Section 9.6

Graphing Quadratic Functions (Parabolas)

The standard form for graphing Quadratic function (Parabola) is

$$y = f(x) = a(x-h)^2 + k$$

and its coordinate of vertex is (h,k)
Note: this is also Horizontal Shift: h and Vertical Shift: k

If $a > 0$, then the parabola opens upward.

If $a < 0$, then the parabola opens downward.

"vertex formula" {note this formula only gives x coordinate of the vertex, value of h}

$$h = x = \frac{-b}{2a}$$ (this is also the axis of symmetry)

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converting the quadratic equation $f(x) = ax^2 + bx + c$

to Standard form $f(x) = a(x-h)^2 + k$

Step 1: Find $h = x = \frac{-b}{2a}$

Step 2: Calculate $k = f(k) = a(h)^2 + b(h) + c$. Now we have the vertex: (h,k)

Step 3: Write in Standard Form using h and k values found in steps 1 and 2 plus a value.

230) $f(x) = x^2 - 3$

$f(x) = (x-(0))^2 + (-3)$

H-shift: 0
V-shift: -3 } vertex: $(0,-3)$

like $y = x^2$

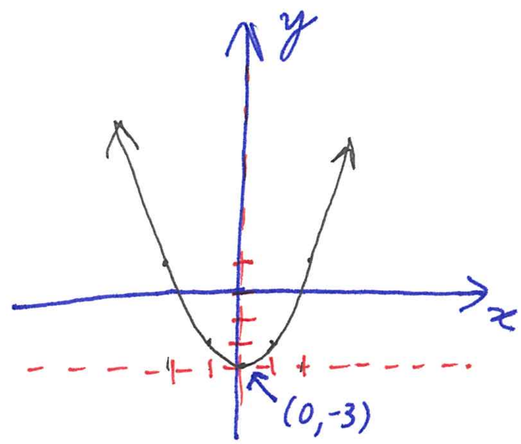
domain: $(-\infty, \infty)$

range: $[-3, \infty)$

axis of symmetry: $x = 0$

$a = 1 > 0$: parabola opens upward

minimum value is $^-3$



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232) $f(x) = -x^2 - 1$

$f(x) = -(x-(0))^2 + (-1)$

H-shift: 0
V-shift: -1 } vertex: $(0,-1)$

like $y = -x^2$

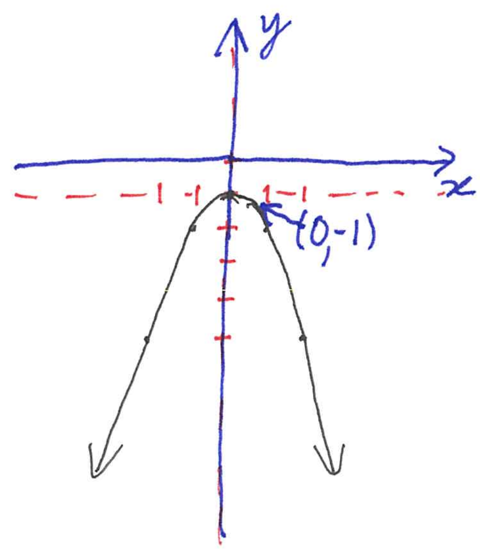
domain: $(-\infty, \infty)$

range: $(-\infty, -1]$

axis of symmetry: $x = 0$

$a = ^-1 < 0$: parabola opens downward

Maximum value is $^-1$



234-a) $f(x)=4x^2+x-4$ $a=4>0$: opens upward

234-b) $f(x)=-9x^2-24x-16$ $a=-9<0$: opens downward

236-a) $f(x)=x^2+3x-4$ $a=1>0$: opens upward

236-b) $f(x)=-4x^2-12x-9$ $a=-4<0$: opens downward

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238) $f(x)=x^2+10x+25$

$a=1 \quad b=10 \quad c=25$

a) $h=x=\frac{-b}{2a}=\frac{-(10)}{2(1)}=\frac{-10}{2}=-5$

axis of symmetry: $\underline{\underline{x=-5}}$

b) $k=f(-5)=(-5)^2+10(-5)+25=25-50+25=0$

vertex (h,k): $\underline{\underline{(-5,0)}}$

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240) $f(x)=-2x^2-8x-3$

$a=-2 \quad b=-8 \quad c=-3$

a) $h=x=\frac{-b}{2a}=\frac{-(-8)}{2(-2)}=\frac{8}{-4}=-2$

axis of symmetry: $\underline{\underline{x=-2}}$

b) $k=f(-2)=-2(-2)^2-8(-2)-3=-8+16-3=5$

vertex (h,k): $\underline{\underline{(-2,5)}}$

242) $f(x) = x^2 + 10x - 11$

y-int: set $x = 0$

$y = f(0) = (0)^2 + 10(0) - 11 = -11$

$y = -11$ or $(0, -11)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = x^2 + 10x - 11$

