

section 9.2

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make sure to make the leading term's (2nd degree, square term's) coefficient is positive one (+1). If not, we cannot use easier method and more complicated technique is needed!

72-a)

$$n^2 - 16n$$

$$\frac{b_n}{2} = \frac{-16}{2} = -8$$

$$= n^2 - 16n + (64)$$

$$\left(\frac{b_n}{2}\right)^2 = (-8)^2 = 64$$

$$= \underline{\underline{(n-8)^2}}$$

72-b)

$$y^2 + 15y$$

$$\frac{b_y}{2} = \frac{15}{2}$$

$$= y^2 + 15y + \left(\frac{225}{4}\right)$$

$$\left(\frac{b_y}{2}\right)^2 = \left(\frac{15}{2}\right)^2 = \frac{225}{4}$$

$$= \underline{\underline{\left(y + \frac{15}{2}\right)^2}}$$

72-c)

$$q^2 + \frac{3}{4}q$$

$$\frac{b_q}{2} = \frac{\left(\frac{3}{4}\right)}{2} = \frac{3}{8}$$

$$= q^2 + \frac{3}{4}q + \left(\frac{9}{64}\right)$$

$$\left(\frac{b_q}{2}\right)^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

$$= \underline{\underline{\left(q + \frac{3}{8}\right)^2}}$$

74-a)

$$\frac{dq}{2} = \frac{-6}{2} = -3$$

$$\left(\frac{dq}{2}\right)^2 = (-3)^2 = 9$$

$$q^2 - 6q$$

$$= q^2 - 6q + (9)$$

$$= \underline{\underline{(q-3)^2}}$$

74-b)

$$\frac{dx}{2} = \frac{-7}{2}$$

$$\left(\frac{dx}{2}\right)^2 = \left(\frac{-7}{2}\right)^2 = \frac{49}{4}$$

$$x^2 - 7x$$

$$= x^2 - 7x + \left(\frac{49}{4}\right)$$

$$= \underline{\underline{\left(x - \frac{7}{2}\right)^2}}$$

74-c)

$$\frac{dn}{2} = \frac{\left(\frac{-2}{3}\right)}{2} = \frac{-1}{3}$$

$$\left(\frac{dn}{2}\right)^2 = \left(\frac{-1}{3}\right)^2 = \frac{1}{9}$$

$$n^2 - \frac{2}{3}n$$

$$= n^2 - \frac{2}{3}n + \left(\frac{1}{9}\right)$$

$$= \underline{\underline{\left(n - \frac{1}{3}\right)^2}}$$

76)

$$z^2 + 12z = -11$$

$$\frac{dz}{2} = \frac{12}{2} = 6 \quad z^2 + 12z + (36) = -11 + (36)$$

$$\left(\frac{dz}{2}\right)^2 = (6)^2 = 36$$

$$(z+6)^2 = 25$$

$$(z+6) = \pm\sqrt{25} = \pm 5$$

$$z+6 = \pm 5$$

$$\begin{array}{r} -6 \quad -6 \\ \hline z = -6 \pm 5 \end{array}$$

$$z = -6 + 5 = -1$$

$$z = -6 - 5 = -11$$

$$\boxed{z = -11, z = -1}$$

$$\begin{array}{r} z^2 + 12z = -11 \\ +11 \quad +11 \\ \hline \end{array}$$

$$z^2 + 12z + 11 = 0$$

$$(z+11)(z+1) = 0$$

$$z+11=0$$

$$\begin{array}{r} -11 \quad -11 \\ \hline z = -11 \end{array}$$

$$z+1=0$$

$$\begin{array}{r} -1 \quad -1 \\ \hline z = -1 \end{array}$$

78)

