section 5.5

it is best to do the numeric part ex. 56 to 60 before any others from this section.
(use Rational Zero Theorem)

56) $f(x) = x^4 + 3x^3 - 4x + 4$

constant term: $4 = (1)(4) = (2)(2) \rightarrow p = \pm 1, \pm 2, \pm 4$ $\qquad x = \frac{p}{q}$

leading term: $1 = (1)(1) \rightarrow q = \pm 1$

possible zeros

$\frac{1}{1} \rightarrow x = \pm 1$, $\frac{2}{1} \rightarrow x = \pm 2$, $\frac{4}{1} \rightarrow x = \pm 4$

---

58) $f(x) = 3x^3 + 5x^2 - 5x + 4$

constant term: $4 = (1)(4) = (2)(2) \rightarrow p = \pm 1, \pm 2, \pm 4$ $\qquad x = \frac{p}{q}$

leading term: $3 = (1)(3) \rightarrow q = \pm 1, \pm 3$

possible zeros

$\frac{1}{1} \rightarrow x = \pm 1$, $\frac{2}{1} \rightarrow x = \pm 2$, $\frac{4}{1} \rightarrow x = \pm 4$, $\frac{1}{3} \rightarrow x = \pm \frac{1}{3}$, $\frac{2}{3} \rightarrow x = \pm \frac{2}{3}$, $\frac{4}{3} \rightarrow x = \pm \frac{4}{3}$

---

60) $f(x) = 4x^5 - 10x^4 + 8x^3 + x^2 - 8$

constant term: $-8 = (-1)(8) = (-2)(4) \rightarrow p = \pm 1, \pm 2, \pm 4, \pm 8$ $\qquad x = \frac{p}{q}$

leading term: $4 = (1)(4) = (2)(2) \rightarrow q = \pm 1, \pm 2, \pm 4$

possible zeros

$\frac{1}{1} = \frac{2}{2} = \frac{4}{4} \rightarrow x = \pm 1$, $\frac{2}{1} = \frac{4}{2} = \frac{8}{4} \rightarrow x = \pm 2$, $\frac{4}{1} = \frac{8}{2} \rightarrow x = \pm 4$, $\frac{8}{1} \rightarrow x = \pm 8$, $\frac{1}{2} = \frac{2}{4} \rightarrow x = \pm \frac{1}{2}$, $\frac{1}{4} \rightarrow x = \pm \frac{1}{4}$

6) $(x^4 - 9x^2 + 14) \div (x-2)$

let $f(x) = x^4 - 9x^2 + 14$

$x - 2 = 0 \rightarrow x = 2$

$f(2) = (2)^4 - 9(2)^2 + 14 = 16 - 18 + 14 = \underline{\underline{12}}$

---

8) $(x^4 + 5x^3 - 4x + 17) \div (x+1)$

$f(x) = x^4 + 5x^3 - 4x + 17$

$x + 1 = 0 \rightarrow x = -1$

$f(-1) = (-1)^4 + 5(-1)^2 - 4(-1) + 17 = 1 - 5 + 4 + 17 = \underline{\underline{17}}$

