of a logarithmic function

$f(x) = a \log_b (x-h) + k$



h: horizontal shift

k: vertical shift

Vertical asymptote: $x - h = 0$
$x = h$

Range: $(-\infty, \infty)$ Domain: solve inequality $(x-h) > 0$

$a > 0$: $+(x-h)$: end behavior as $x \to +\infty$, $f(x) \to +\infty$
as $x \to h^+$, $f(x) \to -\infty$

$-(x-h)$: end behavior as $x \to -\infty$, $f(x) \to +\infty$
"reflection on y-axis" as $x \to h^-$, $f(x) \to -\infty$

$a < 0$: "reflection on x-axis"

$+(x-h)$: end behavior as $x \to +\infty$, $f(x) \to -\infty$
as $x \to h^+$, $f(x) \to +\infty$

$-(x-h)$: end behavior as $x \to -\infty$, $f(x) \to +\infty$
"reflection on y-axis" as $x \to h^-$, $f(x) \to -\infty$

6) $f(x) = \log_3(x+4)$ Domain: $x+4>0$

$x>-4$

$-4<x \rightarrow (-4, \infty)$

Range: $(-\infty, \infty)$

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8) $g(x) = \log_5(2x+9) - 2$ Domain: $2x+9>0$

$2x>-9$

$x>-\frac{9}{2}$

$-\frac{9}{2}<x \rightarrow \left(-\frac{9}{2}, \infty\right)$

Range: $(-\infty, \infty)$

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10) $f(x) = \log_2(12-3x) - 3$ Domain: $12-3x>0$

$12>3x$

$4>x$

$x<4 \rightarrow (-\infty, 4)$

Range: $(-\infty, \infty)$

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12) $g(x) = \ln(3-x)$

domain: $3-x>0$

$3>x$

$x<3$

$(-\infty, 3)$

V.A.: $3-x=0$

$3=x$

14) $f(x) = 3\log(-x)+2$

domain: $-x>0$

$0>x$

$x<0$

$(-\infty, 0)$

V.A.: $-x=0$

$x=0$

16) $f(x) = \ln(2-x)$

domain: $2-x>0$

$2>x$

$x<2$

$\underline{\underline{(-\infty, 2)}}$

V.A.: $2-x=0$

$\underline{\underline{2=x}}$

end behavior:

as $x \to -\infty$, $f(x) \to +\infty$

as $x \to 2^-$, $f(x) \to -\infty$

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18) $h(x) = -\log(3x-4)+3$ $\quad a<0$: reflection on x-axis

domain: $3x-4>0$

$3x>4$

$x>\frac{4}{3}$

$\frac{4}{3}<x$

$\underline{\underline{(\frac{4}{3}, \infty)}}$

V.A.: $3x-4=0$

$3x=4$

$\underline{\underline{x=\frac{4}{3}}}$

end behavior:

as $x \to \infty$, $f(x) \to -\infty$

as $x \to \frac{4}{3}^+$, $f(x) \to +\infty$

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20) $f(x) = \log_3(15-5x)+6$

domain: $15-5x>0$

$15>5x$

$3>x$

$x<3$

$\underline{\underline{(-\infty, 3)}}$

V.A.: $15-5x=0$

$15=5x$

$\underline{\underline{3=x}}$

end behavior:

as $x \to -\infty$, $f(x) \to +\infty$

as $x \to 3^-$, $f(x) \to -\infty$

22) $f(x) = \log(5x+10)+3$ range: $(-\infty,\infty)$

domain: $5x+10>0$
$5x>-10$
$x>-2$
$-2<x$
$\underline{\underline{(-2,\infty)}}$

y-int: $f(0) = \log(5(0)+10)+3 = \log(10)+3$
$= 1+3 = 4$ $\underline{\underline{(0,4)}}$

x-int: $0 = f(x) = \log(5x+10) = 3$
$-3 = \log(5x+10)$
$\Downarrow$
$10^{-3} = 5x+10$
$\frac{1}{10^3} = 5x+10$
$\frac{1}{1000} = 5x+10$