$$y^2 - 2y = 8$$

$$\frac{b_y}{2} = \frac{-2}{2} = -1$$

$$\left(\frac{b_y}{2}\right)^2 = (-1)^2 = 1$$

$$y^2 - 2y + (1) = 8 + (1)$$

$$(y-1)^2 = 9$$

$$(y-1) = \pm\sqrt{9} = \pm 3$$

$$y-1 = \pm 3$$

$$\begin{array}{r} +1 \quad +1 \\ \hline \end{array}$$

$$y = 1 \pm 3$$

$$y = 1+3 = 4$$

$$y = 1-3 = -2$$

$$\boxed{y = -2, y = 4}$$

$$\begin{array}{r} y^2 - 2y = 8 \\ -8 \quad -8 \\ \hline \end{array}$$

$$y^2 - 2y - 8 = 0$$

$$(y+2)(y-4) = 0$$

$$y+2=0$$

$$\begin{array}{r} -2 \quad -2 \\ \hline \end{array}$$

$$y = -2$$

$$y-4=0$$

$$\begin{array}{r} +4 \quad +4 \\ \hline \end{array}$$

$$y = 4$$

80)

$$n^2 - 2n = -3$$

$$\frac{b_n}{2} = \frac{-2}{2} = -1$$

$$\left(\frac{b_n}{2}\right)^2 = (-1)^2 = 1$$

$$n^2 - 2n + (1) = -3 + (1)$$

$$(n-1)^2 = -2$$

$$(n-1) = \pm\sqrt{-2}$$

no real # solution

82)

$$t^2 - 14t = -50$$

$$\frac{b_t}{2} = \frac{-14}{2} = -7$$

$$\left(\frac{b_t}{2}\right)^2 = (-7)^2 = 49$$

$$t^2 - 14t + (49) = -50 + (49)$$

$$(t-7)^2 = -1$$

$$(t-7) = \pm\sqrt{-1}$$

no real # solution

84)

$$b^2 + 6b = 41$$

$$\frac{b}{2} = \frac{6}{2} = 3$$

$$\left(\frac{b}{2}\right)^2 = (3)^2 = 9$$

$$b^2 + 6b + (9) = 41 + (9)$$

$$(b+3)^2 = 50$$

$$(b+3) = \pm\sqrt{50} = \pm\sqrt{25}\sqrt{2} = \pm 5\sqrt{2}$$

$$b+3 = \pm 5\sqrt{2}$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$b = -3 \pm 5\sqrt{2}$$

$$\begin{array}{l} b = -3 + 5\sqrt{2} \\ b = -3 - 5\sqrt{2} \end{array}$$

86)

$$\frac{by}{2} = \frac{-3}{2}$$

$$\left(\frac{by}{2}\right)^2 = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$$

$$y^2 - 3y = 2$$

$$y^2 - 3y + \left(\frac{9}{4}\right) = 2 + \left(\frac{9}{4}\right)$$

$$\left(y - \frac{3}{2}\right)^2 = \frac{8}{4} + \frac{9}{4} = \frac{17}{4}$$

$$\left(y - \frac{3}{2}\right) = \pm\sqrt{\frac{17}{4}} = \frac{\pm\sqrt{17}}{\sqrt{4}} = \frac{\pm\sqrt{17}}{2}$$

$$y - \frac{3}{2} = \frac{\pm\sqrt{17}}{2}$$

$$\begin{array}{r} +\frac{3}{2} \quad +\frac{3}{2} \\ \hline \end{array}$$

$$y = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

$$y = \frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

$$y = \frac{3 \pm \sqrt{17}}{2}$$

$$\begin{array}{l} y = \frac{3 + \sqrt{17}}{2} \\ y = \frac{3 - \sqrt{17}}{2} \end{array}$$

