CCNY MATH 19000

Section 7.6 (Solve Rational Inequality)

Guide to solve rational inequality.

Warning: **DO NOT** use the technique of removing denominators used in regular solving (equality) problems (method used in section 7.4). You will **NOT** get the correct answer.

Step 1: If needed, set one side of inequality to 0.

Step 2: If needed, combine into a single fraction (you may need to find LCD, technique used in section 7.2).

Step 3: If needed, factor both numerator and denominator.

Step 4: Find critical points-the points where the rational expression is zero [this part is from solving numerator = 0] or undefined [this part is from solving denominator = 0]. Note: critical points from the numerator are part of the answer if the inequality contains equal sign (i.e. $\geq$).

Step 5: Critical points divide the number line into intervals.

Step 6: Create a grided sign chart using a test value in each interval for all factors then use this information to conclude a final sign chart.

Step 7: Determine the intervals where the inequality is correct. Write the solution in interval notation.

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340) $\frac{x+6}{x-5} \geq 0$

critical points (step 4)

numerator (top)

$x+6=0$

$-6 \quad -6$

$x = \boxed{-6}$

denominator (bottom)

$x-5=0$

$+5 \quad +5$

$x = 5$

(step 5, 6)

	$(-\infty,-6)$	$(-6,5)$	$(5,\infty)$
$(x+6)$	neg	POS	POS
$(x-5)$	neg	neg	POS
$\frac{(x+6)}{x-5}$	POS	neg	POS

ans: $(-\infty,-6] \cup (5,\infty)$ (step 7)

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342) $\frac{x-4}{x+2} \leq 0$

critical points (step 4)

(top)

$x-4=0$

$+4 \quad +4$

$x = \boxed{4}$

(bottom)

$x+2=0$

$-2 \quad -2$

$x = -2$

(step 5, 6)

	$(-\infty,-2)$	$(-2,4)$	$(4,\infty)$
$(x-4)$	neg	neg	POS
$(x+2)$	neg	POS	POS
$\frac{(x-4)}{(x+2)}$	POS	neg	POS

ans: $(-2,4]$ (step 7)

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344) $\frac{x+8}{x+3} > 0$

critical points (step 4)

(top)

$x+8=0$

$-8 \quad -8$

$x = -8$

(bottom)

$x+3=0$

$-3 \quad -3$

$x = -3$

(step 5, 6)

	$(-\infty,-8)$	$(-8,-3)$	$(-3,\infty)$
$(x+8)$	neg	POS	POS
$(x+3)$	neg	neg	POS
$\frac{(x+8)}{(x+3)}$	POS	neg	POS

ans: $(-\infty,-8) \cup (-3,\infty)$ (step 7)

346) $\frac{x+5}{x-2} < 0$

critical points (step 4)

(top) $x+5=0$, $-5\ -5$, $x=-5$

(bottom) $x-2=0$, $+2\ +2$, $x=2$

(step 5, 6)

	$(-\infty,-5)$	$(-5,2)$	$(2,\infty)$
$(x+5)$	neg	POS	POS
$(x-2)$	neg	neg	POS
$\frac{(x+5)}{(x-2)}$	POS	neg	POS

ans: $(-5, 2)$ (step 7)

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348) $\frac{5x}{x-2} < 1$

$-1\ \ -1$ (step 1)

$\frac{5x}{x-2} - 1 < 0$

LCD $= (x-2)$

$\frac{5x}{(x-2)} - 1\left(\frac{(x-2)}{(x-2)}\right) < 0$ (step 2)

$\frac{5x-1(x-2)}{x-2} < 0$

$\frac{5x-x+2}{x-2} < 0$

$\frac{4x-2}{x-2} < 0$

$\frac{2(2x-1)}{(x-2)} < 0$ (step 3)

critical points (step 4)

(top) $2x-1=0$, $+1\ +1$, $\frac{2x}{2}=\frac{1}{2}$, $x=\frac{1}{2}$

(bottom) $x-2=0$, $+2\ +2$, $x=2$

(step 5, 6)

	$(-\infty,\frac{1}{2})$	$(\frac{1}{2},2)$	$(2,\infty)$
$(2x-1)$	neg	POS	POS
$(x-2)$	neg	neg	POS
$\frac{2(2x-1)}{(x-2)}$	POS	neg	POS

ans: $(\frac{1}{2}, 2)$ (step 7)

350) $\frac{3x}{x-4} > 2$

$-2 \quad -2$ (step 1)

$\frac{3x}{x-4} - 2 > 0$ LCD = $(x-4)$

$\frac{3x}{(x-4)} - 2\left(\frac{(x-4)}{(x-4)}\right) > 0$ (step 2)