$\frac{1}{1000} - 10 = 5x$
$\frac{1}{1000} - \frac{10000}{1000} = 5x$
$\frac{-9999}{1000} = 5x$
$\frac{-9999}{5000} = x$ $\underline{\underline{\left(\frac{-9999}{5000}, 0\right)}}$

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24) $f(x) = \log_2(x+2)-5$

range: $\underline{(-\infty,\infty)}$

domain: $x+2>0$
$x>-2$
$-2<x$
$\underline{\underline{(-2,\infty)}}$

y-int: $f(0) = \log_2((0)+2)-5$
$= \log_2(2)-5$
$= 1-5 = -4$
$\underline{\underline{(0,-4)}}$

x-int: $0 = f(x) = \log_2(x+2)-5$
$5 = \log_2(x+2)$
$\Downarrow$
$2^5 = x+2$
$32 = x+2$
$30 = x$
$\underline{\underline{(30,0)}}$

6.4 $\underline{|5}$

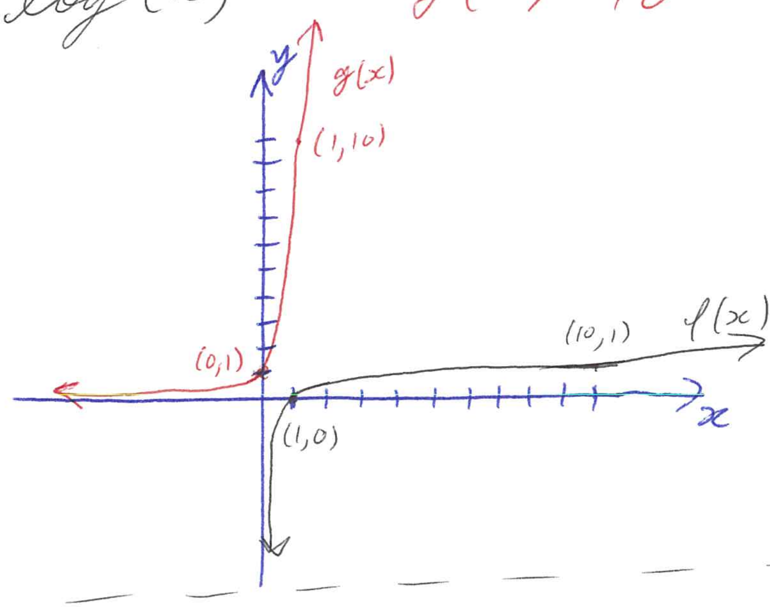
26) $d(x) = \log(x)$ graph D

28) $g(x) = \log_2(x)$ graph A

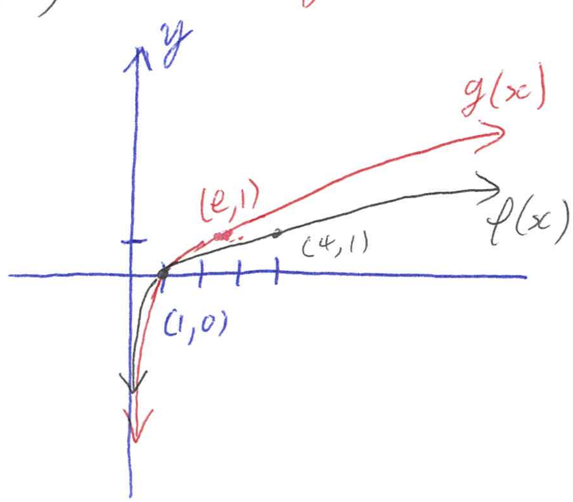
30) $j(x) = \log_{25}(x)$ graph E

32) $g(x) = \log_2(x)$ graph A

34) $f(x) = \log(x)$ $g(x) = 10^x$



36) $f(x) = \log_4(x)$ $g(x) = \ln x$



38) $f(x) = \log_4(-x+2)$ graph B

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40) $h(x) = \log_4(x+2)$ graph A

