section 3.7 3.7 & 5.7

2) In general $f(x) = x^2$ is not one-to-one function. We restrict the domain of the function $f(x) = x^2$ so that it is one-to-one in restricted domain in order to find the function's inverse.

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4) One-to-one functions either always increasing or always decreasing because both $f(\,)$ and $f^{-1}(\,)$ must be functions (must pass vertical & horizontal line tests).

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6) $f(x) = a - x$ show $(f \circ f)(x) = x$

$(f \circ f)(x) = f(f(x)) = f(a-x) = a - (a-x) = a - a + x = x$

---

8) $f(x) = x + 5$

$y = x + 5$

$y - 5 = x$

$\underline{f^{-1}(x) = x - 5}$

10) $f(x) = 3 - x$

$y = 3 - x$

$x = 3 - y$

$\underline{f^{-1}(x) = 3 - x}$

12) $f(x) = \frac{2x+3}{5x+4}$

$y = \frac{2x+3}{5x+4}$

$y(5x+4) = 2x + 3$

$5xy + 4y = 2x + 3$

$5xy - 2x = 3 - 4y$

$x(5y - 2) = 3 - 4y$

$x = \frac{3-4y}{5y-2}$

$\underline{f^{-1}(x) = \frac{3-4x}{5x-2}}$

14) $f(x)=(x-6)^2=(x-(6))^2+(0)$

$h=+6 \quad k=0 \Rightarrow$ vertex: $(6,0)$; parabola opens upward

restrict domain to $[6,\infty)$

$y=(x-6)^2$

$\pm\sqrt{y}=x-6$

$6\pm\sqrt{y}=x$

for this we must choose $+\sqrt{}$

$$f^{-1}(x)=6+\sqrt{x}$$

---

16) $f(x)=\dfrac{x}{2+x} \qquad g(x)=\dfrac{2x}{1-x}$ GLCD $=(1-x)$

a) $f(g(x))=f\left(\dfrac{2x}{1-x}\right)=\dfrac{\left(\frac{2x}{1-x}\right)}{2+\left(\frac{2x}{1-x}\right)}=\left(\dfrac{\frac{2x}{1-x}}{\frac{2}{1}+\frac{2x}{1-x}}\right)\left(\dfrac{\frac{1-x}{1}}{\frac{1-x}{1}}\right)$

$=\dfrac{(2x)}{2(1-x)+(2x)}=\dfrac{2x}{2-2x+2x}=\dfrac{2x}{2}=x$

GLCD $=2+x$

$g(f(x))=g\left(\dfrac{x}{2+x}\right)=\dfrac{2\left(\frac{x}{2+x}\right)}{1-\left(\frac{x}{2+x}\right)}=\left(\dfrac{\frac{2x}{2+x}}{\frac{1}{1}-\frac{x}{2+x}}\right)\left(\dfrac{\frac{2+x}{1}}{\frac{2+x}{1}}\right)$

$=\dfrac{(2x)}{1(2+x)-(x)}=\dfrac{2x}{2+x-x}=\dfrac{2x}{2}=x$

b) $f(x)$ and $g(x)$ are inverse function of each other.

18) $f(x) = -3x+5$ and $g(x) = \frac{x-5}{-3}$

$$f(g(x)) = f\left(\frac{x-5}{-3}\right) = -3\left(\frac{x-5}{-3}\right)+5 = (x-5)+5 = x$$

---

20) $f(x) = \sqrt[3]{3x+1}$

① not higher order, ② odd root

$\downarrow$

$f(x)$ is one-to-one

---

22) $f(x) = x^3 - 27$

① no middle terms, ② odd degree polynomial

$\downarrow$

$f(x)$ is one-to-one

---

24) fails Horizontal line test $\rightarrow$ not one-to-one

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26) $f(x)=0$

look at coordinate where y value is 0: $(2,0)$

$f(2)=0 \longrightarrow x=2$

28) $f^{-1}(x)=0$

look at coordinate where x value is 0: $(0,3)$

$f^{-1}(3)=0 \longrightarrow x=3$

30) look at graph coordinate $(6,2)$

$f(6) = 2$ and $f^{-1}(2) = 6$

---

32) range: $[0, \infty)$

---

34) $f(3) = 2 \rightarrow f^{-1}(2) = 3$

---

36) $f^{-1}(-2) = -1 \rightarrow f(-1) = -2$

---

38) $f(x) = 3 \rightarrow x = 7$ "find 3 on 2nd row"

---

40) $f^{-1}(x) = 7 \rightarrow x = 3$ "find 7 on 1st row"

---

section 5.7

2) because a quadratic function is not one-to-one

---

4) the inverse of a quadratic function will take the form of ($\sqrt{\ }$) square root function and it will open upward or downward depending on restricted domain of a quadratic function.