$\frac{3x-2(x-4)}{(x-4)} > 0$

$\frac{3x-2x+8}{x-4} > 0$

$\frac{x+8}{x-4} > 0$

critical points (step 4)

(top)	(bottom)
$x+8=0$	$x-4=0$
$-8 \quad -8$	$+4 \quad +4$
$x=-8$	$x=4$

(step 5, 6)

	$(-\infty,-8)$	$(-8,4)$	$(4,\infty)$
$(x+8)$	neg	POS	POS
$(x-4)$	neg	neg	POS
$\frac{(x+8)}{(x-4)}$	POS	neg	POS

ans: $(-\infty,-8) \cup (4,\infty)$ (step 7)

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352) $\frac{4x-1}{x-4} \leq 1$

$-1 \quad -1$ (step 1)

$\frac{4x-1}{x-4} - 1 \leq 0$ LCD = $(x-4)$

$\frac{(4x-1)}{(x-4)} - 1\left(\frac{(x-4)}{(x-4)}\right) \leq 0$ (step 2)

$\frac{(4x-1)-1(x-4)}{(x-4)} \leq 0$

$\frac{4x-1-x+4}{(x-4)} \leq 0$ (step 3)

$\frac{3x+3}{x-4} \leq 0 \rightarrow \frac{3(x+1)}{(x-4)} \leq 0$

critical points (step 4)

(top)	(bottom)
$x+1=0$	$x-4=0$
$-1 \quad -1$	$+4 \quad +4$
$x=\boxed{-1}$	$x=4$

(step 5, 6)

	$\boxed{-1}$		4
	$(-\infty,-1)$	$(-1,4)$	$(4,\infty)$
$(x+1)$	neg	POS	POS
$(x-4)$	neg	neg	POS
$\frac{3(x+1)}{(x-4)}$	POS	neg	POS

ans: $[-1,4)$ (step 7)

354) $\frac{4x-3}{x-3} \geq 2$

$-2 \quad -2$ (step 1)

$\frac{(4x-3)}{(x-3)} - 2 \geq 0$ LCD = $(x-3)$

$\frac{(4x-3)}{(x-3)} - 2\left(\frac{(x-3)}{(x-3)}\right) \geq 0$ (step 2)

$\frac{(4x-3) - 2(x-3)}{(x-3)} \geq 0$

$\frac{4x-3-2x+6}{x-3} \geq 0$

$\frac{2x+3}{x-3} \geq 0$

critical points (step 4)

(top)

$2x+3=0$

$-3 \quad -3$

$\frac{2x}{2} = \frac{-3}{2}$

$x = \boxed{\frac{-3}{2}}$

(bottom)

$x-3=0$

$+3 \quad +3$

$x=3$

(step 5, 6)

	$(-\infty, \frac{-3}{2})$	$(\frac{-3}{2}, 3)$	$(3, \infty)$
$(2x+3)$	neg	POS	POS
$(x-3)$	neg	neg	POS
$\frac{(2x+3)}{(x-3)}$	POS	neg	POS

ans: $(-\infty, \frac{-3}{2}] \cup (3, \infty)$ (step 7)

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356) $\frac{1}{x^2-4x-12} > 0$ (step 3)

$\frac{1}{(x+2)(x-6)} > 0$

critical points (step 4)

(bottom)

$x+2=0 \quad | \quad x-6=0$

$-2 \quad -2 \quad | \quad +6 \quad +6$

$x=-2 \quad | \quad x=6$

(step 5, 6)

	$(-\infty, -2)$	$(-2, 6)$	$(6, \infty)$
$(x+2)$	neg	POS	POS
$(x-6)$	neg	neg	POS
$\frac{1}{(x+2)(x-6)}$	POS	neg	POS

ans: $(-\infty, -2) \cup (6, \infty)$ (step 7)

358) $\dfrac{4}{x^2+7x+12} < 0$ (step 3)

$\dfrac{4}{(x+4)(x+3)} < 0$

critical points (step 4)

(bottom)

$x+4=0$	$x+3=0$
$-4\ -4$	$-3\ -3$
$x=-4$	$x=-3$

(step 5, 6)

	$(-\infty,-4)$	$(-4,-3)$	$(-3,\infty)$
$(x+4)$	neg	POS	POS
$(x+3)$	neg	neg	POS
$\dfrac{1}{(x+4)(x+3)}$	POS	neg	POS

ans: $(-4,-3)$ (step 7)

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360) $\dfrac{6}{3x^2-2x-5} \geq 0$

$\dfrac{6}{(x+1)(3x-5)} \geq 0$ (step 3)

critical points (step 4)

(bottom)

$x+1=0$	$3x-5=0$
$-1\ -1$	$+5\ +5$
$x=-1$	$\frac{3x}{3}=\frac{5}{3}$
	$x=\frac{5}{3}$

