Section 9.3

Quadratic Formula

Given a quadratic equality $ax^2+bx+c=0$, the quadratic equation is $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Discriminant: b^2-4ac

Case 1: When $\{b^2-4ac\}>0$, then the quadratic equation has 2 real number solutions. Also, if $\{b^2-4ac\}$ is a perfect square (i.e. 4, 9, 16, 25, 36, 39, etc.), then the quadratic equation is factorable.

Case 2: When $\{b^2-4ac\}=0$, then the quadratic equation has 1 real number solution and it is factorable.

Case 3: When $\{b^2-4ac\}<0$, then the quadratic equation has 2 Complex Number solutions.

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Rule of thumb for solving quadratic equation.

Step 1: Calculate the discriminant to see if the quadratic equation has real number solutions. This step can help us if we should try factoring or not.

Step 2: Try factoring.

Step 3: If factoring takes too long or the discriminant states that it is not factorable, then use the Quadratic Formula or Completing the Square technique.

114) $4n^2 - 9n + 5 = 0$

$a = 4 \quad b = -9 \quad c = 5$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(5)}}{2(4)}$$

$$n = \frac{9 \pm \sqrt{1}}{8}$$

$$n = \frac{9 \pm 1}{8}$$

$$n = \frac{9-1}{8} = \frac{8}{8} = 1, \quad n = \frac{9+1}{8} = \frac{10}{8} = \frac{5}{4}$$

$$\boxed{\begin{array}{l} n = 1 \\ n = \frac{5}{4} \end{array}}$$

discriminant:

$b^2 - 4ac = (-9)^2 - 4(4)(5)$

$= 81 - 4(20) = 81 - 80 = 1$

perfect square, 2 real. solutions

factorable

$4n^2 - 9n + 5 = 0$

$(n-1)(4n-5) = 0$

$n - 1 = 0$	$4n - 5 = 0$
$+1 \quad +1$	$+5 \quad +5$
$n = 1$	$\frac{4n}{4} = \frac{5}{4}$
	$n = \frac{5}{4}$

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116) $3q^2 + 8q - 3 = 0$

$a = 3 \quad b = 8 \quad c = -3$

$$q = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(-3)}}{2(3)}$$

$$q = \frac{-8 \pm \sqrt{100}}{6}$$

$$q = \frac{-8 \pm 10}{6}$$

$$q = \frac{-8-10}{6} = \frac{-18}{6} = -3, \quad q = \frac{-8+10}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\boxed{\begin{array}{l} q = \frac{1}{3} \\ q = -3 \end{array}}$$

$b^2 - 4ac = (8)^2 - 4(3)(-3)$

$= 64 + 36 = 100$

perfect square, 2 real solutions

factorable

$3q^2 + 8q - 3 = 0$

$(q+3)(3q-1) = 0$

$q + 3 = 0$	$3q - 1 = 0$
$-3 \quad -3$	$+1 \quad +1$
$q = -3$	$\frac{3q}{3} = \frac{1}{3}$
	$q = \frac{1}{3}$

118) $q^2+3q-18=0$

$a=1 \quad b=3 \quad c=-18$

$$q=\frac{-(3)\pm\sqrt{(3)^2-4(1)(-18)}}{2(1)}$$

$$q=\frac{-3\pm\sqrt{81}}{2}$$

$$q=\frac{-3\pm 9}{2}$$

$$q=\frac{-3-9}{2}=\frac{-12}{2}=-6, \quad q=\frac{-3+9}{2}=\frac{6}{2}=3$$

$$\boxed{\begin{array}{l} q=3 \\ q=-6 \end{array}}$$

$b^2-4ac=(3)^2-4(1)(-18)$

$=9+72=81$

perfect square, 2 real solutions

factorable

$q^2+3q-18=0$

$(q+6)(q-3)=0$

$$\begin{array}{l} q+6=0 \\ \quad -6\ -6 \\ \hline q=6 \end{array} \qquad \begin{array}{l} q-3=0 \\ \quad +3\ +3 \\ \hline q=3 \end{array}$$

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120) $t^2+13t=-40$

$$\begin{array}{r} t^2+13t=-40 \\ +40\quad +40 \\ \hline t^2+13t+40=0 \end{array}$$

