section 3.4

2) "see 1st gray box of online text"

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4) "see 15th & 16th gray box of online text."

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6) $f(x) = -3x^2 + x$ domain: $(-\infty, \infty)$ $\quad g(x) = 5$ domain: $(-\infty, \infty)$ $\quad$ domain: $(-\infty, \infty)$

$$(f+g)(x) = f(x) + g(x) = (-3x^2+x) + (5) = \underline{\underline{-3x^2+x+5}}$$

$$(f-g)(x) = f(x) - g(x) = (-3x^2+x) - (5) = \underline{\underline{-3x^2+x-5}}$$

$$(fg)(x) = f(x)g(x) = (-3x^2+x)(5) = \underline{\underline{-15x^2+5x}}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{(-3x^2+x)}{(5)} = \underline{\underline{\frac{-3x^2+x}{5} = \frac{-3}{5}x^2 + \frac{1}{5}x}}$$

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8) $f(x) = \frac{1}{x-4}$, domain: $(-\infty, 4) \cup (4, \infty)$; $g(x) = \frac{1}{6-x}$ domain: $(-\infty, 6) \cup (6, \infty)$

$$(f+g)(x) = f(x) + g(x) = \left(\frac{1}{x-4}\right) + \left(\frac{1}{6-x}\right) = \frac{1}{x-4} + \frac{1}{6-x}$$

$$(f-g)(x) = f(x) - g(x) = \left(\frac{1}{x+4}\right) - \left(\frac{1}{6-x}\right) = \frac{1}{x+4} - \frac{1}{6-x}$$

$$(fg)(x) = f(x)g(x) = \left(\frac{1}{x+4}\right)\left(\frac{1}{6-x}\right) = \frac{1}{(x+4)(6-x)}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\left(\frac{1}{x+4}\right)}{\left(\frac{1}{6-x}\right)} = \left(\frac{1}{x+4}\right)\left(\frac{6-x}{1}\right) = \frac{6-x}{x+4}$$

domain: $(-\infty, 4) \cup (4, 6) \cup (6, \infty)$

10) $f(x)=\sqrt{x}$, domain: $[0,\infty)$; $g(x)=|x-3|$ domain: $(-\infty,\infty)$

$\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=\frac{(|x-3|)}{(\sqrt{x})}=\frac{|x-3|}{\sqrt{x}}$ domain: $[0,\infty)$

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12) $f(x)=x^2+1$, $g(x)=\sqrt{x+2}$

$(f\circ g)(x)=f(g(x))=f(\sqrt{x+2})=(\sqrt{x+2})^2+1=(x+2)+1=\underline{\underline{x+3}}$

$(g\circ f)(x)=g(f(x))=g(x^2+1)=\sqrt{(x^2+1)+2}=\underline{\underline{\sqrt{x^2+3}}}$

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14) $f(x)=|x|$, $g(x)=5x+1$

$(f\circ g)(x)=f(g(x))=f(5x+1)=|(5x+1)|=\underline{\underline{|5x+1|}}$

$(g\circ f)(x)=g(f(x))=g(|x|)=5(|x|)+1=\underline{\underline{5|x|+1}}$

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16) $f(x)=\frac{1}{x-6}$, $g(x)=\frac{7}{x}+6$

$(f\circ g)(x)=f(g(x))=f\left(\frac{7}{x}+6\right)=\frac{1}{\left(\frac{7}{x}+6\right)-6}=\frac{1}{\frac{7}{x}}=\underline{\underline{\frac{x}{7}}}$

$(g\circ f)(x)=g(f(x))=g\left(\frac{1}{x-6}\right)=\frac{7}{\left(\frac{1}{x-6}\right)}+6=7(x-6)+6$

$=7x-42+6=\underline{\underline{7x-36}}$

18) $f(x) = x^4 + 6$, $g(x) = x - 6$, $h(x) = \sqrt{x}$

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f((\sqrt{x}) - 6) = f(\sqrt{x} - 6)$