88)

$$\frac{bz}{2} = \frac{2}{2} = 1$$

$$\left(\frac{bz}{2}\right)^2 = (1)^2 = 1$$

$$\begin{array}{r} z^2 + 2z - 5 = 2 \\ +5 \quad +5 \\ \hline \end{array}$$

$$z^2 + 2z = 7$$

$$z^2 + 2z + (1) = 7 + (1)$$

$$(z+1)^2 = 8$$

$$(z+1) = \pm\sqrt{8} = \pm\sqrt{4}\sqrt{2} = \pm 2\sqrt{2}$$

$$z+1 = \pm 2\sqrt{2}$$

$$\begin{array}{r} -1 \quad -1 \\ \hline \end{array}$$

$$z = -1 \pm 2\sqrt{2}$$

$$\begin{array}{l} z = -1 + 2\sqrt{2} \\ z = -1 - 2\sqrt{2} \end{array}$$

90)

$$\frac{b_x}{2} = \frac{-10}{2} = -5$$

$$\left(\frac{b_x}{2}\right)^2 = (-5)^2 = 25$$

$$x^2 - 10x - 6 = 5$$

$$\begin{array}{r} x^2 - 10x - 6 = 5 \\ +6 \quad +6 \\ \hline x^2 - 10x = 11 \end{array}$$

$$x^2 - 10x + (25) = 11 + (25)$$

$$(x - 5)^2 = 36$$

$$(x - 5) = \pm \sqrt{36} = \pm 6$$

$$\begin{array}{r} x - 5 = \pm 6 \\ +5 \quad +5 \\ \hline x = 5 \pm 6 \end{array}$$

$$x = 5 \pm 6$$

$$x = 5 + 6 = 11$$

$$x = 5 - 6 = -1$$

$$x = 11$$

$$x = -1$$

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$$\begin{array}{r} x^2 - 10x - 6 = 5 \\ -5 \quad -5 \\ \hline x^2 - 10x - 11 = 0 \end{array}$$

$$x^2 - 10x - 11 = 0$$

$$(x + 1)(x - 11) = 0$$

$$x + 1 = 0$$

$$\begin{array}{r} -1 \quad -1 \\ \hline x = -1 \end{array}$$

$$x = -1$$

$$x - 11 = 0$$

$$\begin{array}{r} +11 \quad +11 \\ \hline x = 11 \end{array}$$

$$x = 11$$

92)

$$\frac{b_w}{2} = \frac{5}{2}$$

$$\left(\frac{b_w}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$w^2 = 5w - 1$$

$$\begin{array}{r} w^2 = 5w - 1 \\ -5w \quad -5w \\ \hline w^2 - 5w = -1 \end{array}$$

$$w^2 - 5w = -1$$

$$w^2 - 5w + \left(\frac{25}{4}\right) = -1 + \left(\frac{25}{4}\right)$$

$$\left(w - \frac{5}{2}\right)^2 = \frac{-4}{4} + \frac{25}{4} = \frac{21}{4}$$

$$\left(w - \frac{5}{2}\right) = \pm \sqrt{\frac{21}{4}} = \frac{\pm \sqrt{21}}{\sqrt{4}} = \frac{\pm \sqrt{21}}{2}$$

$$w - \frac{5}{2} = \frac{\pm \sqrt{21}}{2}$$

$$\begin{array}{r} + \frac{5}{2} \quad + \frac{5}{2} \\ \hline w = \frac{5}{2} \pm \frac{\sqrt{21}}{2} \end{array}$$

$$w = \frac{5}{2} \pm \frac{\sqrt{21}}{2}$$

$$w = \frac{5}{2} \pm \frac{\sqrt{21}}{2}$$

$$w = \frac{5 \pm \sqrt{21}}{2}$$

$$w = \frac{5 + \sqrt{21}}{2}$$

$$w = \frac{5 - \sqrt{21}}{2}$$

94)

$$\frac{dy}{2} = \frac{-6}{2} = -3$$

$$\left(\frac{dy}{2}\right)^2 = (-3)^2 = 9$$

$$\begin{array}{r} y^2 - 14 = 6y \\ -6y + 14 \quad -6y + 14 \\ \hline \end{array}$$

$$y^2 - 6y = 14$$

$$y^2 - 6y + (9) = 14 + (9)$$

$$(y - 3)^2 = 23$$

$$(y - 3) = \pm \sqrt{23}$$

$$y - 3 = \pm \sqrt{23}$$

$$\begin{array}{r} +3 \quad +3 \\ \hline y = 3 \pm \sqrt{23} \end{array}$$

$$\boxed{\begin{array}{l} y = 3 + \sqrt{23} \\ y = 3 - \sqrt{23} \end{array}}$$

96)