$3x^2-2x-5$

3 $(1)(3)$ $\quad$ 5 $(1)(5)$

$=(1x+1)(3x-5)$

$+(3)(1)-(1)(5)$

$+3-5$

-2

(step 5, 6)

	$(-\infty,-1)$	$(-1,\frac{5}{3})$	$(\frac{5}{3},\infty)$
$(x+1)$	neg	POS	POS
$(3x-5)$	neg	neg	POS
$\dfrac{6}{(x+1)(3x-5)}$	POS	neg	POS

ans: $(-\infty,-1)\cup(\frac{5}{3},\infty)$ (step 7)

362) $\dfrac{-1}{10x^2+11x-6} \le 0$

(step 3)

$\dfrac{-1}{(2x+3)(5x-2)} \le 0$

10
(1)(10)
(2)(5)

$10x^2+11x-6$

6
(1)(6)
(2)(3)

$=(2x+3)(5x-2)$

$+(5)(3)-(2)(2)$

$+15-4$

$+11$

critical points (step 4)

(bottom)

$2x+3=0$	$5x-2=0$
$-3\ -3$	$+2\ +2$
$\frac{2x}{2}=\frac{-3}{2}$	$\frac{5x}{5}=\frac{2}{5}$
$x=\frac{-3}{2}$	$x=\frac{2}{5}$

(step 5,6)	$(-\infty,-\frac{3}{2})$	$(-\frac{3}{2},\frac{2}{5})$	$(\frac{2}{5},\infty)$
$(2x+3)$	neg	POS	POS
$(5x-2)$	neg	neg	POS
$\frac{-1}{(2x+3)(5x-2)}$	neg	POS	neg

ans: $(-\infty,-\frac{3}{2})\cup(\frac{2}{5},\infty)$ (step 7)

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364) $\dfrac{1}{3}+\dfrac{1}{x^2} > \dfrac{4}{3x}$ (step 1)

$\frac{-4}{3x}\quad\frac{-4}{3x}$

$\dfrac{1}{3}-\dfrac{4}{3x}+\dfrac{1}{x^2}>0$ LCD $=3x^2$

$\dfrac{1}{3}\left(\dfrac{x^2}{x^2}\right)-\dfrac{4}{3x}\left(\dfrac{x}{x}\right)+\dfrac{1}{x^2}\left(\dfrac{3}{3}\right)>0$

$\dfrac{x^2}{3x^2}-\dfrac{4x}{3x^2}+\dfrac{3}{3x^2}>0$

$\dfrac{x^2-4x+3}{3x^2}>0$ (step 3)

$\dfrac{(x-1)(x-3)}{3x^2}>0$

critical points (step 4)

(top)

$x-1=0$	$x-3=0$
$+1\ +1$	$+3\ +3$
$x=1$	$x=3$

(bottom)

$3x^2=0$

$x^2=0$

$x=0$

(step 2)

(step 5,6)	$(-\infty,0)$	$(0,1)$	$(1,3)$	$(3,\infty)$
$(x-1)$	neg	neg	POS	POS
$(x-3)$	neg	neg	neg	POS
x	neg	POS	POS	POS
$\frac{(x-1)(x-3)}{3x^2}$	POS	POS	neg	POS

ans: $(-\infty,0)\cup(0,1)\cup(3,\infty)$ (step 7)

366) $\frac{1}{2} - \frac{3}{2x^2} \geq \frac{1}{x}$ (step 1)

$\frac{-1}{x} \qquad -\frac{1}{x}$

$\frac{1}{2} - \frac{1}{x} - \frac{3}{2x^2} \geq 0$ LCD $= 2x^2$ (step 2)

$\frac{1}{2}\left(\frac{x^2}{x^2}\right) - \frac{1}{x}\left(\frac{2x}{2x}\right) - \frac{3}{2x^2} \geq 0$

$\frac{x^2}{2x^2} - \frac{2x}{2x^2} - \frac{3}{2x^2} \geq 0$

$\frac{x^2 - 2x - 3}{2x^2} \geq 0$

$\frac{(x+1)(x-3)}{2x^2} \geq 0$ (step 3)

critical points (step 4)

(top)

$x+1=0$ | $x-3=0$

$-1\ -1$ | $+3\ +3$

$x = \boxed{-1}$ | $x = \boxed{3}$

(bottom)

$2x^2 = 0$

$x^2 = 0$

$x = 0$

(step 5,6)	$(-\infty,-1)$	$(-1,0)$	$(0,3)$	$(3,\infty)$
		$\boxed{-1}$	0	$\boxed{3}$
$(x+1)$	neg	POS	POS	POS
$(x-3)$	neg	neg	neg	POS
x	neg	neg	POS	POS
$\frac{(x+1)(x-3)}{3x^2}$	POS	neg	neg	POS

ans: $(-\infty, -1] \cup [3, \infty)$ (step 7)