$= \underline{\underline{(\sqrt{x} - 6)^4 + 6}}$

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20) $f(x) = \frac{1}{x}$, domain: $(-\infty, 0) \cup (0, \infty)$; $g(x) = x - 3$, domain: $(-\infty, \infty)$

a) $(f \circ g)(x) = f(g(x)) = f(x-3) = \frac{1}{(x-3)} = \underline{\underline{\frac{1}{x-3}}}$

b) $x - 3 = 0$, $x = 3$ $\Rightarrow (-\infty, 3) \cup (3, \infty)$ $\quad$ $\underline{\underline{\text{domain: } (-\infty, 3) \cup (3, \infty)}}$

c) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right) - 3 = \underline{\underline{\frac{1}{x} - 3}}$

d) $x = 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$ $\quad$ $\underline{\underline{\text{domain: } (-\infty, 0) \cup (0, \infty)}}$

e) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\left(\frac{1}{x}\right)}{x-3} = \left(\frac{1}{x}\right)\left(\frac{1}{x-3}\right) = \underline{\underline{\frac{1}{x(x-3)}}}$

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22) $f(x) = \frac{1-x}{x}$, $g(x) = \frac{1}{1+x^2}$

a) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1-x}{x}\right) = \frac{1}{1 + \left(\frac{1-x}{x}\right)^2} = \frac{1}{1 + \left(\frac{1-2x+x^2}{x^2}\right)}$

$= \frac{1}{1\left(\frac{x^2}{x^2}\right) + \left(\frac{1-2x+x^2}{x^2}\right)} = \frac{1}{\frac{1-2x+2x^2}{x^2}} = \underline{\underline{\frac{x^2}{1-2x+2x^2}}}$

b) $(g \circ f)(2) = g(f(2)) = \frac{(2)^2}{1-2(2)+2(2)^2} = \frac{4}{1-4+8} = \underline{\underline{\frac{4}{5}}}$

using part a)

24) $q(x)=\frac{1}{\sqrt{x}}$, domain: $(0,\infty)$; $h(x)=x^2-9$, domain: $(-\infty,\infty)$

a) $\frac{q(x)}{h(x)}=\frac{\left(\frac{1}{\sqrt{x}}\right)}{(x^2-9)}=\frac{1}{\sqrt{x}(x^2-9)}$

$\sqrt{x}(x^2-9)=0$

$\sqrt{x}(x+3)(x-3)$

$x+3=0 \mid x-3=0$

$x=-3 \mid x=3$

for $\sqrt{x}$ on bottom

$x>0$

$0<x$

$(0,\infty)$

domain: $(0,3)\cup(3,\infty)$

b) $q(h(x))=q(x^2-9)=\sqrt{(x^2-9)}=\sqrt{x^2-9}$

$x^2-9\geq 0$

$(x+3)(x-3)\geq 0$

$x+3=0 \mid x-3=0$

$x=\boxed{-3} \mid x=\boxed{3}$

	$(-\infty,-3)$	$(-3,3)$	$(3,\infty)$
$(x+3)$	neg	POS	POS
$(x-3)$	neg	neg	POS
$(x+3)(x-3)$	POS	neg	POS

domain: $(-\infty,-3]\cup[-3,\infty)$

c) $h(q(x))=h\left(\frac{1}{\sqrt{x}}\right)=\left(\frac{1}{\sqrt{x}}\right)^2-9=\frac{1}{x}-9$

$x=0 \rightarrow (-\infty,0)\cup(0,\infty)$

domain: $(0,\infty)$

$h(x)=f(g(x))$ find $f(x)$ and $g(x)$

26) $h(x)=(x+2)^2 \rightarrow f(x)=x^2$, $g(x)=x+2$

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28) $h(x)=\frac{3}{x-5} \rightarrow f(x)=\frac{3}{x}$, $g(x)=x-5$

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30) $h(x)=4+\sqrt[3]{x} \rightarrow f(x)=4+x$, $g(x)=\sqrt[3]{x}$

32) $h(x) = \frac{1}{(3x^2-4)^{-3}} \rightarrow f(x) = \frac{1}{x^{-3}}$, $g(x) = 3x^2-4$

34) $h(x) = \left(\frac{8+x^3}{8-x^3}\right)^4 \rightarrow f(x) = x^4$, $g(x) = \frac{8+x^3}{8-x^3}$

36) $h(x) = (5x-1)^3 \rightarrow f(x) = x^3$, $g(x) = 5x-1$

38) $h(x) = |x^2+7| \rightarrow f(x) = |x|$, $g(x) = x^2+7$

40) $h(x) = \left(\frac{1}{2x-3}\right)^2 \rightarrow f(x) = x^2$, $g(x) = \frac{1}{2x-3}$

