

2) we determine the domain of a function by an equation by evaluating all values that we can calculate/compute by the equation given.

a) polynomial: all real numbers $(-\infty, \infty)$

b) rational function: since a fraction with

$$f(x) = \frac{2}{x-3}$$

$$\begin{aligned} x-3 &= 0 \\ x &= 3 \leftarrow \text{denominator} \\ &\quad \text{will be 0} \end{aligned}$$

$$\boxed{(-\infty, 3) \cup (3, \infty)}$$

domain

denominator 0 cannot be evaluated, the domain will be all real numbers except those values that will make fraction have a zero value on denominator

c) radical function: since we cannot evaluate radical (even roots)

$$f(x) = \sqrt{x-2}$$

$$x-2 \geq 0$$

$$x \geq 2$$

$$2 \leq x$$

$$\boxed{\begin{array}{l} \text{domain;} \\ [2, \infty) \end{array}}$$

equation (even roots) where inside is negative, we solve the inequality of inside expression of radical function ≥ 0 .

d) radical function: all real numbers $(-\infty, \infty)$ (odd roots)

$$f(x) = \sqrt[3]{x}, \quad f(x) = \sqrt[5]{x}, \text{ etc.}$$

$$6) f(x) = -2x(x-1)(x-2) \text{ polynomial}$$

$$\text{domain: } (-\infty, \infty)$$

$$8) f(x) = 3\sqrt{x-2} \text{ radical (even root)}$$

$$x-2 \geq 0$$

$$x \geq 2$$

$$\text{domain: } [2, \infty)$$

4) parenthesis: value is not included

bracket: value is included

$$10) f(x) = \sqrt{4-3x} \text{ radical (even root)}$$

$$4-3x \geq 0$$

$$4 \geq 3x$$

$$\frac{4}{3} \geq x$$

$$\Rightarrow x \leq \frac{4}{3} \text{ domain: } (-\infty, \frac{4}{3}]$$

$$12) f(x) = \sqrt[3]{1-2x} \text{ radical (odd root)}$$

$$\text{domain: } (-\infty, \infty)$$

$$14) f(x) = \frac{9}{x-6} \text{ rational}$$

$$x-6 = 0$$

$$x = 6$$

$$\text{domain: } (-\infty, 6) \cup (6, \infty)$$

$$16) f(x) = \frac{\sqrt{x+4}}{x-4}$$

radical (even root) $\rightarrow$ on top
rational $\rightarrow$ bottom

radical part

$$x+4 \geq 0$$

$$x \geq -4 \quad [-4, \infty)$$

$$-4 \leq x$$

rational part

$$x-4 = 0$$

$$x = 4 \rightarrow (-\infty, 4) \cup (4, \infty)$$

intersection
 $\swarrow$

domain: $[-4, 4) \cup (4, \infty)$

$$18) f(x) = \frac{1}{x^2 - x - 6}$$

rational

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$x+2 = 0 \quad | \quad x-3 = 0$$

$$x = -2 \quad | \quad x = 3$$

domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$20) f(x) = \frac{5}{\sqrt{x-3}}$$

radical (even root) & rational
on bottom

$$x-3 > 0$$

$$x > 3$$

$$3 < x$$

domain: $(3, \infty)$

$$22) f(x) = \frac{\sqrt{x-4}}{\sqrt{x-6}}$$

radical (even root) $\rightarrow$ on top
 rational & radical (even root) $\rightarrow$ on bottom

radical top

$$x-4 \geq 0$$

$$x \geq 4 \quad [4, \infty)$$

$$4 \leq x$$

rational & radical on bottom

$$x-6 > 0$$

$$x > 6 \quad (6, \infty)$$

$$6 < x$$

intersection

domain: $(6, \infty)$

$$24) f(x) = \frac{x}{x} = 1$$

rational

$$x \neq 0$$

domain: $(-\infty, 0) \cup (0, \infty)$

$$26) a) f(x) = \sqrt{2x^3 - 50x}$$

radical (even root)

$$2x^3 - 50x \geq 0$$

$$2x(x^2 - 25) \geq 0$$

$$(x+5)(2x)(x-5) \geq 0$$

$$x+5=0 \quad | \quad 2x=0 \quad | \quad x-5=0$$

$$x = \boxed{-5} \quad | \quad x = \boxed{0} \quad | \quad x = \boxed{5}$$

	$\boxed{-5}$	$\boxed{0}$	$\boxed{5}$	
	$(-\infty, -5)$	$(-5, 0)$	$(0, \infty)$	$(5, \infty)$
$(x+5)$	neg	POS	POS	POS
$(2x)$	neg	neg	POS	POS
$(x-5)$	neg	neg	neg	POS
$(x+5)(2x)(x-5)$	neg	POS	neg	POS