---

10) $(5x^5 - 4x^4 + 3x^3 - 2x^2 + x - 1) \div (x+6)$

$f(x) = 5x^5 - 4x^4 + 3x^3 - 2x^2 + x - 1$

$x + 6 = 0 \rightarrow x = -6$

$f(-6) = 5(-6)^5 - 4(-6)^4 + 3(-6)^3 - 2(-6)^2 + (-6) - 1$

$= 5(-7776) - 4(1296) + 3(-216) - 2(36) - 6 - 1$

$= \underbrace{-38880 - 5184}\ \underbrace{-648 - 72 - 7}$

$= -44064 - 727 = \underline{\underline{-44791}}$

---

12) $(3x^3 + 4x^2 - 8x + 2) \div (x-3)$

$f(x) = 3x^3 + 4x^2 - 8x + 2$

$x - 3 = 0 \rightarrow x = 3$

$f(3) = 3(3)^3 + 4(3)^2 - 8(3) + 2$

$= 3(27) + 4(9) - 24 + 2 = 81 + 36 - 24 + 2 = \underline{\underline{95}}$

---

15) $f(x) = 2x^3 + x^2 - 5x + 2$; $x+2$ {extra example}

$$
\begin{array}{r l}
 & 2x^2 - 3x + 1 \\
x+2 & \overline{)\,2x^3 + x^2 - 5x + 2} \\
 & -(2x^3 + 4x^2) \\
 & \quad -3x^2 - 5x \\
 & \quad -(-3x^2 - 6x) \\
 & \qquad +x + 2 \\
 & \qquad -(+x + 2) \\
 & \qquad\quad 0
\end{array}
$$

$f(x) = 2x^3 + x^2 - 5x + 2$

$= (x+2)(2x^2 - 3x + 1)$

$= (x+2)(2x-1)(x-1)$

$x + 2 = 0$

$\underline{\underline{x = -2}}$

$2x - 1 = 0$

$2x = 1$

$\underline{\underline{x = \frac{1}{2}}}$

$x - 1 = 0$

$\underline{\underline{x = 1}}$

14) $f(x) = 2x^3 - 9x^2 + 13x - 6$; $x - 1$

$$
\begin{array}{r}
2x^2 - 7x + 6 \\
x-1 \,\big)\, 2x^3 - 9x^2 + 13x - 6 \\
-(2x^3 - 2x^2) \\
\hline
-7x^2 + 13x \\
-(-7x^2 + 7x) \\
\hline
+6x - 6 \\
-(+6x - 6) \\
\hline
0
\end{array}
$$

$f(x) = 2x^3 - 9x^2 + 13x - 6$

$= (x-1)(2x^2 - 7x + 6)$

$= (x-1)(2x-1)(x-3)$

$2x - 1 = 0$	$x - 1 = 0$	$x - 3 = 0$
$2x = 1$	$\underline{\underline{x = 1}}$	$\underline{\underline{x = 3}}$
$\underline{\underline{x = \frac{1}{2}}}$		

---

16) $f(x) = 3x^3 + x^2 - 20x + 12$; $x + 3$

$$
\begin{array}{r}
3x^2 - 8x + 4 \\
x+3 \,\big)\, 3x^3 + x^2 - 20x + 12 \\
-(3x^3 + 9x^2) \\
\hline
-8x^2 - 20x \\
-(-8x^2 - 24x) \\
\hline
+4x + 12 \\
-(+4x + 12) \\
\hline
0
\end{array}
$$

$f(x) = 3x^3 + x^2 - 20x + 12$

$= (x+3)(3x^2 - 8x + 4)$

$= (x+3)(3x-2)(x-2)$

$x + 3 = 0$	$3x - 2 = 0$	$x - 2 = 0$
$\underline{\underline{x = -3}}$	$3x = 2$	$\underline{\underline{x = 2}}$
	$\underline{\underline{x = \frac{2}{3}}}$	

---

18) $f(x) = -5x^3 + 16x^2 - 9$; $x - 3$

$$
\begin{array}{r}
-5x^2 + x + 3 \\
x-3 \,\big)\, -5x^3 + 16x^2 + 0x - 9 \\
-(-5x^3 + 15x^2) \\
\hline
+1x^2 + 0x \\
-(+1x^2 - 3x) \\
\hline
+3x - 9 \\
-(+3x - 9) \\
\hline
0
\end{array}
$$

$f(x) = -5x^3 + 16x^2 - 9$

$= (x-3)(-5x^2 + x + 3) = (x-3)(-1)(5x^2 - x - 3)$

we need to use quadratic formula for

$5x^2 - x - 3 \Rightarrow a = 5 \quad b = -1 \quad c = -3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-3)}}{2(5)}$

