section 9.8

for this section, use the guide given in section 7.6.

only solving inequality algebraically shown here.

364) $x^2+4x-12<0$

(step 3) $(x+6)(x-2)<0$

(step 4) critical points

top

$x+6=0$, $-6\ -6$, $x=-6$ | $x-2=0$, $+2\ +2$, $x=2$

step 5,6	$(-\infty,-6)$	$(-6,2)$	$(2,\infty)$
$(x+6)$	neg	POS	POS
$(x-2)$	neg	neg	POS
$(x+6)(x-2)$	POS	neg	POS

(step 7) ans: $(-6,2)$

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366) $x^2-6x+8\geq 0$

(step 3) $(x-2)(x-4)\geq 0$

(step 4) critical points

top

$x-2=0$, $+2\ +2$, $x=\boxed{2}$ | $x-4=0$, $+4\ +4$, $x=\boxed{4}$

step 5,6	$(-\infty,2)$	$(2,4)$	$(4,\infty)$
$(x-2)$	neg	POS	POS
$(x-4)$	neg	neg	POS
$(x-2)(x-4)$	POS	neg	POS

(step 7) ans: $(-\infty,2]\cup[4,\infty)$

---

368) $-x^2+2x+24<0$

$+x^2-2x-24 \quad +x^2-2x-24$

$0<x^2-2x-24$

(step 3) $0<(x+4)(x-6)$

(step 4) critical points

top

$x+4=0$, $-4\ -4$, $x=-4$ | $x-6=0$, $+6\ +6$, $x=6$

step 5,6	$(-\infty,-4)$	$(-4,6)$	$(6,\infty)$
$(x+4)$	neg	POS	POS
$(x-6)$	neg	neg	POS
$(x+4)(x-6)$	POS	neg	POS

(step 7) ans: $(-\infty,-4)\cup(6,\infty)$

370) $-x^2+2x+15>0$

$\quad +x^2-2x-15 \quad +x^2-2x-15$

$0 > x^2-2x-15$

(step 3) $0 > (x+3)(x-5)$

(step 4) critical points

top

$x+3=0$, $-3\ -3$, $x=-3$ | $x-5=0$, $+5\ +5$, $x=5$

step 5,6	$(-\infty,-3)$	$(-3,5)$	$(5,\infty)$
$(x+3)$	neg	POS	POS
$(x-5)$	neg	neg	POS
$(x+3)(x-5)$	POS	neg	POS

(step 7) ans: $(-3,5)$

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372) $x^2+x-6 \le 0$

(step 3) $(x+3)(x-2) \le 0$

(step 4) critical points

top

$x+3=0$, $-3\ -3$, $x=\boxed{-3}$ | $x-2=0$, $+2\ +2$, $x=\boxed{2}$

step 5,6	$(-\infty,-3)$	$(-3,2)$	$(2,\infty)$
$(x+3)$	neg	POS	POS
$(x-2)$	neg	neg	POS
$(x+3)(x-2)$	POS	neg	POS

(step 7) ans: $[-3,2]$

---

374) $x^2-4x+3>0$

(step 3) $(x-1)(x-3)>0$

(step 4) critical points

top

$x-1=0$, $+1\ +1$, $x=1$ | $x-3=0$, $+3\ +3$, $x=3$

step 5,6	$(-\infty,1)$	$(1,3)$	$(3,\infty)$
$(x-1)$	neg	POS	POS
$(x-3)$	neg	neg	POS
$(x-1)(x-3)$	POS	neg	POS

(step 7) ans: $(-\infty,1) \cup (3,\infty)$

376) $x^2+8x<-12$

(step 1) $x^2+8x+12<-12+12$

$x^2+8x+12<0$

(step 3) $(x+6)(x+2)<0$

(step 4) critical points

top

$x+6=0$, $x=-6$ | $x+2=0$, $x=-2$

step 5, 6

	$(-\infty,-6)$	$(-6,-2)$	$(-2,\infty)$
$(x+6)$	neg	POS	POS
$(x+2)$	neg	neg	POS
$(x+6)(x+2)$	POS	neg	POS

(step 7) ans: $(-6,-2)$

---

378) $-x^2+8x-11<0$

$+x^2-8x+11<+x^2-8x+11$

$0<x^2-8x+11$

this trinomial is not factorable

$a=1$ $b=-8$ $c=11$

$b^2-4ac=(-8)^2-4(1)(11)=64-44=20$

not perfect square, 2 real sol.