$a=1 \quad b=13 \quad c=40$

$$t=\frac{-(13)\pm\sqrt{(13)^2-4(1)(40)}}{2(1)}$$

$$t=\frac{-13\pm\sqrt{9}}{2}$$

$$t=\frac{-13\pm 3}{2}$$

$$t=\frac{-13-3}{2}=\frac{-16}{2}=-8, \quad t=\frac{-13+3}{2}=\frac{-10}{2}=-5$$

$$\boxed{\begin{array}{l} t=-5 \\ t=-8 \end{array}}$$

$b^2-4ac=(13)^2-4(1)(40)$

$=169-160=9$

perfect square, 2 real solutions

factorable

$t^2+13t+40=0$

$(t+8)(t+5)=0$

$$\begin{array}{l} t+8=0 \\ \quad -8\ -8 \\ \hline t=-8 \end{array} \qquad \begin{array}{l} t+5=0 \\ \quad -5\ -5 \\ \hline t=-5 \end{array}$$

122) $2p^2 + 8p + 5 = 0$

$a = 2 \quad b = 8 \quad c = 5$

$b^2 - 4ac = (8)^2 - 4(2)(5)$

$= 64 - 40 = 24$

not perfect square, 2 real solutions

$$p = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(5)}}{2(2)}$$

$$p = \frac{-8 \pm \sqrt{24}}{4}$$

$$p = \frac{-8 \pm \sqrt{4}\sqrt{6}}{4}$$

$$p = \frac{-8 \pm 2\sqrt{6}}{4}$$

$$p = \frac{\cancel{2}^{1}(-4 \pm \sqrt{6})}{\cancel{4}_{2}}$$

$$p = \frac{-4 \pm \sqrt{6}}{2}$$

$$\boxed{p = \frac{-4 + \sqrt{6}}{2} \qquad p = \frac{-4 - \sqrt{6}}{2}}$$

124) $5b^2 + 2b - 4 = 0$

$a = 5 \quad b = 2 \quad c = -4$

$b^2 - 4ac = (2)^2 - 4(5)(-4)$

$= 4 + 80 = 84$

not perfect square, 2 real solutions

$$b = \frac{-(2) \pm \sqrt{(2)^2 - 4(5)(-4)}}{2(5)}$$

$$b = \frac{-2 \pm \sqrt{84}}{10}$$

$$b = \frac{-2 \pm \sqrt{4}\sqrt{21}}{10}$$

$$b = \frac{-2 \pm 2\sqrt{21}}{10}$$

$$b = \frac{\cancel{2}^{1}(-1 \pm \sqrt{21})}{\cancel{10}_{5}}$$

$$b = \frac{-1 \pm \sqrt{21}}{5}$$

$$\boxed{b = \frac{-1 + \sqrt{21}}{5} \qquad b = \frac{-1 - \sqrt{21}}{5}}$$

126) $y^2+4y-4=0$

$a=1 \quad b=4 \quad c=-4$

$b^2-4ac=(4)^2-4(1)(-4)$

$=16+16=32$

not perfect square, 2 real. sol.

$$y=\frac{-(4)\pm\sqrt{(4)^2-4(1)(-4)}}{2(1)}$$

$$y=\frac{-4\pm\sqrt{32}}{2}$$

$$y=\frac{-4\pm\sqrt{16}\sqrt{2}}{2}$$

$$y=\frac{-4\pm4\sqrt{2}}{2}$$

$$y=\frac{\cancel{2}^{1}(-2\pm2\sqrt{2})}{\cancel{2}_{1}}$$

$$y=-2\pm2\sqrt{2}$$

$$\boxed{\begin{aligned} y&=-2+2\sqrt{2} \\ y&=-2-2\sqrt{2} \end{aligned}}$$

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128) $6x^2+2x-20=0$

$a=6 \quad b=2 \quad c=-20$

$b^2-4ac=(2)^2-4(6)(-20)$

$=4+480=484=(4)(121)$

perfect square, 2 real sol.
factorable

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(6)(-20)}}{2(6)}$$

$$x=\frac{-2\pm\sqrt{484}}{12}$$

$$x=\frac{-2\pm\sqrt{4}\sqrt{121}}{12}$$

$$x=\frac{-2\pm(2)(11)}{12}$$

$$x=\frac{-2\pm22}{12}$$

$$x=\frac{\cancel{2}^{1}(-1\pm11)}{\cancel{12}_{6}}$$

$$x=\frac{-1\pm11}{6}$$

$$x=\frac{-1-11}{6}=\frac{-12}{6}=-2$$

$$x=\frac{-1+11}{6}=\frac{10}{6}=\frac{5}{3}$$

$$\boxed{\begin{aligned} x&=\frac{5}{3} \\ x&=-2 \end{aligned}}$$

$6x^2+2x-20=0$

$2(3x^2+x-10)=0$

$2(x+2)(3x-5)=0$

$x+2=0$	$3x-5=0$
$-2 \quad -2$	$+5 \quad +5$
$x=-2$	$\frac{3x}{3}=\frac{5}{3}$
	$x=\frac{5}{3}$