6) $f(x)=(x+2)^2$, $[-2,\infty)$ → right side interval from vertex

$\Downarrow$

we take $+\sqrt{}$

$y=(x+2)^2$

$\pm\sqrt{y}=x+2$

$2\pm\sqrt{y}=x$

$$\underline{\underline{f^{-1}(x)=2+\sqrt{x}}}$$

---

8) $f(x)=3x^2+5$, $(-\infty,0]$ → left side interval from vertex

$\Downarrow$

we take $-\sqrt{}$

$y=3x^2+5$

$y-5=3x^2$

$\frac{y-5}{3}=x^2$

$\pm\sqrt{\frac{y-5}{3}}=x$

$$\underline{\underline{f^{-1}(x)=-\sqrt{\frac{x-5}{3}}}}$$

---

10) $f(x)=9-x^2$, $[0,\infty)$ → right side interval from vertex

$\Downarrow$

we take $+\sqrt{}$

$y=9-x^2$

$x^2=9-y$

$x=\pm\sqrt{9-y}$

$$\underline{\underline{f^{-1}(x)=+\sqrt{9-x}}}$$

---

12) $f(x)=x^3+5$

$y=x^3+5$

$y-5=x^3$

$\sqrt[3]{y-5}=x$

$$\underline{f^{-1}(x)=\sqrt[3]{x-5}}$$

14) $f(x) = 4 - x^3$

$y = 4 - x^3$

$x^3 = 4 - y$

$x = \sqrt[3]{4-y}$

$$\underline{\underline{f^{-1}(x) = \sqrt[3]{4-x}}}$$

---

16) $f(x) = \sqrt{2x+1}$ this is $+\sqrt{\ }$

$y = \sqrt{2x+1}$

$y^2 = 2x+1$

$y^2 - 1 = 2x$

$\frac{y^2-1}{2} = x$

$f^{-1}(x) = \frac{x^2-1}{2} = \frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{2}(x-(0))^2 + (\frac{-1}{2})$

$h = 0 \quad k = \frac{-1}{2} \rightarrow$ vertex: $(0, \frac{-1}{2})$

take right side interval from vertex

$[0, \infty) \longrightarrow x \geq 0$

$$\underline{\underline{f^{-1}(x) = \frac{1}{2}x^2 - \frac{1}{2}, \quad x \geq 0}}$$