$= \dfrac{1 \pm \sqrt{1 + 60}}{10} = \dfrac{1 \pm \sqrt{61}}{10}$

$x - 3 = 0$	$\underline{\underline{x = \frac{1 - \sqrt{61}}{10}}}$	$\underline{\underline{x = \frac{1 + \sqrt{61}}{10}}}$
$\underline{\underline{x = 3}}$		

20) $\overbrace{4x^3-7x+3}^{f(x)=}$; $x-1$

$$
\begin{array}{r}
4x^2+4x-3 \\
x-1\,\overline{)\,4x^3+0x^2-7x+3} \\
-(4x^3-4x^2) \\
\hline
+4x^2 \\
-(+4x^2-4x) \\
\hline
-3x+3 \\
-(-3x+3) \\
\hline
0
\end{array}
$$

$f(x)=4x^3-7x+3$

$=(x-1)(4x^2+4x-3)$

$=(x-1)(2x+3)(2x-1)$

$2x+3=0$	$2x-1=0$	$x-1=0$
$2x=-3$	$2x=1$	$\underline{\underline{x=1}}$
$\underline{\underline{x=\frac{-3}{2}}}$	$\underline{\underline{x=\frac{1}{2}}}$	

---

22) $x^3-3x^2-10x+24=0$

$f(x)=x^3-3x^2-10x+24$

try $x=1$: $f(1)=(1)^3-3(1)^2-10(1)+24=1-3-10+24=12$: no

try $x=-1$: $f(-1)=(-1)^3-3(-1)^2-10(-1)+24=-1-3+10+24=20$: no

try $x=2$: $f(2)=(2)^3-3(2)^2-10(2)+24=8-12-20+24=0$; $x=2 \longrightarrow x-2=0$

$$
\begin{array}{r}
x^2-x-12 \\
x-2\,\overline{)\,x^3-3x^2-10x+24} \\
-(x^3-2x^2) \\
\hline
-x^2-10x \\
-(-x^2+2x) \\
\hline
-12x+24 \\
(-12x+24) \\
\hline
0
\end{array}
$$

$f(x)=x^3-3x^2-10x+24$

$=(x-2)(x^2-x-12)$

$=(x-2)(x+3)(x-4)$

$x+3=0$	$x-2=0$	$x-4=0$
$\underline{\underline{x=-3}}$	$\underline{\underline{x=2}}$	$\underline{\underline{x=4}}$

---

24) $x^3+2x^2-9x-18=0$

$f(x)=x^3+2x^2-9x-18$

try $x=1$: $f(1)=(1)^3+2(1)^2-9(1)-18=1+2-9-18=-24$: no

24) continued...

try $x=-1$: $f(-1)=(-1)^3+2(-1)^2-9(-1)-18=-1+2+9-18=-28$: no

try $x=2$: $f(2)=(2)^3+2(2)^2-9(2)-18=8+8-18-18=-20$: no

try $x=-2$: $f(-2)=(-2)^3+2(-2)^2-9(-2)-18=-8+8+18-18=0$; $x=-2 \rightarrow x+2=0$

$$\begin{array}{r|l} & x^2-9 \\ x+2 & x^3+2x^2-9x-18 \\ & -(x^3+2x^2) \\ \hline & 0x^2-9x-18 \\ & -(-9x-18) \\ \hline & 0 \end{array}$$

$f(x)=x^3+2x^2-9x-18$
$=(x+2)(x^2-9)$
$=(x+2)(x+3)(x-3)$

$x+3=0$	$x+2=0$	$x-3=0$
$x=-3$	$x=-2$	$x=3$

---

26) $x^3-3x^2-25x+75=0$

$f(x)=x^3-3x^2-25x+75$

try $x=1$: $f(1)=(1)^3-3(1)^2-25(1)+75=1-3-25+75=48$: no

try $x=-1$: $f(-1)=(-1)^3-3(-1)^2-25(-1)+75=-1-3+25+75=96$: no

try $x=2$: $f(2)=(2)^3-3(2)^2-25(2)+75=8-12-50+75=21$; no

try $x=-2$: $f(-2)=(-2)^3-3(-2)^2-25(-2)+75=-8-12+50+75=105$; no

try $x=3$: $f(3)=(3)^3-3(3)^2-25(3)+75: 27-27-75+75=0$: $x=3 \rightarrow x-3=0$

$$\begin{array}{r|l} & x^2-25 \\ x-3 & x^3-3x^2-25x+75 \\ & -(x^3-3x^2) \\ \hline & 0x^2-25x+75 \\ & -(-25x+75) \\ \hline & 0 \end{array}$$

