

Perfect Square Trinomials

to use the quick form we need to set up the trinomial into $a^2 + 2ab + b^2$ so that the factors are

$$(a+b)(a+b) = (a+b)^2$$

note: if you don't or can't rewrite as pattern above then just use the technique of section 6.2

Difference of Squares

these expressions are always 2 terms with even powered variables

$$a^2 - b^2 = (a+b)(a-b)$$

note: if 2 terms have same sign, it is not factorable
i.e. $a^2 + b^2$ or $-a^2 - b^2 = -1(a^2 + b^2)$

Sums and Differences of Cubes

the pattern below shares the same form
(written in black) sum part (in blue) difference part (in red)

$$a^3 \pm b^3 = (a \pm b) \underbrace{(a^2 \mp ab + b^2)}$$

this trinomial is not factorable

$$\begin{aligned}
 160) \quad & 25v^2 + 20v + 4 \\
 &= (5v)^2 + 20v + (+2)^2 \\
 &= ((5v) + (+2))^2 \\
 &= \underline{\underline{(5v + 2)^2}}
 \end{aligned}$$

"middle term +"

6.3 2

$$\begin{aligned}
 a &= 5v, \quad b = +2 \\
 ab &= (5v)(+2) = +10v \\
 2ab &= 2(+10v) = \underline{\underline{+20v}}
 \end{aligned}$$

$$\begin{aligned}
 162) \quad & 49s^2 + 154s + 121 \\
 &= (7s)^2 + 154s + (+11)^2 \\
 &= ((7s) + (+11))^2 \\
 &= \underline{\underline{(7s + 11)^2}}
 \end{aligned}$$

"middle term +"

$$\begin{aligned}
 a &= 7s, \quad b = +11 \\
 ab &= (7s)(+11) = +77s \\
 2ab &= 2(+77s) = \underline{\underline{+154s}}
 \end{aligned}$$

$$\begin{aligned}
 164) \quad & 64z^2 - 16z + 1 \\
 &= (8z)^2 - 16z + (-1)^2 \\
 &= ((8z) + (-1))^2 \\
 &= \underline{\underline{(8z - 1)^2}}
 \end{aligned}$$

"middle term -"

$$\begin{aligned}
 a &= 8z, \quad b = -1 \\
 ab &= (8z)(-1) = -8z \\
 2ab &= 2(-8z) = \underline{\underline{-16z}}
 \end{aligned}$$

$$\begin{aligned}
 166) \quad & 4p^2 - 52p + 169 \\
 &= (2p)^2 - 52p + (-13) \\
 &= ((2p) + (-13))^2 \\
 &= \underline{\underline{(2p - 13)^2}}
 \end{aligned}$$

"middle term -"

$$a = 2p, \quad b = \downarrow -13$$

$$ab = (2p)(-13) = -26p$$

$$2ab = 2(-26p) = \underline{\underline{-52p}}$$

$$\begin{aligned}
 168) \quad & 25r^2 + 60rs + 36s^2 \\
 &= (5r)^2 + 60rs + (+6s)^2 \\
 &= ((5r) + (+6s))^2 \\
 &= \underline{\underline{(5r + 6s)^2}}
 \end{aligned}$$

"middle term +"

$$a = 5r, \quad b = \downarrow +6s$$

$$ab = (5r)(+6s) = +30rs$$

$$2ab = 2(+30rs) = \underline{\underline{+60rs}}$$

$$\begin{aligned}
 170) \quad & 64m^2 - 16m + 1 \\
 &= (8m)^2 - 16m + (-1)^2 \\
 &= ((8m) + (-1))^2 \\
 &= \underline{\underline{(8m - 1)^2}}
 \end{aligned}$$

"middle term -"

$$a = 8m, \quad b = \downarrow -1$$

$$ab = (8m)(-1) = -8m$$

$$2ab = 2(-8m) = \underline{\underline{-16m}}$$

$$172) 64x^2y - 96xy + 36y$$

$$= 4y(16x^2 - 24x + 9)$$

$$= 4y((4x)^2 - 24x + (-3)^2)$$

$$= 4y((4x) + (-3))^2$$

$$= \underline{\underline{4y(4x-3)^2}}$$

"middle term -"