$$\frac{dy}{2} = \frac{16}{2} = 8$$

$$\left(\frac{dy}{2}\right)^2 = (8)^2 = 64$$

$$(y+9)(y+7) = 80$$

$$\begin{array}{r} y^2 + 16y + 63 = 80 \\ -63 \quad -63 \\ \hline \end{array}$$

$$y^2 + 16y = 17$$

$$y^2 + 16y + (64) = 17 + (64)$$

$$(y + 8)^2 = 81$$

$$(y + 8) = \pm \sqrt{81} = \pm 9$$

$$y + 8 = \pm 9$$

$$\begin{array}{r} -8 \quad -8 \\ \hline y = -8 \pm 9 \end{array}$$

$$y = -8 + 9 = 1$$

$$y = -8 - 9 = -17$$

$$\boxed{\begin{array}{l} y = 1 \\ y = -17 \end{array}}$$

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$$\begin{array}{r} y^2 + 16y + 63 = 80 \\ -80 \quad -80 \\ \hline \end{array}$$

$$y^2 + 16y - 17 = 0$$

$$(y + 17)(y - 1) = 0$$

$$y + 17 = 0$$

$$\begin{array}{r} -17 \quad -17 \\ \hline y = -17 \end{array}$$

$$y - 1 = 0$$

$$\begin{array}{r} +1 \quad +1 \\ \hline y = 1 \end{array}$$

98)

$$\frac{bx}{2} = \frac{-8}{2} = -4$$

$$\left(\frac{bx}{2}\right)^2 = (-4)^2 = 16$$

$$(x-2)(x-6) = 5$$

$$\begin{array}{r} x^2 - 8x + 12 = 5 \\ -12 \quad -12 \\ \hline \end{array}$$

$$x^2 - 8x = -7$$

$$x^2 - 8x + (16) = -7 + (16)$$

$$(x-4)^2 = 9$$

$$(x-4) = \pm\sqrt{9} = \pm 3$$

$$x-4 = \pm 3$$

$$\begin{array}{r} +4 \quad +4 \\ \hline \end{array}$$

$$x = 4 \pm 3$$

$$x = 4 + 3 = 7$$

$$x = 4 - 3 = 1$$

$$\boxed{x=7, x=1}$$

$$\begin{array}{r} x^2 - 8x + 12 = 5 \\ -5 \quad -5 \\ \hline \end{array}$$

$$x^2 - 8x + 7 = 0$$

$$(x-1)(x-7) = 0$$

$$x-1=0$$

$$\begin{array}{r} +1 \quad +1 \\ \hline \end{array}$$

$$x=1$$

$$x-7=0$$

$$\begin{array}{r} +7 \quad +7 \\ \hline \end{array}$$

$$x=7$$

100)

$$\frac{bx}{2} = \frac{-7}{2}$$

$$\left(\frac{bx}{2}\right)^2 = \left(\frac{-7}{2}\right)^2 = \frac{49}{4}$$

$$\begin{array}{r} 2x^2 - 14x + 12 = 0 \\ -12 \quad -12 \\ \hline \end{array}$$

$$2x^2 - 14x = -12$$

$$\begin{array}{r} 2(x^2 - 7x) = \frac{-12}{2} \\ \hline \end{array}$$

$$x^2 - 7x = -6$$

$$x^2 - 7x + \left(\frac{49}{4}\right) = -6 + \left(\frac{49}{4}\right)$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{-24}{4} + \frac{49}{4} = \frac{25}{4}$$

$$\left(x - \frac{7}{2}\right) = \pm\sqrt{\frac{25}{4}} = \frac{\pm\sqrt{25}}{\sqrt{4}} = \frac{\pm 5}{2}$$