$f(x)=x^3-3x^2-25x+75=(x-3)(x^2-25)$
$=(x+3)(x+5)(x-5)$

$x+5=0$	$x+3=0$	$x-5=0$
$x=-5$	$x=-3$	$x=5$

28) $2x^3+x^2-7x-6=0$

$f(x)=2x^3+x^2-7x-6$

try $x=1$: $f(1)=2(1)^3+(1)^2-7(1)-6=2+1-7-6=-10$: no

try $x=-1$: $f(-1)=2(-1)^3+(-1)^2-7(-1)-6=-3+1+7-6=-1$: no

try $x=2$: $f(2)=2(2)^3+(2)^2-7(2)-6=16+4-14-6=0$: $x=2 \rightarrow x-2=0$

$$\begin{array}{r|l} x-2 & \underline{\;2x^2+5x+3\;} \\ & 2x^3+x^2-7x-6 \\ & \underline{-(2x^3-4x^2)} \\ & +5x^2-7x \\ & \underline{-(+5x^2-10x)} \\ & +3x-6 \\ & \underline{-(+3x-6)} \\ & 0 \end{array}$$

$f(x)=2x^3+x^2-7x-6$
$=(x-2)(2x^2+5x+3)$
$=(x-2)(2x+3)(x+1)$

$2x+3=0$	$x+1=0$	$x-2=0$
$2x=-3$	$\underline{\underline{x=-1}}$	$\underline{\underline{x=2}}$
$\underline{\underline{x=\frac{-3}{2}}}$		

30) $3x^3-x^2-11x-6=0$

$f(x)=3x^3-x^2-11x-6$

try $x=1$: $f(1)=3(1)^3-(1)^2-11(1)-6=3-1-11-6=-15$; no

try $x=-1$: $f(-1)=3(-1)^3-(-1)^2-11(-1)-6=-3-1+11-6=1$: no

try $x=2$: $f(2)=3(2)^3-(2)^2-11(2)-6=24-4-22-6=-8$: no

try $x=-2$: $f(-2)=3(-2)^3-(-2)^2-11(-2)-6=-24-4+22-6=-12$; no

try $x=3$; $f(3)=3(3)^3-(3)^2-11(3)-6=81-9-33-6=33$: no

try $x=-3$; $f(-3)=3(-3)^3-(-3)^2-11(-3)-6=-81-9+33-6=-63$; no

30) continued...

using Rational Zero Theorem

constant term: $-6 = (-1)(+6) = (+1)(-6) = (-2)(3) = (2)(-3)$

$p = \pm 1, \pm 2, \pm 3, \pm 6$

leading term: $3 = (1)(3) = (-1)(-3)$ $\qquad x = \frac{p}{q}$

$q = \pm 1, \pm 3$

$x = \frac{+1}{+1} = +1$, $x = \frac{+1}{-1} = \frac{-1}{+1} = -1$, $x = \frac{+2}{+1} = \frac{-2}{-1} = +2$, $x = \frac{-2}{+1} = \frac{+2}{-1} = -2$,