$0 = (x+11)(x-1)$

$x + 11 = 0$	$x - 1 = 0$
$-11 \; -11$	$+1 \; +1$
$x = -11$	$x = 1$
$(-11, 0)$	$(1, 0)$

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244) $f(x) = x^2 + 5x + 6$

y-int: set $x = 0$

$y = f(0) = (0)^2 + 5(0) + 6 = 6$

$y = 6$ $(0, 6)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = x^2 + 5x + 6$

$0 = (x+3)(x+2)$

$x + 3 = 0$	$x + 2 = 0$
$-3 \; -3$	$-2 \; -2$
$x = -3$	$x = -2$
$(-3, 0)$	$(-2, 0)$

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246) $f(x) = -3x^2 + x - 1$

y-int: set $x = 0$

$y = f(0) = -3(0)^2 + (0) - 1 = -1$

$y = -1$ $(0, -1)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = -3x^2 + x - 1$

$a = -3$ $b = 1$ $c = -1$

$b^2 - 4ac = (1)^2 - 4(-3)(-1) = 1 - 12 = -11$

2 complex number solutions

no x-intercept

248) $f(x) = x^2 + 8x + 12$

y-int: set $x = 0$

$y = f(0) = (0)^2 + 8(0) + 12 = 12$

$y = 12 \quad (0, 12)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = x^2 + 8x + 12$

$0 = (x+6)(x+2)$

$x + 6 = 0$
$-6 \;\; -6$
$x = -6$
$(-6, 0)$

$x + 2 = 0$
$-2 \;\; -2$
$x = -2$
$(-2, 0)$

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250) $f(x) = -x^2 - 14x - 49$

y-int: set $x = 0$

$y = f(0) = -(0)^2 - 14(0) - 49 = -49$

$y = -49 \quad (0, -49)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = -x^2 - 14x - 49$

$x^2 + 14x + 49 = 0$

$(x+7)(x+7) = 0$

$(x+7)^2 = 0$

$x + 7 = 0$
$-7 \;\; -7$
$x = -7$
$(-7, 0)$

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252) $f(x) = 4x^2 + 4x + 1$

y-int: set $x = 0$

$y = f(0) = 4(0)^2 + 4(0) + 1 = 1$

$y = 1 \quad (0, 1)$

x-int: set $y = f(x) = 0$, solve for x

$0 = f(x) = 4x^2 + 4x + 1$

$0 = (2x+1)(2x+1)$

$0 = (2x+1)^2$

$2x + 1 = 0$
$-1 \;\; -1$
$\frac{2x}{2} = \frac{-1}{2}$
$x = -\frac{1}{2}$
$\left(-\frac{1}{2}, 0\right)$

254) $f(x) = x^2 + 4x - 12$ axis of symmetry:

$a = 1 \quad b = 4 \quad c = -12$

$h = x = \frac{-b}{2a} = \frac{-(4)}{2(1)} = -2$

$k = f(-2) = (-2)^2 + 4(-2) - 12 = 4 - 8 - 12 = -16$

standard form: $f(x) = a(x-h)^2 + k$

$= 1(x-(-2))^2 + (-16) = (x+2)^2 + 16$

vertex $(h, k) = (-2, 16)$

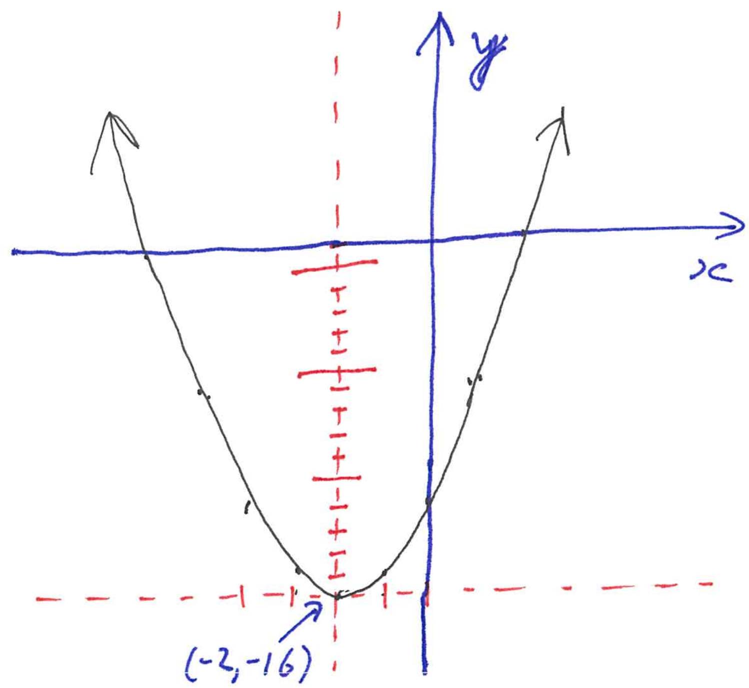
like

$y = x^2$

min. value: -16

domain: $(-\infty, \infty)$

range: $[-16, \infty)$



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256) $f(x) = x^2 - 6x + 8$ axis of symmetry:

$a = 1 \quad b = -6 \quad c = 8$

$h = x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = 3$

$k = f(3) = (3)^2 - 6(3) + 8 = 9 - 18 + 8 = -1$

standard form: $f(x) = a(x-h)^2 + k$

$= 1(x-(3))^2 + (-1) = (x-3)^2 - 1$

vertex (h, k): $(3, -1)$

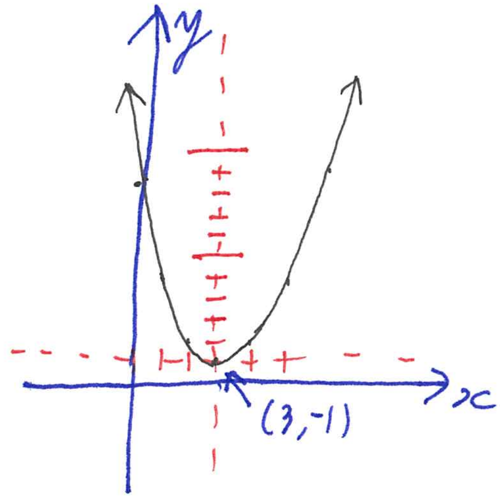
256) continued...

like $y = x^2$

min. value: -1

domain: $(-\infty, \infty)$

range: $[-1, \infty)$



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258) $f(x) = -x^2 + 8x - 16$

$a = -1 \quad b = 8 \quad c = -16$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(8)}{2(-1)} = 4$

$k = f(4) = -(4)^2 + 8(4) - 16 = -16 + 32 - 16 = 0$

standard form: $f(x) = a(x-h)^2 + k$

$= (-1)(x-(4))^2 + (0) = -(x-4)^2$

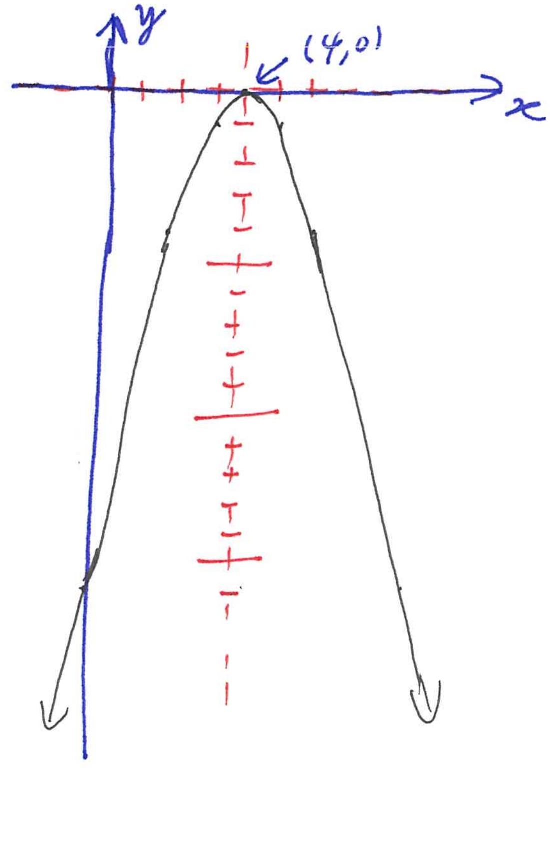
vertex (h,k): $(4,0)$

like $y = -x^2$

Max. value: 0

domain: $(-\infty, \infty)$

range: $(-\infty, 0]$



260) $f(x) = 5x^2 + 2 = 5x^2 + 0x + 2$

$a = 5 \quad b = 0 \quad c = 2$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(0)}{2(5)} = 0$