42) $f(g(3)) = f(4) = \underline{\underline{0}}$

44) $g(f(1)) = g(3) = \underline{\underline{4}}$

46) $f(f(5)) = f(1) = \underline{\underline{3}}$

48) $g(g(2)) = g(0) = \underline{\underline{2}}$

50) $g(f(1)) = g(1) = \underline{\underline{1}}$

52) $f(g(4)) = f(2) = \underline{\underline{4}}$

54) $f(h(2)) = f(2) = \underline{\underline{4}}$

56) $f(g(h(4)) = f(g(4)) = f(2) = \underline{\underline{4}}$

58) $f(g(8)) = f(4) = \underline{\underline{4}}$

60) $g(f(5)) = g(0) = \underline{\underline{9}}$

62) $f(f(4)) = f(4) = \underline{\underline{4}}$

62) $g(g(2)) = g(6) = \underline{\underline{7}}$

66) $(f \circ g)(1) = f(g(1)) = f(0) = \underline{\underline{5}}$

68) $(g \circ f)(2) = g(f(2)) = g(1) = \underline{\underline{0}}$

70) $(g \circ g)(1) = g(g(1)) = g(0) = \underline{\underline{1}}$

72) $f(x) = 4x+8$, $g(x) = 7 - x^2$

$f(g(0)) = f(7-(0)^2) = f(7) = 4(7)+8 = 28+8 = \underline{\underline{36}}$

$g(f(0)) = g(4(0)+8) = g(8) = 7-(8)^2 = 7-64 = \underline{\underline{-57}}$

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74) $f(x) = \sqrt{x+4}$, $g(x) = 12 - x^3$

$f(g(0)) = f(12-(0)^3) = f(12) = \sqrt{(12)+4} = \sqrt{16} = \underline{\underline{4}}$

$g(f(0)) = g(\sqrt{(0)+4}) = g(\sqrt{4}) = g(2) = 12-(2)^3 = 12-8 = \underline{\underline{4}}$

$f(x) = 2x^2+1 \quad g(x) = 3x+5$

76) $f(g(2)) = f(3(2)+5) = f(11) = 2(11)^2+1 = 2(121)+1$

$= 242+1 = \underline{\underline{243}}$

---

78) $g(f(-3)) = g(2(-3)^2+1) = g(2(9)+1) = g(18+1) = g(19)$

$= 3(19)+5 = 57+5 = \underline{\underline{62}}$

$f(x) = x^3+1$, domain: $(-\infty, \infty)$; $g(x) = \sqrt[3]{x-1}$, domain: $(-\infty, \infty)$

80) $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1 = (x-1)+1 = x$

$(g \circ f)(x) = g(f(x)) = g(x^3+1) = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$

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82) domain of $(g \circ f)(x)$ is $\underline{\underline{(-\infty, \infty)}}$

84) $f(x)=\frac{1}{x}$

a) $(f\circ f)(x)=f(f(x))=f\left(\frac{1}{x}\right)=\frac{1}{\left(\frac{1}{x}\right)}=x$

b) no because if $f(x)=\frac{1}{x+1}$ then

$(f\circ f)(x)=f(f(x))=f\left(\frac{1}{x+1}\right)=\frac{1}{\left(\frac{1}{x+1}\right)+1}=\frac{1}{\frac{1}{x+1}+1\left(\frac{x+1}{x+1}\right)}$

$=\frac{1}{\frac{1}{x+1}+\frac{x+1}{x+1}}=\frac{1}{\frac{1+x+1}{x+1}}=\frac{1}{\frac{x+2}{x+1}}=\frac{x+1}{x+2}\neq x$

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86) True

$f(x)=x^2+2$ for $x\geq 0$ $[0,\infty)$ and $g(x)=\sqrt{x-2}$

88) $(g\circ f)(a)=g(f(a))=g((a)^2+2)=g(a^2+2)=\sqrt{(a^2+2)-2}$

$=\sqrt{a^2}=a$

$(f\circ g)(a)=f(g(a))=f(\sqrt{(a)-2})=f(\sqrt{a-2})=(\sqrt{a-2})^2+2$

$=(a-2)+2=a$