$x = \frac{+3}{+1} = \frac{-3}{-1} = +3$, $x = \frac{-3}{+1} = \frac{+3}{-1} = -3$, $x = \frac{+6}{+3} = \frac{-6}{-3} = +2$, $x = \frac{+6}{-3} = \frac{-6}{+3} = -2$

for these see above. (previous page)

try $x = \frac{+1}{+3} = \frac{-1}{-3} = \frac{1}{3}$; $f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 11\left(\frac{1}{3}\right) - 6 = \frac{1}{9} - \frac{1}{9} - \frac{11}{3} - 6 = \frac{-29}{3}$; no

try $x = \frac{-1}{+3} = \frac{+1}{-3} = \frac{-1}{3}$; $f\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right)^3 - \left(\frac{-1}{3}\right)^2 - 11\left(\frac{-1}{3}\right) - 6 = \frac{-1}{9} - \frac{1}{9} + \frac{11}{3} - 6 = \frac{-2}{9} + \frac{11}{3} - 6 \neq 0$; no

try $x = \frac{+2}{+3} = \frac{-2}{-3} = \frac{2}{3}$; $f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 - 11\left(\frac{2}{3}\right) - 6 = \frac{8}{9} - \frac{4}{9} - \frac{22}{3} - 6 = \frac{4}{9} - \frac{66}{9} - \frac{54}{9} \neq 0$; no

try $x = \frac{-2}{+3} = \frac{+2}{-3} = \frac{-2}{3}$; $f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^3 - \left(\frac{-2}{3}\right)^2 - 11\left(\frac{-2}{3}\right) - 6 = \frac{-8}{9} - \frac{4}{9} + \frac{22}{3} - 6 = \frac{-12}{9} + \frac{22}{3} - 6$

$= \frac{-4}{3} + \frac{22}{3} - \frac{18}{3} = 0$; $x = \frac{-2}{3} \rightarrow 3x = -2 \rightarrow 3x + 2 = 0$

$$
\begin{array}{r}
x^2 - x - 3 \\
3x+2 \,\overline{)\, 3x^3 - x^2 - 11x - 6} \\
-(3x^3 + 2x^2) \\
\hline
-3x^2 - 11x \\
-(-3x^2 - 2x) \\
\hline
-9x - 6 \\
-(-9x - 6) \\
\hline
0
\end{array}
$$

$f(x) = 3x^3 - x^2 - 11x - 6 = (3x+2)(x^2 - x - 3)$

we need to use quadratic formula for

$x^2 - x - 3 \Rightarrow a = 1 \quad b = -1 \quad c = -3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$

$3x + 2 = 0$ | $x = \frac{1-\sqrt{13}}{2}$ | $x = \frac{1+\sqrt{13}}{2}$

$x = \frac{-2}{3}$

32) $2x^3-3x^2+4x+3=0$

$f(x)=2x^3-3x^2+4x+3$

try $x=1$: $f(1)=2(1)^3-3(1)^2+4(1)+3=2-3+4+3=2$: no

try $x=-1$: $f(-1)=2(-1)^3-3(-1)^2+4(-1)+3=-2-3-4+3=-6$: no

use Rational Zero Theorem

constant term: $3=(1)(3)=(-3)(-1)$

$p=\pm1, \pm3$ $\qquad x=\frac{p}{q}$

leading term: $2=(1)(2)=(-1)(-2)$

$q=\pm1, \pm2$

$x=\frac{+1}{+1}=\frac{-1}{-1}=+1$ and $x=\frac{-1}{+1}=\frac{+1}{-1}=-1$ see above

try $x=\frac{+3}{+1}=\frac{-3}{-1}=+3$; $f(3)=2(3)^3-3(3)^2+4(3)+3=54-27+12+3\neq0$: no

try $x=\frac{-3}{+1}=\frac{+3}{-1}=-3$; $f(-3)=2(-3)^3-3(-3)^2+4(-3)+3=-54-27-12+3\neq0$: no

try $x=\frac{+1}{+2}=\frac{-1}{-2}=\frac{1}{2}$: $f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)+3=\frac{1}{4}-\frac{3}{4}+2+3\neq0$: no

try $x=\frac{-1}{+2}=\frac{+1}{+2}=\frac{-1}{2}$: $f\left(\frac{-1}{2}\right)=2\left(\frac{-1}{2}\right)^3-3\left(\frac{-1}{2}\right)^2+4\left(\frac{-1}{2}\right)+3=\frac{-1}{4}-\frac{3}{4}-2+3=\frac{-4}{4}-2+3$