$k = f(0) = 5(0)^2 + 2 = 2$

standard form: $f(x) = a(x-h)^2 + k$

$= 5(x-(0))^2 + (2)$

vertex (h,k): $(0,2)$

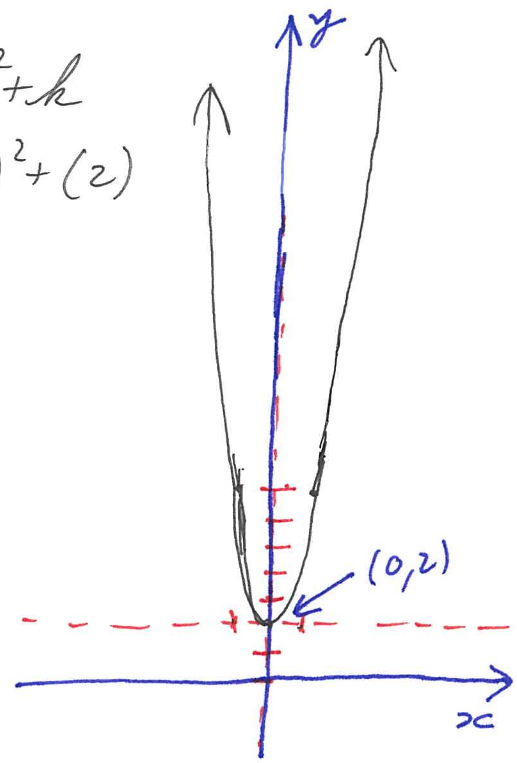
like $y = 5x^2$

x	y
2	20
1	5
0	0
-1	5
-2	20

min. value: 2

domain: $(-\infty, \infty)$

range: $[2, \infty)$



262) $f(x) = 3x^2 - 6x - 1$

$a = 3 \quad b = -6 \quad c = -1$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$

$k = f(1) = 3(1)^2 - 6(1) - 1 = 3 - 6 - 1 = -4$

standard form: $f(x) = a(x-h)^2 + k$

$= 3(x-(1))^2 + (-4) = 3(x-1)^2 - 4$

vertex (h,k): $(1,-4)$

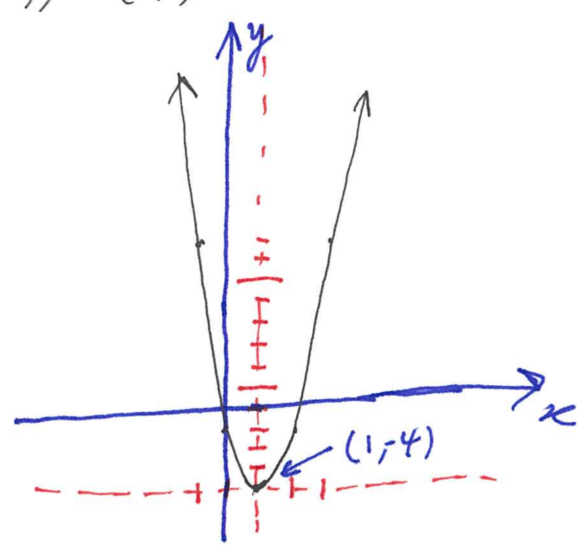
like $y = 3x^2$

x	y
2	12
1	3
0	0
-1	3
-2	12

min. value: -4

domain: $(-\infty, \infty)$

range: $[-4, \infty)$



264) $f(x) = -4x^2 - 6x - 2$

$a = -4 \quad b = -6 \quad c = -2$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(-6)}{2(-4)} = \frac{6}{-8} = \frac{-3}{4}$

