section 5.1

6) $f(x) = x^2 - 12x + 32$

$a=1 \quad b=-12 \quad c=32$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-12)}{2(1)} = 6 \rightarrow h = +6$

$f(6) = (6)^2 - 12(6) + 32 = 36 - 72 + 32 = -4 \rightarrow k = -4$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = x^2 - 12x + 32 = 1(x - (+6))^2 + (-4)$

$= (x-6)^2 - 4$ vertex: $(h,k) = (6,-4)$

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8) $f(x) = x^2 - x = x^2 - x + 0$

$a=1 \quad b=-1 \quad c=0$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2} \rightarrow h = +\frac{1}{2}$

$f(\frac{1}{2}) = (\frac{1}{2})^2 - (\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4} \longrightarrow k = \frac{-1}{4}$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = x^2 - x = 1\left(x - (\frac{+1}{2})\right)^2 + (\frac{-1}{4})$

$= (x - \frac{1}{2})^2 - \frac{1}{4}$ vertex: $(h,k) = (\frac{1}{2}, \frac{-1}{4})$

10) $h(x) = 2x^2 + 8x - 10$

$a = 2 \quad b = 8 \quad c = -10$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(8)}{2(2)} = \frac{-8}{4} = -2 \rightarrow h = -2$

$h(-2) = 2(-2)^2 + 8(-2) - 10 = 8 - 16 - 10 = -18 \rightarrow k = -18$

plug h and k into $y = a(x-h)^2 + k$

$h(x) = 2x^2 + 8x - 10 = 2(x-(-2))^2 + (-18)$

$= 2(x+2)^2 - 18$ vertex: $(h, k) = (-2, -18)$

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12) $f(x) = 2x^2 - 6x = 2x^2 - 6x + 0$

$a = 2 \quad b = -6 \quad c = 0$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-6)}{2(2)} = \frac{6}{4} = \frac{3}{2} \rightarrow h = \frac{+3}{2}$

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) = 2\left(\frac{9}{4}\right) - 9 = \frac{9}{2} - 9 = \frac{-9}{2} \rightarrow k = \frac{-9}{2}$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = 2x^2 - 6x = 2\left(x - \left(\frac{3}{2}\right)\right)^2 + \left(\frac{-9}{2}\right)$

$= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2}$ vertex: $(h, k) = \left(\frac{3}{2}, \frac{-9}{2}\right)$

14) $y(x) = 2x^2 + 10x + 12$

$a = 2 \quad b = 10 \quad c = 12$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(10)}{2(2)} = \frac{-5}{2} \rightarrow h = \frac{-5}{2}$

$y\left(\frac{-5}{2}\right) = 2\left(\frac{-5}{2}\right)^2 + 10\left(\frac{-5}{2}\right) + 12 = \frac{25}{2} - 25 + 12 = \frac{25}{2} - 13 = \frac{25}{2} - \frac{26}{2} = \frac{-1}{2}$

plug h and k into $y = a(x-h)^2 + k$ $\qquad k = \frac{-1}{2}$

$y(x) = 2x^2 + 10x + 12 = 2\left(x - \left(\frac{-5}{2}\right)\right)^2 + \left(\frac{-1}{2}\right)$

$= 2\left(x + \frac{5}{2}\right)^2 - \frac{1}{2}$ $\qquad$ vertex: $(h, k) = \left(\frac{-5}{2}, \frac{-1}{2}\right)$

$a > 0$ opens upward $\rightarrow$ Abs. min value of $k = \frac{-1}{2}$

axis of symmetry: $\underline{\underline{x = h = \frac{-5}{2}}}$

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16) $f(x) = -x^2 + 4x + 3$

$a = -1 \quad b = 4 \quad c = 3$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(4)}{2(-1)} = \frac{-4}{-2} = 2 \rightarrow h = +2$

$f(2) = -(2)^2 + 4(2) + 3 = -4 + 8 + 3 = 7 \rightarrow k = +7$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = -x^2 + 4x + 3 = -1(x - (2))^2 + (7)$

$= -(x-2)^2 + 7$ $\qquad$ vertex: $(h, k) = (2, 7)$

$a < 0$ opens downward $\rightarrow$ Abs Max value of $k = 7$

axis of symmetry: $\underline{\underline{x = h = 2}}$

18) $h(t) = -4t^2 + 6t - 1$

$a = -4 \quad b = 6 \quad c = -1$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(6)}{2(-4)} = \frac{-6}{-8} = \frac{3}{4} \rightarrow h = \frac{+3}{4}$