$=-1-2+3=0$; $x=\frac{-1}{2}\rightarrow 2x=-1\rightarrow 2x+1=0$

32) continued...

$$\begin{array}{r} x^2-2x+6 \\ 2x+1 \overline{\big) 2x^3-3x^2+4x+3} \\ -(2x^3+x^2) \\ \hline -4x^2+4x \\ -(-2x^2-2x) \\ \hline +6x+3 \\ (+6x+3) \\ \hline 0 \end{array}$$

$f(x)=2x^3-3x^2+4x+3$

$= (2x+1)(x^2-2x+6)$

we need to use quadratic formula for $x^2-2x+6 \Rightarrow a=1 \quad b=-2 \quad c=6$

$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(6)}}{2(1)} = \frac{2\pm\sqrt{4-24}}{2}$

$x=\frac{2\pm\sqrt{-20}}{2}$ no real number solutions

$2x+1=0 \rightarrow \underline{\underline{x=\frac{-1}{2}}}$

---

34) $x^4+2x^3-9x^2-2x+8=0$

$f(x)=x^4+2x^3-9x^2-2x+8$

try $x=1$: $f(1)=(1)^4+2(1)^3-9(1)^2-2(1)+8=1+2-9-2+8=0$: $x=1 \rightarrow x-1=0$

$$\begin{array}{r} x^3+3x^2-6x-8 \\ x-1 \overline{\big) x^4+2x^3-9x^2-2x+8} \\ -(x^4-x^3) \\ \hline +3x^3-9x^2 \\ -(+3x^3-3x^2) \\ \hline -6x^2-2x \\ -(-6x^2+6x) \\ \hline -8x+8 \\ -(-8x+8) \\ \hline 0 \end{array}$$

$f(x)=x^4+2x^3-9x^2-2x+8$

$=(x-1)(x^3+3x^2-6x-8)$

let $g(x)=x^3+3x^2-6x-8$

try $x=1$: $g(1)=(1)^3+3(1)^2-6(1)-8=1+3-6-8$: no

---

try $x=-1$: $g(-1)=(-1)^3+3(-1)^2-6(-1)-8=-1+3+6-8=0$:

$x=-1 \rightarrow x+1=0$

34) continued...

$$\begin{array}{r} x^2+2x-8 \\ x+1 \overline{\smash{)}\, x^3+3x^2-6x-8} \\ -(x^3+x^2) \\ \hline +2x^2-6x \\ -(+2x^2+2x) \\ \hline -8x-8 \\ -(-8x-8) \\ \hline 0 \end{array}$$

$f(x)=x^4+2x^3-9x^2-2x+8$

$f(x)=(x-1)(x^3+3x^2-6x-8)$

$f(x)=(x-1)(x+1)(x^2+2x-8)$

$=(x-1)(x+1)(x+4)(x-2)$

$x+4=0$	$x+1=0$	$x-1=0$	$x-2=0$
$\underline{\underline{x=-4}}$	$\underline{\underline{x=-1}}$	$\underline{\underline{x=1}}$	$\underline{\underline{x=2}}$

36) $2x^4-3x^3-15x^2+32x-12=0$

$f(x)=2x^4-3x^3-15x^2+32x-12$

use Rational Zero Theorem

constant term: $-12=(1)(-12)=(-2)(6)=(-3)(4)$

$p=\pm1, \pm2, \pm3, \pm4, \pm6$

$x=\dfrac{p}{q}$

leading term: $2=(1)(2)$

$q=\pm1, \pm2$

try $x=\frac{+1}{+1}=\frac{+2}{+2}=\frac{-1}{-1}=\frac{-2}{-1}=+1$; $f(1)=2(1)^4-3(1)^3-15(1)^2+32(1)-12\neq0$; no

try $x=\frac{-1}{1}=-1$; $f(-1)=2(-1)^4-3(-1)^3-15(-1)^2+3(-1)-12=2+3-15-3-12\neq0$; no

try $x=\frac{+2}{+1}=\frac{+4}{+2}=2$; $f(2)=2(2)^4-3(2)^3-15(2)^2+32(2)-12=32-24-60+64-12$