domain: $[-5, 0] \cup [5, \infty)$

28) domain: $[4, 8)$

range: $(2, 8]$

30) domain: $[2, 6]$

range: $[-6, -2]$

32) domain: $[-3, 2)$

range: $[-5, 4]$

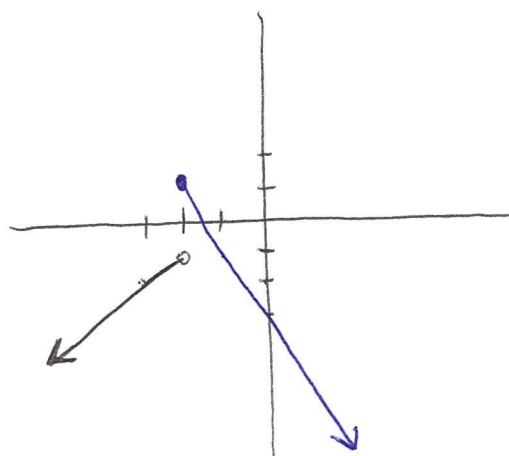
34) domain: $[-4, \infty)$

range: $[-2, \infty)$

36) domain: $(-2, 2, \infty)$

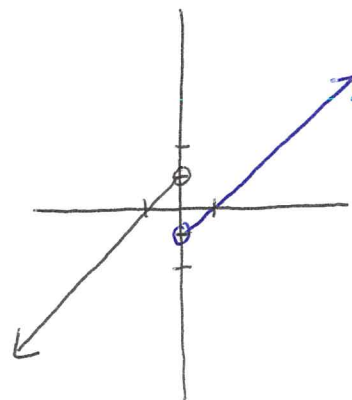
range: $[-4, \infty)$

38) $f(x) = \begin{cases} x+1 & \text{if } x < -2 \\ -2x-3 & \text{if } x \geq -2 \end{cases}$



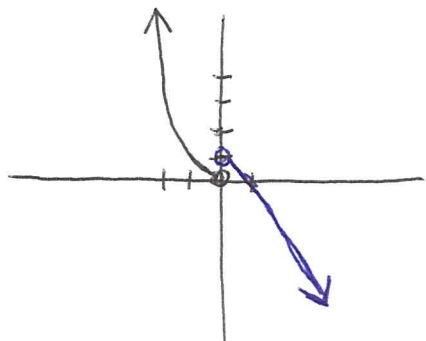
domain: $(-\infty, \infty)$

40) $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } x > 0 \end{cases}$



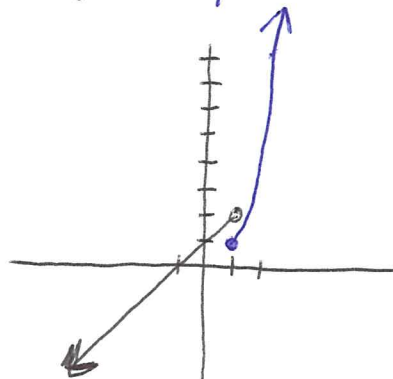
domain: $(-\infty, 0) \cup (0, \infty)$

$$42) f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1-x & \text{if } x > 0 \end{cases}$$



domain: $(-\infty, 0) \cup (0, \infty)$

$$44) f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ x^3 & \text{if } x \geq 1 \end{cases}$$



domain: $(-\infty, \infty)$

$$46) f(x) = \begin{cases} x+1 & \text{if } x < -2 \\ -2x+3 & \text{if } x \geq -2 \end{cases}$$

$$f(-3) = (-2) + 1 = \underline{\underline{-1}}$$

$$f(-2) = -2(-2) + 1 = 4 + 1 = \underline{\underline{5}}$$

$$f(-1) = -2(-1) + 1 = 2 + 1 = \underline{\underline{3}}$$

$$f(0) = -2(0) + 1 = \underline{\underline{1}}$$

$$48) f(x) = \begin{cases} 2x^2+3 & \text{if } x \leq -1 \\ 5x+7 & \text{if } x > -1 \end{cases}$$

$$f(-3) = 2(-3)^2 + 3 = 2(9) + 3 = 18 + 3 = \underline{\underline{21}}$$

$$f(-2) = 2(-2)^2 + 3 = 2(4) + 3 = 8 + 3 = \underline{\underline{11}}$$

$$f(-1) = 2(-1)^2 + 3 = 2(1) + 3 = \underline{\underline{5}}$$

$$f(0) = 5(0) + 7 = \underline{\underline{7}}$$

$$50) f(x) = \begin{cases} x^2-2 & \text{if } x < 2 \\ 4+|x-5| & \text{if } x \geq 2 \end{cases}$$

$$f(-1) = (-1)^2 - 2 = 1 - 2 = \underline{\underline{-1}}$$

$$f(0) = (0)^2 - 2 = \underline{\underline{-2}}$$

$$f(2) = 4 + |(2) - 5| = 4 + |-3| = 4 + 3 = \underline{\underline{7}}$$

$$f(4) = 4 + |(4) - 5| = 4 + |-1| = 4 + 1 = \underline{\underline{5}}$$

$$52) \text{ domain: } (-\infty, \infty)$$

$$54) \text{ domain: }$$

$$(-\infty, 0) \cup (2, \infty)$$