130) $2x^2 - x + 1 = 0$

$a=2 \quad b=-1 \quad c=1$

$b^2-4ac = (-1)^2 - 4(2)(1)$

$= 1 - 8 = -7$

2 Complex number solution

$\boxed{\text{no solution in real number}}$

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132) $8x^2 - 4x + 1 = 0$

$a=8 \quad b=-4 \quad c=1$

$b^2-4ac = (-4)^2 - 4(8)(1)$

$= 16 - 32 = -16$

2 Complex number solution

$\boxed{\text{no solution in real number}}$

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134) $(x+1)(x-3) = 2$

$x^2 - 2x - 3 = 2$

$\quad\ \ -2 \quad -2$

$x^2 - 2x - 5 = 0$

$a=1 \quad b=-2 \quad c=-5$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = \frac{2 \pm \sqrt{4}\sqrt{6}}{2}$$

$$x = \frac{2 \pm 2\sqrt{6}}{2}$$

$b^2-4ac = (-2)^2 - 4(1)(-5)$

$= 4 + 20 = 24$

not perfect square, 2 real sol.

$$x = \frac{\cancel{2}^{1}(1 \pm \sqrt{6})}{\cancel{2}_{1}}$$

$x = 1 \pm \sqrt{6}$

$$\boxed{\begin{array}{l} x = 1 + \sqrt{6} \\ x = 1 - \sqrt{6} \end{array}}$$

136) $(x+2)(x+6)=21$

$$x^2+8x+12=21$$
$$\quad -21 \quad -21$$
$$x^2+8x-9=0$$

$a=1 \quad b=8 \quad c=-9$

$$x=\frac{-(8)\pm\sqrt{(8)^2-4(1)(-9)}}{2(1)}$$

$$x=\frac{-8\pm\sqrt{100}}{2}$$

$$x=\frac{-8\pm 10}{2}$$

$$x=\frac{\cancel{2}^{1}(-4\pm 5)}{\cancel{2}_{1}}$$

$x=-4\pm 5$

$x=-4+5=1$

$x=-4-5=-9$

$\boxed{x=1 \quad x=-9}$

$b^2-4ac=(8)^2-4(1)(-9)$
$=64+36=100$

perfect square, 2 real sol. factorable

$x^2+8x-9=0$

$(x+9)(x-1)=0$

$x+9=0$	$x-1=0$
$-9 \; -9$	$+1 \; +1$
$x=-9$	$x=1$

138) $\frac{1}{3}n^2+n=\frac{-1}{2}$

LCD $=(2)(3)=6$

$$\left(\frac{(2)(3)}{1}\right)\left(\frac{n^2}{3}+\frac{n}{1}\right)=\left(\frac{-1}{2}\right)\left(\frac{(2)(3)}{1}\right)$$

$$n^2(2)+n(2)(3)=(-1)(3)$$

$$2n^2+6n=-3$$
$$\quad +3 \quad +3$$
$$2n^2+6n+3=0$$

$a=2 \quad b=6 \quad c=3$

$$n=\frac{-(6)\pm\sqrt{(6)^2-4(2)(3)}}{2(2)}$$

$$n=\frac{-6\pm\sqrt{12}}{4}$$

$$n=\frac{-6\pm\sqrt{4}\sqrt{3}}{4}$$

$$n=\frac{-6\pm 2\sqrt{3}}{4}$$

$$n=\frac{\cancel{2}^{1}(-3\pm\sqrt{3})}{\cancel{4}_{2}}$$

$$n=\frac{-3\pm\sqrt{3}}{2}$$

$b^2-4ac=(6)^2-4(2)(3)$
$=36-24=12$

not perfect square, 2 real sol.

$$\boxed{n=\frac{-3+\sqrt{3}}{2} \quad n=\frac{-3-\sqrt{3}}{2}}$$

140) $\frac{1}{9}c^2+\frac{2}{3}c=3$

$LCD=(3)^2=9$

$\left(\frac{(3)^2}{1}\right)\left(\frac{c^2}{(3)^2}+\frac{2c}{(3)}\right)=\left(\frac{3}{1}\right)\left(\frac{(3)^2}{1}\right)$