$h\left(\frac{3}{4}\right) = -4\left(\frac{3}{4}\right)^2 + 6\left(\frac{3}{4}\right) - 1 = \frac{-9}{4} + \frac{18}{4} - \frac{4}{4} = \frac{5}{4} \rightarrow k = \frac{+5}{4}$

plug h and k into $y = a(x-h)^2 + k$

$h(t) = -4t^2 + 6t - 1 = -4\left(t - \left(\frac{3}{4}\right)\right)^2 + \left(\frac{5}{4}\right)$

$= -4\left(t - \frac{3}{4}\right)^2 + \frac{5}{4}$ vertex: $(h, k) = \left(\frac{3}{4}, \frac{5}{4}\right)$

$a < 0$ opens downward $\rightarrow$ Abs Max value of $k = \frac{5}{4}$

axis of symmetry: $x = h = \frac{3}{4}$

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20) $f(x) = \frac{-1}{3}x^2 - 2x + 3$

$a = \frac{-1}{3} \quad b = -2 \quad c = 3$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-2)}{2\left(\frac{-1}{3}\right)} = -3 \rightarrow h = -3$

$f(-3) = \frac{-1}{3}(-3)^2 - 2(-3) + 3 = -3 + 6 + 3 = 6 \rightarrow k = +6$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = \frac{-1}{3}x - 2x + 3 = \frac{-1}{3}\left(x - (-3)\right)^2 + (+6)$

$= \frac{-1}{3}(x+3)^2 + 6$ vertex: $(h, k) = (3, 6)$

$a < 0$ opens downward $\rightarrow$ Abs. Max value of $k = 6$

axis of symmetry: $x = h = 3$

22) $f(x) = -2(x+3)^2 - 6 = -2(x-(-3))^2 + (-6)$

$h = -3 \quad k = -6 \quad$ vertex: $(-3, -6)$

domain: $(-\infty, \infty)$

$a < 0$ opens downward $\rightarrow$ range: $(-\infty, -6]$

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24) $f(x) = 2x^2 - 4x + 2$

$a = 2 \quad b = -4 \quad c = 2$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1 \rightarrow h = +1$

$f(1) = 2(1)^2 - 4(1) + 2 = 2 - 4 + 2 = 0 \rightarrow k = 0$

plug h and k into $\quad y = a(x-h)^2 + k$

$f(x) = 2x^2 - 4x + 2 = 2(x-(+1))^2 + (0)$ domain: $(-\infty, \infty)$

$= 2(x-1)^2$ vertex: $(1, 0)$

$a > 0$ opens upward $\rightarrow$ range: $[0, \infty)$

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26) $(h, k) = (2, 0) \; ; \; (x, y) = (4, 4)$ use $y = a(x-h)^2 + k$ to find a

$(4) = a((4) - (2))^2 + (0)$

$4 = a(2)^2$

$4 = 4a$

$1 = a$

$y = 1(x-(2))^2 + (0)$

$y = (x-2)^2$

$\underline{\underline{y = x^2 - 4x + 4}}$

28) $(h,k)=(0,1)$; $(x,y)=(2,5)$

$(5)=a((2)-(0))^2+(1)$

$5=a(2)^2+1$

$4=4a$

$1=a$

$y=1(x-(0))^2+(1)$

$\underline{\underline{y=x^2+1}}$

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30) $(h,k)=(-5,3)$; $(x,y)=(2,9)$

$(9)=a((2)-(-5))^2+(3)$

$9=a(7)^2+3$

$6=49a$

$\frac{6}{49}=a$

$y=\frac{6}{49}(x-(-5))^2+(3)$

$y=\frac{6}{49}(x+5)^2+3$

$y=\frac{6}{49}(x^2+10x+25)+3$

$y=\frac{6}{49}x^2+\frac{60}{49}x+\frac{150}{49}+\frac{147}{49}$

$\underline{\underline{y=\frac{6}{49}x^2+\frac{60}{49}x+\frac{297}{49}}}$

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32) $(h,k)=(1,0)$; $(x,y)=(0,1)$

$(1)=a((0)-(1))^2+(0)$

$1=a(1)^2$

$1=a$

$y=1(x-(1))^2+(0)$

$y=(x-1)^2$

$\underline{\underline{y=x^2-2x+1}}$

34) $f(x) = x^2 - 2x = x^2 - 2x + 0$

$a = 1 \quad b = -2 \quad c = 0$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1 \rightarrow h = +1$