---

18) $f(x) = 9 + \sqrt{4x-4}$ this is $+\sqrt{\ }$

$y = 9 + \sqrt{4x-4}$

$y - 9 = \sqrt{4x-4}$

$(y-9)^2 = 4x - 4$

$(y-9)^2 + 4 = 4x$

$\frac{(y-9)^2+4}{4} = x$

$f^{-1}(x) = \frac{(x-9)^2+4}{4} = \frac{1}{4}(x-9)^2 + 1$

$= \frac{1}{4}(x-(9))^2 + (1)$

$h = +9, \quad k = +1 \rightarrow$ vertex: $(9, 1)$

take right side interval from vertex

$[9, \infty) \longrightarrow x \geq 9$

$$\underline{\underline{f^{-1}(x) = \frac{1}{4}(x-9)^2 + 1, \quad x \geq 9}}$$

20) $f(x) = 9 + 2\sqrt[3]{x}$

$y = 9 + 2\sqrt[3]{x}$

$y - 9 = 2\sqrt[3]{x}$

$\frac{y-9}{2} = \sqrt[3]{x}$

$\left(\frac{y-9}{2}\right)^3 = x$

$f^{-1}(x) = \left(\frac{x-9}{2}\right)^3$

22) $f(x) = \frac{2}{x+8}$

$y = \frac{2}{x+8}$

$y(x+8) = 2$

$xy + 8y = 2$

$xy = 2 - 8y$

$x = \frac{2-8y}{y}$

$f^{-1}(x) = \frac{2-8x}{x}$

24) $f(x) = \frac{x+3}{x+7}$

$y = \frac{x+3}{x+7}$

$y(x+7) = x+3$

$xy + 7y = x + 3$

$xy - x = 3 - 7y$

$x(y-1) = 3 - 7y$

$x = \frac{3-7y}{y-1}$

$f^{-1}(x) = \frac{3-7x}{x-1}$

26) $f(x) = \frac{3x+4}{5-4x}$

$y = \frac{3x+4}{5-4x}$

$y(5-4x) = 3x+4$

$5y - 4xy = 3x + 4$

$5y - 4 = 3x + 4xy$

$5y - 4 = x(3+4y)$

$\frac{5y-4}{3+4y} = x$

$f^{-1}(x) = \frac{5x-4}{3+4x}$

28) $f(x) = x^2 + 2x$, $[-1, \infty) \rightarrow$ right side

$= x^2 + 2x + 0$

$a=1$, $b=2$, $c=0$

by vertex formula $x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = -1$

$h = -1$

$f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1$

$k = -1$

plug h and k into $y = a(x-h)^2 + k$

$f(x) = x^2 + 2x = 1(x-(-1))^2 + (-1) = (x+1)^2 - 1$

$y = (x+1)^2 - 1$

$y + 1 = (x+1)^2$

$\pm\sqrt{y+1} = x + 1$

$-1 \pm \sqrt{y+1} = x$

$[-1, \infty) \rightarrow$ right side

$\Downarrow$

take $+\sqrt{\ }$

$f^{-1}(x) = -1 + \sqrt{x+1}$

30) $f(x)=x^2-6x+3$, $[3,\infty) \rightarrow$ right side

$a=1 \quad b=-6 \quad c=3$

by vertex formula $x=\frac{-b}{2a}=\frac{-(-6)}{2(1)}=3 \longrightarrow h=+3$

$f(3)=(3)^2-6(3)+3=9-18+3=-6 \longrightarrow k=-6$

plug h and k into $y=a(x-h)^2+k$

$f(x)=x^2-6x+3=1(x-(3))^2+(-6)=(x-3)^2-6$

$y=(x-3)^2-6$

$y+6=(x-3)^2$

$\pm\sqrt{y+6}=x-3$

$3\pm\sqrt{y+6}=x$

$[3,\infty) \rightarrow$ right side

$\Downarrow$

take $+\sqrt{}$

$f^{-1}(x)=3+\sqrt{x+6}$

---

32) $f(x)=4-x^2$, $x\geq 0$

$y=4-x^2$

$x^2=4-y$

$x=\pm\sqrt{4-y}$

$x\geq 0 \rightarrow$ right side

$\Downarrow$

take $+\sqrt{}$

$f^{-1}(x)=+\sqrt{4-x}$

34) $f(x)=(x-4)^2$, $x\geq 4$

$y=(x-4)^2$

$\pm\sqrt{y}=x-4$

$4\pm\sqrt{y}=x$

$x\geq 4 \rightarrow$ right side

$\Downarrow$

take $+\sqrt{}$

$f^{-1}(x)=4+\sqrt{x}$

36) $f(x)=1-x^3$

$y=1-x^3$

$x^3=1-y$

$x=\sqrt[3]{1-y}$

$\underline{\underline{f^{-1}(x)=\sqrt[3]{1-x}}}$

---

38) $f(x)=x^2-6x+1$, $x\geq 3$

$a=1$ $b=-6$, $c=1$

by vertex formula $x=\frac{-b}{2a}=\frac{-(-6)}{2(1)}=3 \rightarrow h=+3$

$f(3)=(3)^2-6(3)+1 = 9-18+1=-8 \rightarrow k=-8$

plug h and k into $y=a(x-h)^2+k$

$f(x)=x^2-6x+1 = 1(x-(3))^2+(-8) = (x-3)^2-8$

$y=(x-3)^2-8$

$y+8=(x-3)^2$

$\pm\sqrt{y+8}=x-3$

$3\pm\sqrt{y+8}=x$

$x\geq 3 \rightarrow$ right side

$\Downarrow$

take $+\sqrt{\ }$

$\underline{\underline{f^{-1}(x)=3+\sqrt{x+8}}}$

---

40) $f(x)=\frac{1}{x^2}$, $x\geq 0$

$y=\frac{1}{x^2}$

$x^2y=1$

$x^2=\frac{1}{y}$

$x=\pm\sqrt{\frac{1}{y}}$

$x\geq 0 \rightarrow$ right side

$\Downarrow$

take $+\sqrt{\ }$

$\underline{\underline{f^{-1}(x)=+\sqrt{\frac{1}{x}}}}$

42) $f(x) = \sqrt{\dfrac{(x+2)(x-3)}{x-1}}$

$$\frac{(x+2)(x-3)}{x-1} \geq 0$$

$x+2=0$ | $x-3=0$ | $x-1=0$

$x=\boxed{-2}$ | $x=\boxed{3}$ | $x=1$

	$(-\infty,-2)$	$(-2,1)$	$(1,3)$	$(3,\infty)$
$(x+2)$	neg	POS	POS	POS
$(x-3)$	neg	neg	neg	POS
$(x-1)$	neg	neg	POS	POS
$\frac{(x+2)(x-3)}{(x-1)}$	neg	POS	neg	POS

domain: $[-2,1) \cup [3,\infty)$

---

44) $f(x) = \sqrt{\dfrac{x^2-x-20}{x-2}}$

$$\frac{x^2-x-20}{x-2} \geq 0$$

$$\frac{(x+4)(x-5)}{(x-2)} \geq 0$$

$x+4=0$ | $x-5=0$ | $x-2=0$

$x=\boxed{-4}$ | $x=\boxed{5}$ | $x=2$

	$(-\infty,-4)$	$(-4,2)$	$(2,5)$	$(5,\infty)$
$(x+4)$	neg	POS	POS	POS
$(x-5)$	neg	neg	neg	POS
$(x-2)$	neg	neg	POS	POS
$\frac{(x+4)(x-5)}{(x-2)}$	neg	POS	neg	POS

domain: $[-4,2) \cup [5,\infty)$