$=96-96=0$; $x=2\rightarrow x-2=0$

36) continued...

$$\begin{array}{r|l} & 2x^3+x^2-13x+6 \\ \hline x-2 & 2x^4-3x^3-15x^2+32x-12 \\ & -(2x^4-4x^3) \\ \hline & +x^3-15x^2 \\ & -(+x^3-2x^2) \\ \hline & -13x^2+32x \\ & -(-13x^2+26x) \\ \hline & +6x-12 \\ & -(+6x-12) \\ \hline & 0 \end{array}$$

$f(x)=2x^4-3x^3-15x^2+32x-12$

$=(x-2)(2x^3+x^2-13x+6)$

let $g(x)=2x^3+x^2-13x+6$

try $x=-2$: $g(-2)=2(-2)^3+(-2)^2-13(-2)+6$

$=-16+4+26+6$

$\neq 0$: no

try $x=3$: $g(3)=2(3)^3+(3)^2-13(2)+6=54+9-39+6\neq 0$: no

try $x=-3$: $g(-3)=2(-3)^3+(-3)^2-13(-3)+6=-54+9+39+6=-54+54=0$:

$x=-3 \rightarrow x+3=0$

$$\begin{array}{r|l} & 2x^2-5x+2 \\ \hline x+3 & 2x^3+x^2-13x+6 \\ & -(2x^3+6x^2) \\ \hline & -5x^2-13x \\ & -(-5x^2-15x) \\ \hline & +2x+6 \\ & -(+2x+6) \\ \hline & 0 \end{array}$$

$f(x)=2x^4-3x^3-15x^2+32x-12$

$=(x-2)(2x^3+x^2-13x+6)$

$=(x-2)(x+3)(2x^2-5x+2)$

$=(x-2)(x+3)(2x-1)(x-2)$

$=(x+3)(2x-1)(x-2)^2$

$x+3=0$	$2x-1=0$	$(x-2)^2=0$
$\underline{\underline{x=-3}}$	$2x=1$	$x-2=0$
	$\underline{\underline{x=\frac{1}{2}}}$	$\underline{\underline{x=2}}$

38) $4x^3-3x+1=0$

$f(x)=4x^3-3x+1$

use Rational Zero Theorem

constant term: $1=(1)(1)$ $\quad p=\pm 1$ $\qquad x=\frac{p}{q}$

leading term: $4=(1)(4)=(2)(2)$

$q=\pm 1, \pm 2, \pm 4$

try $x=1$: $f(1)=4(1)^3-3(1)+1=4-3+1\neq 0$: no

try $x=-1$: $f(-1)=4(-1)^3-3(-1)+1=-4+3+1=0$: $x=-1 \rightarrow x+1=0$

$$
\begin{array}{r|l}
 & 4x^2-4x+1 \\ \hline
x+1 & 4x^3-0x^2-3x+1 \\
 & -(4x^3+4x^2) \\ \hline
 & -4x^2-3x \\
 & -(-4x^2-4x) \\ \hline
 & +x+1 \\
 & -(+x+1) \\ \hline
 & 0
\end{array}
$$

$f(x)=4x^3-3x+1$

$=(x+1)(4x^2-4x+1)$

$=(x+1)(2x-1)(2x-1)=(x+1)(2x-1)^2$

$x+1=0$

$\underline{\underline{x=-1}}$

$(2x-1)^2=0$

$2x-1=0$

$2x=1$

$\underline{\underline{x=\frac{1}{2}}}$

exercises 40 to 55 are skipped