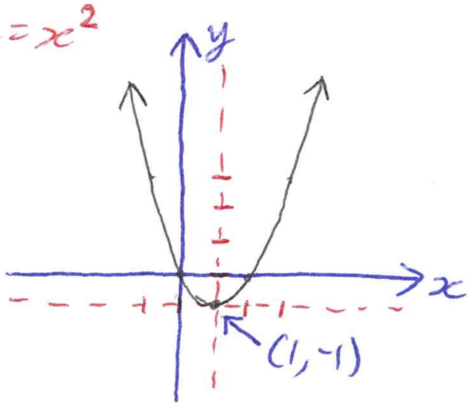
$f(1) = (1)^2 - 2(1) = 1 - 2 = -1 \rightarrow k = -1$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = x^2 - 2x = 1(x - (+1))^2 + (-1)$

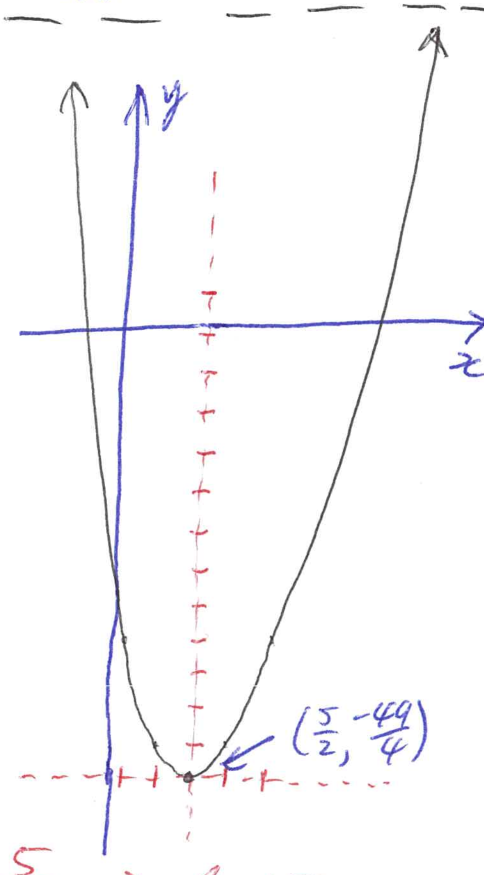
$= (x-1)^2 - 1 \qquad$ vertex: $(1, -1)$

like $y = x^2$



36) $f(x) = x^2 - 5x - 6$

$a = 1 \quad b = -5 \quad c = -6$



by using vertex formula: $x = \frac{-b}{2a} = \frac{-(-5)}{2(1)} = \frac{5}{2} \rightarrow h = \frac{+5}{2}$

$f(\frac{5}{2}) = (\frac{5}{2})^2 - 5(\frac{5}{2}) - 6 = \frac{25}{4} - \frac{25}{2} - 6 = \frac{-25}{4} - 6 = \frac{-25}{4} - \frac{24}{4} = \frac{-49}{4}$

plug h and k into $y = a(x-h)^2 + k \qquad k = \frac{-49}{4}$

$f(x) = x^2 - 5x - 6 = 1(x - (\frac{+5}{2}))^2 + (\frac{-49}{4})$

$= (x - \frac{5}{2})^2 - \frac{49}{4} \qquad$ vertex: $(\frac{5}{2}, \frac{-49}{4})$

like $y = x^2$

38) $f(x) = -2x^2 + 5x - 8$

$a = -2 \quad b = 5 \quad c = -8$

by using vertex formula: $x = \frac{-b}{2a} = \frac{-(5)}{2(-2)} = \frac{5}{4} \rightarrow h = \frac{+5}{4}$

$f\left(\frac{5}{4}\right) = -2\left(\frac{5}{4}\right)^2 + 5\left(\frac{5}{4}\right) - 8 = \frac{-25}{8} + \frac{25}{4} - 8 = \frac{25}{8} - 8 = \frac{25}{8} - \frac{64}{8} = \frac{-39}{8}$

plug h and k into $y = a(x-h)^2 + k$ $\qquad k = \frac{-39}{8}$

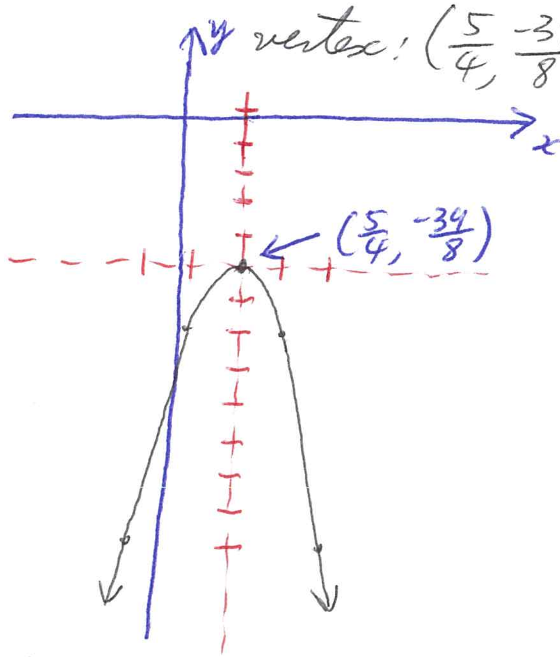
$f(x) = -2x^2 + 5x - 8 = -2\left(x - \left(\frac{+5}{4}\right)\right)^2 + \left(\frac{-39}{8}\right)$