$k = f\left(\frac{-3}{4}\right) = -4\left(\frac{-3}{4}\right)^2 - 6\left(\frac{-3}{4}\right) - 2 = \frac{-9}{4} + \frac{18}{4} - \frac{8}{4} = \frac{-9+18-8}{4} = \frac{1}{4}$

standard form: $f(x) = a(x-h)^2 + k$

$= -4\left(x - \left(\frac{-3}{4}\right)\right)^2 + \left(\frac{1}{4}\right) = -4\left(x + \frac{3}{4}\right)^2 + \frac{1}{4}$

vertex (h, k): $\left(\frac{-3}{4}, \frac{1}{4}\right)$

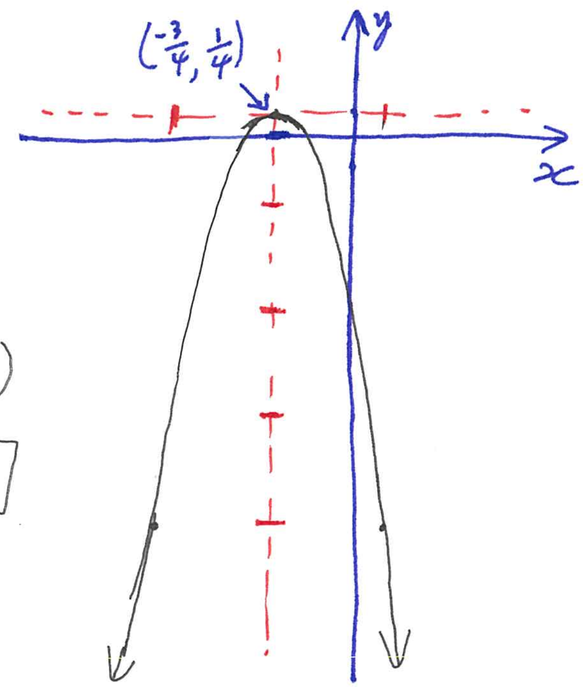
like $y = -4x^2$

x	y
2	-16
1	-4
0	0
-1	-4
-2	-16

Max. value: $\frac{1}{4}$

domain: $(-\infty, \infty)$

range: $\left(-\infty, \frac{1}{4}\right]$



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266) $f(x) = x^2 + 6x + 8$

$a = 1 \quad b = 6 \quad c = 8$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$

$k = f(-3) = (-3)^2 + 6(-3) + 8 = 9 - 18 + 8 = -1$

standard form: $f(x) = a(x-h)^2 + k$

$= 1(x - (-3))^2 + (-1) = (x+3)^2 - 1$

vertex (h, k): $(-3, -1)$

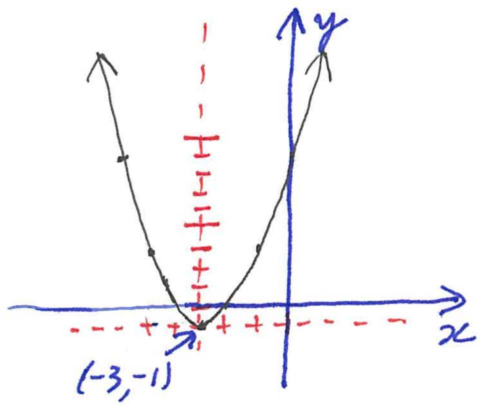
like $y = x^2$

266) continued...

min value: -1

domain: $(-\infty, \infty)$

range: $[-1, \infty)$



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268) $f(x) = -16x^2 + 24x - 9$

$a = -16 \quad b = 24 \quad c = -9$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(24)}{2(-16)} = \frac{-12}{-16} = \frac{3}{4}$

$k = f\left(\frac{3}{4}\right) = -16\left(\frac{3}{4}\right)^2 + 24\left(\frac{3}{4}\right) - 9 = -9 + 18 - 9 = 0$

standard form: $f(x) = a(x-h)^2 + k$

$= -16\left(x - \left(\frac{3}{4}\right)\right)^2 + (0) = -16\left(x - \frac{3}{4}\right)^2$