$$x - \frac{7}{2} = \pm \frac{5}{2}$$

$$\begin{array}{r} +\frac{7}{2} \quad +\frac{7}{2} \\ \hline \end{array}$$

$$x = \frac{7}{2} \pm \frac{5}{2}$$

$$x = \frac{7 \pm 5}{2}$$

$$x = \frac{7+5}{2} = \frac{12}{2} = 6$$

$$x = \frac{7-5}{2} = \frac{2}{2} = 1$$

$$\boxed{x=1, x=6}$$

$$2x^2 - 14x + 12 = 0$$

$$2(x^2 - 7x + 6) = 0$$

$$2(x-1)(x-6) = 0$$

$$x-1=0$$

$$\begin{array}{r} +1 \quad +1 \\ \hline \end{array}$$

$$x=1$$

$$x-6=0$$

$$\begin{array}{r} +6 \quad +6 \\ \hline \end{array}$$

$$x=6$$

102)

$$\frac{dx}{2} = \frac{4}{2} = 2$$

$$\left(\frac{dx}{2}\right)^2 = (2)^2 = 4$$

$$5x^2 + 20x = 15$$

$$\frac{5(x^2 + 4x)}{5} = \frac{15}{5}$$

$$x^2 + 4x = 3$$

$$x^2 + 4x + (4) = 3 + (4)$$

$$(x + 2)^2 = 7$$

$$(x + 2) = \pm\sqrt{7}$$

$$x + 2 = \pm\sqrt{7}$$

$$\begin{array}{r} -2 \quad -2 \\ \hline x = -2 \pm \sqrt{7} \end{array}$$

$$x = -2 + \sqrt{7}$$

$$x = -2 - \sqrt{7}$$

104)

$$\frac{dy}{2} = \left(\frac{-4}{3}\right) = \frac{-2}{3}$$

$$\left(\frac{dy}{2}\right)^2 = \left(\frac{-2}{3}\right)^2 = \frac{4}{9}$$

$$3d^2 - 4d = 15$$

$$\frac{3(d^2 - \frac{4}{3}d)}{3} = \frac{15}{3}$$

$$d^2 - \frac{4}{3}d = 5$$

$$d^2 - \frac{4}{3}d + \left(\frac{4}{9}\right) = 5 + \left(\frac{4}{9}\right)$$

$$\left(d - \frac{2}{3}\right)^2 = \frac{45}{9} + \frac{4}{9} = \frac{49}{9}$$

$$\left(d - \frac{2}{3}\right) = \pm\sqrt{\frac{49}{9}} = \pm\frac{\sqrt{49}}{\sqrt{9}} = \pm\frac{7}{3}$$

$$d - \frac{2}{3} = \pm\frac{7}{3}$$

$$\begin{array}{r} + \frac{2}{3} \quad + \frac{2}{3} \\ \hline \end{array}$$

$$d = \frac{2}{3} \pm \frac{7}{3}$$

$$d = \frac{2 \pm 7}{3}$$

$$d = \frac{2+7}{3} = \frac{9}{3} = 3$$

$$d = \frac{2-7}{3} = \frac{-5}{3}$$

$$\boxed{\begin{array}{l} d = 3 \\ d = \frac{-5}{3} \end{array}}$$

$$\begin{array}{r} 3d^2 - 4d = 15 \\ -15 \quad -15 \\ \hline \end{array}$$

$$3d^2 - 4d - 15 = 0$$

$$(3d+5)(d-3) = 0$$

$$3d+5=0$$

$$\begin{array}{r} -5 \quad -5 \\ \hline \end{array}$$

$$3d = -5$$

$$\frac{3d}{3} = \frac{-5}{3}$$

$$d = \frac{-5}{3}$$

$$d-3=0$$

$$\begin{array}{r} +3 \quad +3 \\ \hline \end{array}$$

$$d = 3$$

106)