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(11)}}{2(1)}$$

$$x=\frac{8\pm\sqrt{20}}{2}=\frac{8\pm\sqrt{4}\sqrt{5}}{2}=\frac{8\pm2\sqrt{5}}{2}$$

$$x=\frac{\cancel{2}^{1}(4\pm\sqrt{5})}{\cancel{2}_{1}}=4\pm\sqrt{5}$$

critical points (step 4)

top

$x=4-\sqrt{5}$ $\Downarrow$ $x=(4-\sqrt{5})$, $-(4-\sqrt{5})\ -(4-\sqrt{5})$, $x-(4-\sqrt{5})=0$, $\{x-(4-\sqrt{5})\}=0$ | $x=4+\sqrt{5}$ $\Downarrow$ $x=(4+\sqrt{5})$, $-(4+\sqrt{5})-(4+\sqrt{5})$, $x-(4+\sqrt{5})=0$, $\{x-(4+\sqrt{5})\}=0$

"step 3" $0<\{x-(4-\sqrt{5})\}\{x-(4+\sqrt{5})\}$

step 5, 6

	$(-\infty,4-\sqrt{5})$	$(4-\sqrt{5},4+\sqrt{5})$	$(4+\sqrt{5},\infty)$
$\{x-(4-\sqrt{5})\}$	neg	POS	POS
$\{x+(4+\sqrt{5})\}$	neg	neg	POS
$\{x-(4-\sqrt{5})\}\{x-(4+\sqrt{5})\}=x^2-8x+11$	POS	neg	POS

(step 7) ans: $(-\infty,4-\sqrt{5})\cup(4+\sqrt{5},\infty)$

380) $x^2 + 6x < -3$

(step 1)
$$\begin{array}{r} x^2+6x<-3 \\ +3 \quad +3 \\ \hline x^2+6x+3<0 \end{array}$$

this trinomial is not factorable

$a=1 \quad b=6 \quad c=3$

$b^2-4ac=(6)^2-4(1)(3)=36-12=28$

not perfect square, 2 real number solutions

$$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(3)}}{2(1)}=\frac{-6\pm\sqrt{28}}{2}=\frac{-6\pm\sqrt{4}\sqrt{7}}{2}=\frac{-6\pm2\sqrt{7}}{2}$$

$$x=\frac{\cancel{2}^{1}(-3\pm\sqrt{7})}{\cancel{2}_{1}}=-3\pm\sqrt{7}$$

$x^2+6+3<0$

"step 3"
$\{x+3+\sqrt{7}\}\{x+3-\sqrt{7}\}<0$

(step 4) critical points

top

$x=-3-\sqrt{7}$	$x=-3+\sqrt{7}$
$\Downarrow$	$\Downarrow$
$\begin{array}{r} x=(-3-\sqrt{7}) \\ -(-3-\sqrt{7}) \quad -(-3-\sqrt{7}) \\ \hline x-(-3-\sqrt{7})=0 \end{array}$	$\begin{array}{r} x=(-3+\sqrt{7}) \\ -(-3+\sqrt{7})-(-3+\sqrt{7}) \\ \hline x-(-3+\sqrt{7})=0 \end{array}$
$\{x-(-3-\sqrt{7})\}=0$	$\{x-(-3+\sqrt{7})\}=0$
$\{x+3+\sqrt{7}\}=0$	$\{x+3-\sqrt{7}\}=0$

(step 7) ans: $\underline{\underline{(-3-\sqrt{7}, -3+\sqrt{7})}}$

step 5,6

	$(-\infty, -3-\sqrt{7})$	$(-3-\sqrt{7}) \quad (-3-\sqrt{7}, -3+\sqrt{7})$	$(-3+\sqrt{7}) \quad (-3+\sqrt{7}, \infty)$
$\{x+3+\sqrt{7}\}$	neg	POS	POS
$\{x+3-\sqrt{7}\}$	neg	neg	POS
$\{x+3+\sqrt{7}\}\{x+3-\sqrt{7}\} = x^2+6x+3$	POS	neg	POS