vertex (h, k): $\left(\frac{3}{4}, 0\right)$

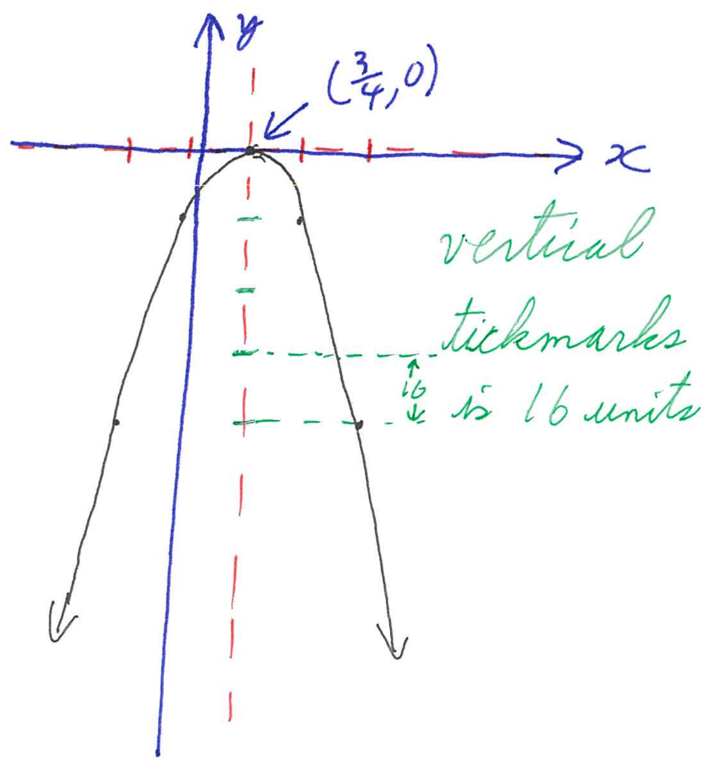
like $y = -16x^2$

x	y
2	-64
1	-16
0	0
-1	-16
-2	-64

Max value: 0

domain: $(-\infty, \infty)$

range: $(-\infty, 0]$



270) $f(x) = -2x^2 + 8x - 10$

$a = -2 \quad b = 8 \quad c = -10$

axis of symmetry:

$h = x = \frac{-b}{2a} = \frac{-(8)}{2(-2)} = \frac{-8}{-4} = 2$

$k = f(2) = -2(2)^2 + 8(2) - 10 = -8 + 16 - 10 = -2$

standard form: $f(x) = a(x-h)^2 + k$

$= -2(x-(2))^2 + (-2) = -2(x-2)^2 - 2$

vertex (h, k): $(2, -2)$

like

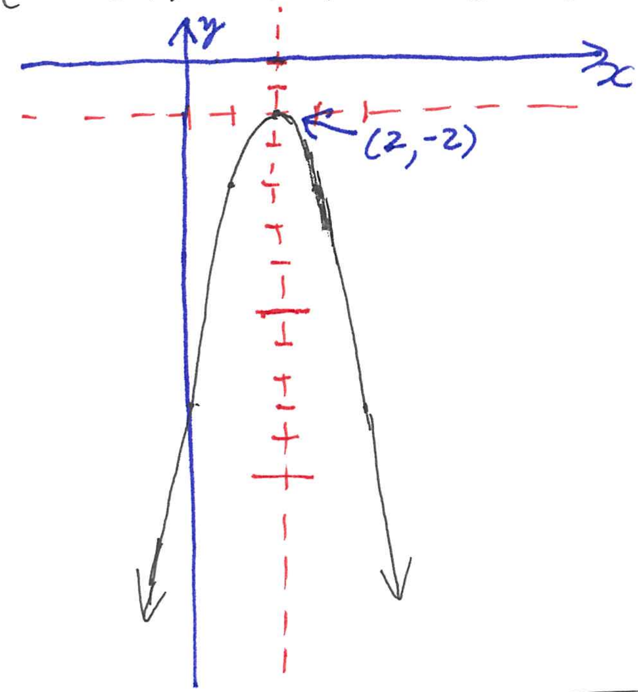
$y = -2x^2$

x	y
2	-8
1	-2
0	0
-1	-2
-2	-8

Max. value: -2

domain: $(-\infty, \infty)$

range: $(-\infty, -2]$



272) $y = -4x^2 + 12x - 5$

$a = -4 \quad b = 12 \quad c = -5$

$h = x = \frac{-b}{2a} = \frac{-(12)}{2(-4)} = \frac{-12}{-8} = \frac{3}{2}$

$k = f(\frac{3}{2}) = -4(\frac{3}{2})^2 + 12(\frac{3}{2}) - 5 = -9 + 18 - 5 = 4$ Max. value: 4

274) $y = -x^2 + 4x - 5$

$h = x = \frac{-b}{2a} = \frac{-(4)}{2(-1)} = \frac{-4}{-2} = 2$

$a = -1 \quad b = 4 \quad c = -5$

$k = f(2) = -(2)^2 + 4(2) - 5 = -4 + 8 - 5 = -1$ Max. value: -1

276) $y = 4x^2 - 49 = 4x^2 + 0x - 49$

$h = x = \frac{-b}{2a} = \frac{-(0)}{2(4)} = 0$

$a = 4 \quad b = 0 \quad c = -49$

$k = f(0) = 4(0)^2 - 49 = -49$ min. value: -49