$$\frac{b_x}{2} = \frac{\left(-\frac{14}{3}\right)}{2} = -\frac{7}{3}$$

$$\left(\frac{b_x}{2}\right)^2 = \left(-\frac{7}{3}\right)^2 = \frac{49}{9}$$

$$3x^2 - 14x + 8 = 0$$

$$\frac{-8 \quad -8}{3x^2 - 14x = -8}$$

$$\frac{3\left(x^2 - \frac{14}{3}x\right) = -8}{3}$$

$$x^2 - \frac{14}{3}x = -\frac{8}{3}$$

$$x^2 - \frac{14}{3}x + \left(\frac{49}{9}\right) = -\frac{8}{3} + \left(\frac{49}{9}\right)$$

$$\left(x - \frac{7}{3}\right)^2 = \frac{-24}{9} + \frac{49}{9} = \frac{25}{9}$$

$$\left(x - \frac{7}{3}\right) = \pm \sqrt{\frac{25}{9}} = \frac{\pm \sqrt{25}}{\sqrt{9}} = \frac{\pm 5}{3}$$

$$x - \frac{7}{3} = \frac{\pm 5}{3}$$

$$+ \frac{7}{3} \quad + \frac{7}{3}$$

$$x = \frac{7}{3} \pm \frac{5}{3} = \frac{7 \pm 5}{3}$$

$$x = \frac{7+5}{3} = \frac{12}{3} = 4$$

$$x = \frac{7-5}{3} = \frac{2}{3}$$

$$\boxed{x = 4}$$

$$\boxed{x = \frac{2}{3}}$$

$$3x^2 - 14x + 8 = 0$$

$$(3x-2)(x-4) = 0$$

$$\begin{array}{l|l} 3x-2=0 & x-4=0 \\ +2 \quad +2 & +4 \quad +4 \end{array}$$

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

$$x = 4$$

108)

$$\frac{b_x}{2} = \frac{\left(-\frac{5}{3}\right)}{2} = -\frac{5}{6}$$

$$\left(\frac{b_x}{2}\right)^2 = \left(-\frac{5}{6}\right)^2 = \frac{25}{36}$$

$$3q^2 - 5q = 9$$

$$\frac{3\left(q^2 - \frac{5}{3}q\right) = 9}{3}$$

$$q^2 - \frac{5}{3}q = 3$$

$$q^2 - \frac{5}{3}q + \left(\frac{25}{36}\right) = 3 + \left(\frac{25}{36}\right)$$

$$\left(q - \frac{5}{6}\right)^2 = \frac{108}{36} + \frac{25}{36} = \frac{133}{36}$$

$$\left(q - \frac{5}{6}\right) = \pm \sqrt{\frac{133}{36}} = \frac{\pm \sqrt{133}}{\sqrt{36}} = \frac{\pm \sqrt{133}}{6}$$

$$q - \frac{5}{6} = \frac{\pm \sqrt{133}}{6}$$

$$\begin{array}{l|l} q - \frac{5}{6} = \frac{\pm \sqrt{133}}{6} & \\ + \frac{5}{6} & + \frac{5}{6} \end{array}$$

$$q = \frac{5}{6} \pm \frac{\sqrt{133}}{6} = \frac{5 \pm \sqrt{133}}{6}$$

$$\boxed{q = \frac{5 + \sqrt{133}}{6}}$$

$$\boxed{q = \frac{5 - \sqrt{133}}{6}}$$

110)

$$7x^2 + 4x = -3$$

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$$\frac{b_x}{2} = \frac{\left(\frac{4}{7}\right)}{2} = \frac{2}{7}$$

$$\frac{7\left(x^2 + \frac{4}{7}x\right)}{7} = \frac{-3}{7}$$

$$\left(\frac{b_x}{2}\right)^2 = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$$

$$x^2 + \frac{4}{7}x = \frac{-3}{7}$$

$$x^2 + \frac{4}{7}x + \left(\frac{4}{49}\right) = \frac{-3}{7} + \left(\frac{4}{49}\right)$$

$$\left(x + \frac{2}{7}\right)^2 = \frac{-21}{49} + \frac{4}{49} = \frac{-17}{49}$$

$$\left(x + \frac{2}{7}\right) = \pm \sqrt{\frac{-17}{49}}$$

no real # solution