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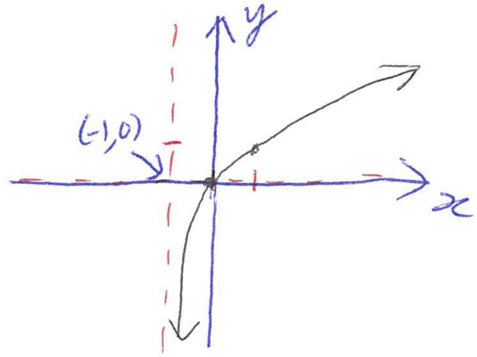
42) $f(x) = \log_2(x+1) = \log_2(x-(-1)) + (0)$ $a>0$

$h=-1$ $k=0$

like $y=\log_2(x)$

x	y
1	0
2	1
4	2



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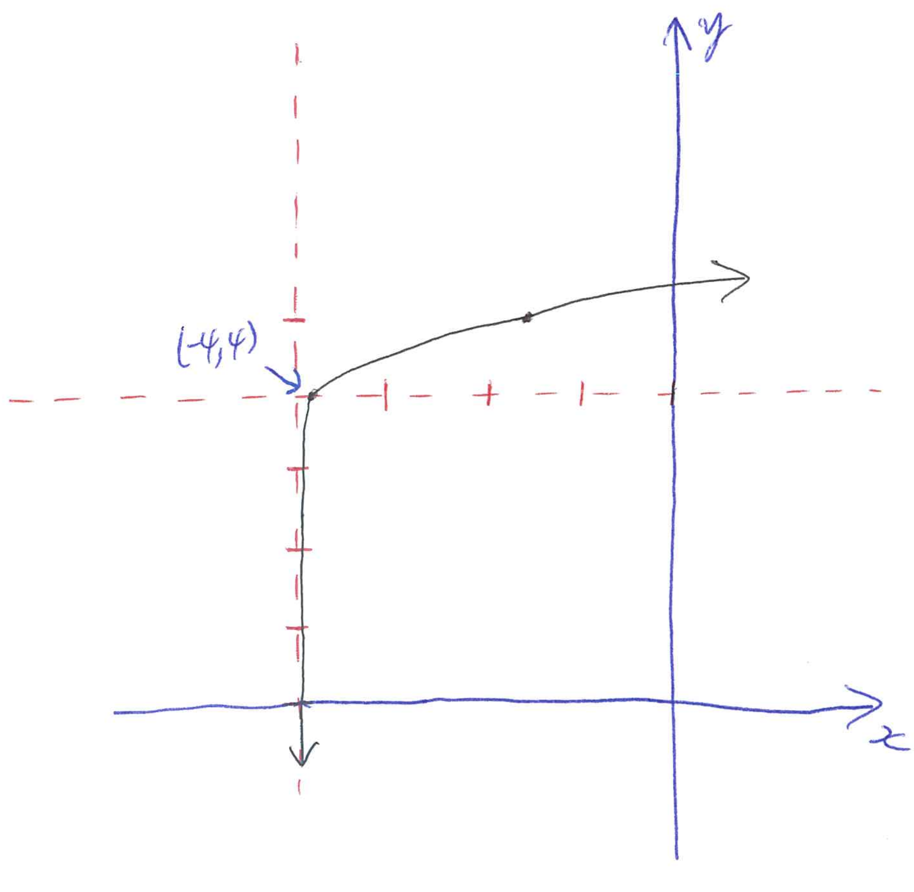
44) $g(x) = \log(4x+16)+4 = \log(4(x+4))+4$