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368) $\frac{4}{x^2-25} > 0$ (step 3)

$\frac{4}{(x+5)(x-5)} > 0$

critical points (step 4)

(bottom)

$x+5=0$ | $x-5=0$

$-5\ -5$ | $+5\ +5$

$x=-5$ | $x=5$

(step 5,6)	$(-\infty,-5)$	$(-5,5)$	$(5,\infty)$
		-5	5
$(x+5)$	neg	POS	POS
$(x-5)$	neg	neg	POS
$\frac{4}{(x+5)(x-5)}$	POS	neg	POS

ans: $(-\infty, -5) \cup (5, \infty)$ (step 7)

370) $\frac{5}{x-1} \leq \frac{4}{x+2}$ (step 1)

$\frac{-4}{x+2}$ $\frac{-4}{x+2}$

$\frac{5}{(x-1)} - \frac{4}{(x+2)} \leq 0$ LCD $= (x+2)(x-1)$ (step 2)

$\frac{5}{(x-1)}\left(\frac{(x+2)}{(x+2)}\right) - \frac{4}{(x+2)}\left(\frac{(x-1)}{(x-1)}\right) \leq 0$

$\frac{5(x+2) - 4(x-1)}{(x+2)(x-1)} \leq 0$

$\frac{5x+10-4x+4}{(x+2)(x-1)} \leq 0$

$\frac{x+14}{(x+2)(x-1)} \leq 0$

critical points (step 4)

(top)

$x+14=0$

$-14 \quad -14$

$x = \boxed{-14}$

(bottom)

$x+2=0$ | $x-1=0$

$-2 \quad -2$ | $+1 \quad +1$

$x=-2$ | $x=1$

(step 5,6)	$(-\infty, -14)$	$(-14, -2)$	$(-2, 1)$	$(1, \infty)$
$(x+14)$	neg	POS	POS	POS
$(x+2)$	neg	neg	POS	POS
$(x-1)$	neg	neg	neg	POS
$\frac{(x+14)}{(x+2)(x-1)}$	neg	POS	neg	POS

ans: $(-\infty, -14] \cup (-2, 1)$ (step 7)

372) $R(x) = \frac{x+1}{x+3}$, $R(x) \geq 0$

$\frac{x+1}{x+3} \geq 0$

critical points (step 4)

(top)

$x+1=0$

$-1 \quad -1$

$x = \boxed{-1}$

(bottom)

$x+3=0$

$-3 \quad -3$

$x=-3$

(step 5,6)	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
$(x+1)$	neg	neg	POS
$(x+3)$	neg	POS	POS
$\frac{(x+1)}{(x+3)}$	POS	neg	POS

ans: $(-\infty, -3) \cup [-1, \infty)$ (step 7)

374) $R(x) = \frac{x+1}{x-4}$, $R(x) \le 0$

$\frac{x+1}{x-4} \le 0$

critical points (step 4)

(top) $x+1=0$, $-1\ -1$, $x=\boxed{-1}$

(bottom) $x-4=0$, $+4\ +4$, $x=4$

(step 5,6)	$(-\infty,-1)$	$(-1,4)$	$(4,\infty)$
$(x+1)$	neg	POS	POS
$(x-4)$	neg	neg	POS
$\frac{(x+1)}{(x-4)}$	POS	neg	POS

(-1 boxed, 4)

ans: $[-1, 4)$ (step 7)

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376) Create a rational inequality whose solution is $(-\infty, -2] \cup [4, \infty)$.

Hint for our solution is in exercise 366 starting from answer and do reverse work to find an equation.

① Since both -2 and 4 are included, the factors containing these values must be in the numerator (top).

② Pick an easy value between -2 and 4 which is 0 $\rightarrow x=0$

$x=0 \Rightarrow x^2=0$

③ table for $(-\infty,-2] \cup [4,\infty)$

factor	$(-\infty,-2)$	$(-2,0)$	$(0,4)$	$(4,\infty)$
for -2: $x=-2 \rightarrow (x+2)$, $+2\ +2$, $x+2=0$	neg	POS	POS	POS
for 4: $x=4$, $-4\ -4$, $x-4=0 \rightarrow (x-4)$	neg	neg	neg	POS
x^2	POS	POS	POS	POS
$\frac{(x+2)(x-4)}{x^2}$	POS	neg	neg	POS

(-2 boxed, 0, 4 boxed)

one possible inequality is

$$\frac{(x+2)(x-4)}{x^2} \ge 0$$