$= -2\left(x - \frac{5}{4}\right)^2 - \frac{39}{8}$

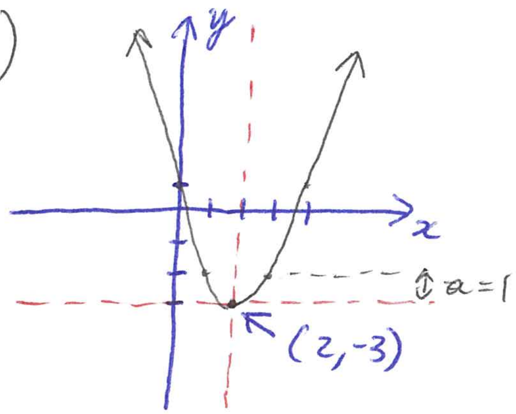
like $y = -2x^2$

x	y
2	-8
1	-2
0	0
-1	-2
-2	-8

vertex: $\left(\frac{5}{4}, \frac{-39}{8}\right)$



40)

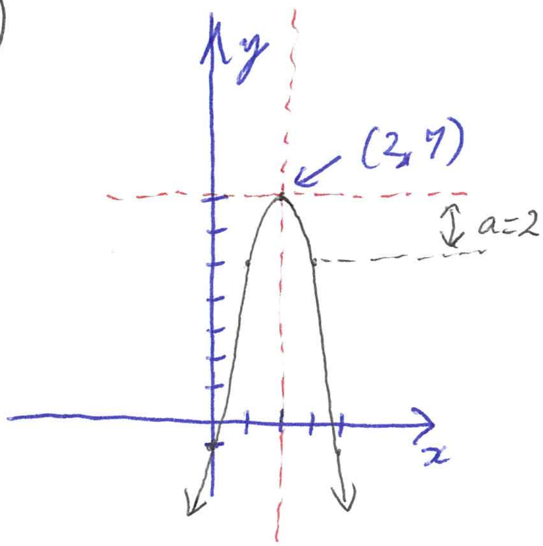


$h = +2 \quad k = -3$

$y = 1\left(x - (+2)\right)^2 + (-3)$

$y = (x-2)^2 - 3$

42)



$h = +2 \quad k = +7$

$y = 2(x - (+2))^2 + (+7)$

$\underline{\underline{y = 2(x-2)^2 + 7}}$

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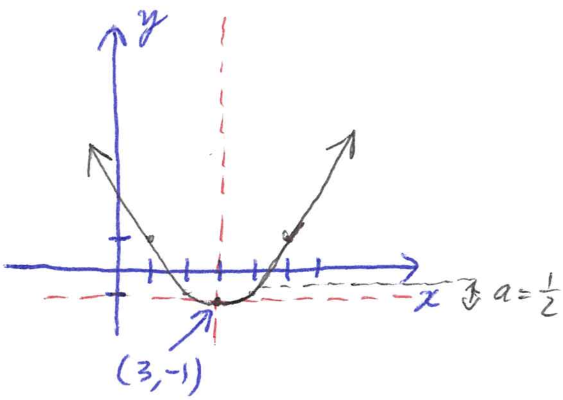
44)



$h = +3 \quad k = -1$

$y = \frac{1}{2}(x - (+3))^2 + (-1)$

$\underline{\underline{y = \frac{1}{2}(x-3)^2 - 1}}$

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46) vertex: $\left.\begin{array}{l}(h, k) = (0, 1) \\ (x, y) = (1, 2)\end{array}\right\} \rightarrow a = +1$

$y = 1(x-(0))^2 + (+1)$

$\underline{\underline{y = x^2 + 1}}$

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48) vertex: $\left.\begin{array}{l}(h, k) = (0, 2) \\ (x, y) = (1, 1)\end{array}\right\} \rightarrow a = -1$

$y = -1(x-(0))^2 + (+2)$

$\underline{\underline{y = -x^2 + 2}}$

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50) vertex: $\left.\begin{array}{l}(h, k) = (0, 0) \\ (x, y) = (1, 2)\end{array}\right\} \rightarrow a = +2$

$y = 2(x-(0))^2 + (0)$

$\underline{\underline{y = 2x^2}}$