$= \log(4(x-(-4))) + (+4)$ $a>0$

$h=-4$, $k=+4$

like $y=\log(4x)$

x	y
$\frac{1}{4}$	0
$\frac{5}{2}$	1
25	2

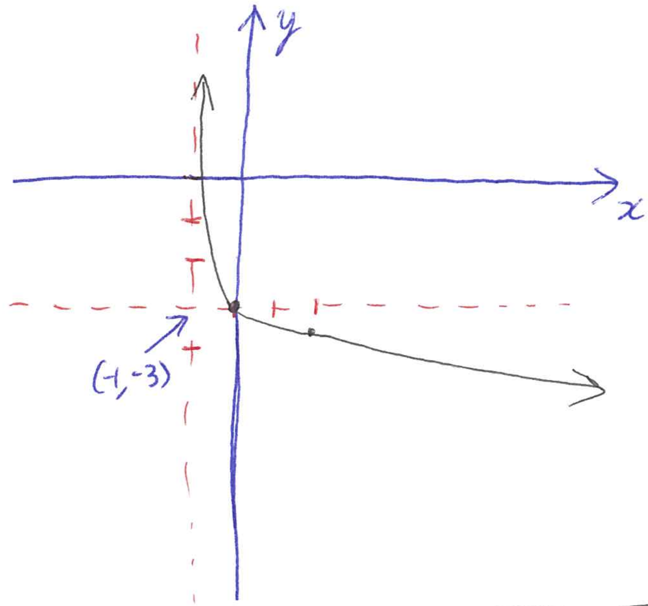


46) $h(x) = \frac{-1}{2}\ln(x+1) - 3 = \frac{-1}{2}\ln(x-(-1)) + (-3)$

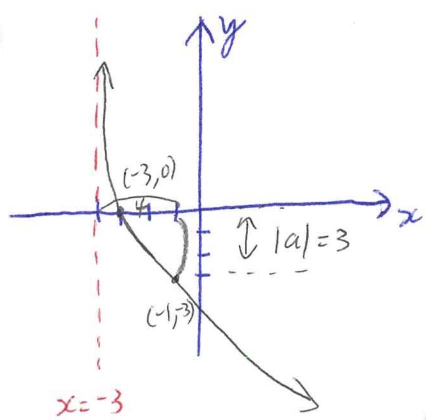
$a < 0 \quad h = -1 \quad k = -3$

like $y = \frac{-1}{2}\ln(x)$

x	y
1	0
e	$\frac{-1}{2}$
e^2	-1



48)



reflection on x-axis: $a < 0$

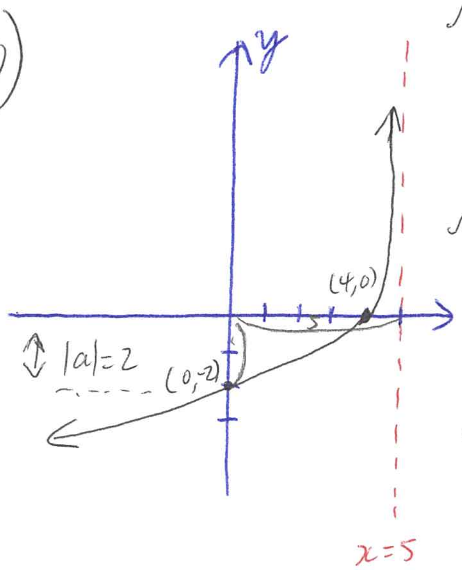
$a = -3$

$h = -3 \quad , k = 0$

using $f(x) = \log_3(x)$

$g(x) = (-3)\log_3(x-(-3)) + 0 = \underline{\underline{-3\log_3(x+3)}}$

50)



reflection on x-axis: $a < 0$

$a = -2$

$h = +5 \qquad k = 0$

reflection on y-axis: $-(x-h)$

using $f(x) = \log_5(x)$

$g(x) = (-2)\log_5(-(x-(+5))) + (0)$

$= -2\log_5(-(x-5)) = -2\log_5(-x+5)$

$\underline{\underline{= -2\log_5(5-x)}}$