382) $-3x^2 - 4x + 4 \le 0$

$\quad +3x^2 + 4x - 4 \qquad +3x^2 + 4x - 4$

$0 \le 3x^2 + 4x - 4$

$a = 3 \quad b = 4 \quad c = -4$

$b^2 - 4ac = (4)^2 - 4(3)(-4)$

$= 16 + 48 = 64$

perfect square, 2 real sol.
factorable

(step 3) $\quad 0 \le (x+2)(3x-2)$

(step 4) critical points

top

$x + 2 = 0$
$\quad -2 \ \ -2$
$x = \boxed{-2}$

$3x - 2 = 0$
$\quad +2 \ \ +2$
$\frac{3x}{3} = \frac{2}{3}$
$x = \boxed{\frac{2}{3}}$

$\boxed{-2} \qquad \boxed{\frac{2}{3}}$

step 5, 6	$(-\infty, -2)$	$(-2, \frac{2}{3})$	$(\frac{2}{3}, \infty)$
$(x+2)$	neg	POS	POS
$(3x-2)$	neg	neg	POS
$(x+2)(3x-2)$	POS	neg	POS

(step 7) ans: $\underline{\underline{(-\infty, -2] \cup [\frac{2}{3}, \infty)}}$

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384) $2x^2 + 5x - 12 > 0$

(step 3) $(x+4)(2x-3) > 0$

(step 4) critical points

top

$x + 4 = 0$
$\quad -4 \ \ -4$
$x = -4$

$2x - 3 = 0$
$\quad +3 \ \ +3$
$\frac{2x}{2} = \frac{3}{2}$
$x = \frac{3}{2}$

$-4 \qquad \frac{3}{2}$

step 5, 6	$(-\infty, -4)$	$(-4, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
$(x+4)$	neg	POS	POS
$(2x-3)$	neg	neg	POS
$(x+4)(2x-3)$	POS	neg	POS

(step 7) ans: $\underline{\underline{(-\infty, -4) \cup (\frac{3}{2}, \infty)}}$

386) $x^2 - 3x + 6 \le 0$ this trinomial is not factorable

$a=1 \quad b=-3 \quad c=6$

$b^2-4ac = (-3)^2-4(1)(6) = 9-24 = -15$

2 complex number sol.

<u>no critical points</u>

step 5, 6	$(-\infty, \infty)$
x^2-3x+6	use test value $x=0$ $(0)^2-3(0)+6=6$ POS

(step 7) ans: <u>none</u>

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388) $-x^2-4x-5 < 0$ this trinomial is not factorable

$$\begin{array}{r} -x^2-4x-5 < 0 \\ +x^2+4x+5 \quad +x^2+4x+5 \\ \hline 0 < x^2+4x+5 \end{array}$$

$a=1 \quad b=4 \quad c=5$

$b^2-4ac = (4)^2-4(1)(5) = 16-20 = -4$

2 complex number sol.

<u>no critical points</u>

step 5, 6	$(-\infty, \infty)$
x^2+4x+5	use test value $x=0$ $(0)^2+4(0)+5=5$ POS

(step 7) ans: $(-\infty, \infty)$

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390) $-x^2+2x-7 \ge 0$ this trinomial is not factorable

$$\begin{array}{r} -x^2+2x-7 \ge 0 \\ +x^2-2x+7 \quad +x^2-2x+7 \\ \hline 0 \ge x^2-2x+7 \end{array}$$

$a=1 \quad b=-2 \quad c=7$

$b^2-4ac = (-2)^2-4(1)(7) = 4-28 = -24$

2 complex number sol.

<u>no critical points</u>

step 5, 6	$(-\infty, \infty)$
x^2-2x+7	use test value $x=0$ $(0)^2-2(0)+7=7$ POS

(step 7) ans: <u>none</u>