$c^2+2c(3)=3(3)^2$

$c^2+6c=27$

$-27\ -27$

$c^2+6c-27=0$

$a=1\quad b=6\quad c=-27$

$c=\frac{-(6)\pm\sqrt{(6)^2-4(1)(-27)}}{2(1)}$

$c=\frac{-6\pm\sqrt{144}}{2}$

$c=\frac{-6\pm12}{2}$

$c=\frac{\cancel{2}(-3\pm6)}{\cancel{2}}$

$c=-3\pm6$

$c=-3+6=3$

$c=-3-6=-9$

$\boxed{c=3 \quad c=-9}$

$b^2-4ac=(6)^2-4(1)(-27)$

$=36+108=144$

perfect square, 2 real sol.
factorable

$c^2+6c-27=0$

$(c+9)(c-3)=0$

$c+9=0$ | $c-3=0$
$-9\ -9$ | $+3\ +3$
$c=-9$ | $c=3$

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142) $25d^2-60d+36=0$

$a=25\quad b=-60\quad c=36$

$d=\frac{-(-60)\pm\sqrt{(-60)^2-4(25)(36)}}{2(25)}$

$d=\frac{60\pm\sqrt{0}}{50}$

$d=\frac{60}{50}=\frac{6}{5}$

$\boxed{d=\frac{6}{5}}$

$b^2-4ac=(-60)^2-4(25)(36)$

$=3600-3600=0$

1 real number solution
factorable

$25d^2-60d+36=0$

$(5d)^2-60d+(6)^2=0$

$(5d-6)^2=0$

$5d-6=0$

$+6\ +6$

$\frac{5d}{5}=\frac{6}{5}$

$d=\frac{6}{5}$

144) $16y^2+8y+1=0$

$a=16 \quad b=8 \quad c=1$

$$y=\frac{-(8)\pm\sqrt{(8)^2-4(16)(1)}}{2(16)}$$

$$y=\frac{-8\pm\sqrt{0}}{32}$$

$$y=\frac{-8}{32}=\frac{-1}{4}$$

$$\boxed{y=\frac{-1}{4}}$$

$b^2-4ac=(8)^2-4(16)(1)$
$=64-64=0$

1 real number solution
factorable

$16y^2+8y+1=0$
$(4y+1)(4y+1)=0$
$(4y+1)^2=0$

$4y+1=0$
$\quad -1 \; -1$
$\frac{4y}{4}=\frac{-1}{4}$
$y=\frac{-1}{4}$

146-a) $9v^2-15v+25=0$

$a=9 \quad b=-15 \quad c=25$

$b^2-4ac=(-15)^2-4(9)(25)$
$=225-900=-675$

2 complex number sol.

146-b) $100w^2+60w+9=0$

$a=100 \quad b=10 \quad c=9$

$b^2-4ac=(60)^2-4(100)(9)$
$=3600-3600=0$

1 real number solution, factorable

146-c) $5c^2+7c-10=0$

$a=5 \quad b=7 \quad c=-10$

$b^2-4ac=(7)^2-4(5)(-10)$
$=49+200=249$

not perfect square, 2 real number sol.

148-a) $25p^2+10p+1=0$

$a=25 \quad b=10 \quad c=1$

$b^2-4ac=(10)^2-4(25)(1)$
$=100-100=0$

1 real number solution, factorable

148-b) $7q^2-3q-6=0$

$a=7 \quad b=-3 \quad c=-6$

$b^2-4ac=(-3)^2-4(7)(-6)$
$=9+168=177$

not perfect square, 2 real number sol.

148-c) $7y^2+2y+8=0$

$a=7 \quad b=2 \quad c=8$

$b^2-4ac=(2)^2-4(7)(8)$
$=4-224=-220$

2 complex number sol.

150-a) $(8v+3)^2=81$ Square Root

150-b) $w^2-9w-22=0$ Factoring

150-c) $4n^2-10n=6$ Factoring

152-a) $8b^2+15b=4$

$\quad\quad\quad\quad\quad -4 \;\; -4$

$8b^2+15b-4=0$

$a=8 \quad b=15 \quad c=-4$

$b^2-4ac=(15)^2-4(8)(-4)$
$=225+128=353$

Quadratic Formula

152-b) $\frac{5}{9}v^2-\frac{2}{3}v=1$ $LCD=(3)^2=9$

$\left(\frac{(3)^2}{1}\right)\left(\frac{5}{(3)^2}v^2-\frac{2}{3}v\right)=\left(\frac{1}{1}\right)\left(\frac{(3)^2}{1}\right)$

$5v^2-2v(3)=1(3)^2$

$5v^2-6v=9$

$\quad\quad -9 \;\; -9$

$5v^2-6v-9=0$

$a=5 \quad b=-6 \quad c=-9$

$b^2-4ac=(-6)^2-4(5)(-9)$
$=36+180=216$

Quadratic Formula

152-c) $\left(w+\frac{4}{3}\right)^2=\frac{2}{9}$ Square Root