$$a = 4x, b = -3$$

$$ab = (4x)(-3) = -12x$$

$$2ab = 2(-12x) = \underline{\underline{-24x}}$$

$$174) 90p^4 + 300p^3q + 250p^2q^2$$

$$= 10p^2(9p^2 + 30pq + 25q^2)$$

$$= 10p^2((3p)^2 + 30pq + (+5q)^2)$$

$$= 10p^2((3p) + (5q))^2$$

$$= \underline{\underline{10p^2(3p+5q)^2}}$$

"middle term +"

$$a = 3p, b = +5q$$

$$ab = (3p)(+5q) = +15pq$$

$$2ab = 2(+15pq) = \underline{\underline{+30pq}}$$

$$176) 169q^2 - 1$$

$$= (13q)^2 - (1)^2$$

$$= \underline{\underline{(13q+1)(13q-1)}}$$

$$178) 121 - 25s^2$$

$$= (11)^2 - (5s)^2$$

$$= \underline{\underline{(11+5s)(11-5s)}}$$

$$\begin{aligned}
 180) \quad & 98z^3 - 72z \\
 &= 2z(49z^2 - 36) \\
 &= 2z((7z)^2 - (6)^2) \\
 &= \underline{\underline{2z(7z+6)(7z-6)}}
 \end{aligned}$$

$$\begin{aligned}
 184) \quad & 49x^2 - 81y^2 \\
 &= (7x)^2 - (9y)^2 \\
 &= \underline{\underline{(7x+9y)(7x-9y)}}
 \end{aligned}$$

$$\begin{aligned}
 188) \quad & m^4 - n^4 \\
 &= (m^2)^2 - (n^2)^2 \\
 &= (m^2+n^2)(m^2-n^2) \\
 &= \underline{\underline{(m^2+n^2)(m+n)(m-n)}}
 \end{aligned}$$

6.3 5

$$\begin{aligned}
 182) \quad & 20b^2 + 140 \\
 &= \underline{\underline{20(b^2 + 7)}} \\
 &(b^2+7) \rightarrow \text{same sign,} \\
 &\quad \text{not factorable}
 \end{aligned}$$

$$\begin{aligned}
 186) \quad & 36p^2 - 49q^2 \\
 &= (6p)^2 - (7q)^2 \\
 &= \underline{\underline{(6p+7q)(6p-7q)}}
 \end{aligned}$$

$$\begin{aligned}
 190) \quad & 48m^4n^2 - 243n^2 \\
 &= 3n^2(16m^4 - 81) \\
 &= 3n^2((4m^2)^2 - (9)^2) \\
 &= 3n^2(4m^2+9)(4m^2-9) \\
 &= 3n^2(4m^2+9)((2m)^2 - (3)^2) \\
 &= \underline{\underline{3n^2(4m^2+9)(2m+3)(2m-3)}}
 \end{aligned}$$

$$192) p^2 + 14p + 49 - q^2$$

$$= \{p^2 + 14p + (+7)^2\} - q^2$$

$$= \{(p) + (+7)\}^2 - q^2$$

$$= (p+7)^2 - q^2$$

$$= ((p+7) + q)((p+7) - q)$$

$$= \underline{\underline{(p+7-q)(p+7+q)}}$$

$$\left\{ \begin{array}{l} \text{"middle term +"} \\ a = p, \quad b = +7 \\ ab = (p)(+7) = +7p \\ 2ab = 2(+7p) = \underline{+14p} \end{array} \right\}$$

$$194) m^2 - 6m + 9 - 16n^2$$

$$= \{m^2 - 6m + (-3)^2\} - 16n^2$$

$$= \{(m) + (-3)\}^2 - 16n^2$$

$$= (m-3)^2 - 16n^2$$

$$= (m-3)^2 - (4n)^2$$

$$= ((m-3) + (4n))((m-3) - (4n))$$

$$= \underline{\underline{(m-3+4n)(m-3-4n)}}$$

$$\left\{ \begin{array}{l} \text{"middle term -"} \\ a = m, \quad b = -3 \\ ab = (m)(-3) = -3m \\ 2ab = 2(-3m) = \underline{-6m} \end{array} \right\}$$

$$(196) \quad n^6 + 512$$

$$= (n^2)^3 + (8)^3 = (n^2 + 8) \left((n^2)^2 - (n^2)(8) + (8)^2 \right) \\ = \underline{\underline{(n^2 + 8)(n^4 - 8n^2 + 64)}}$$

$$(198) \quad v^3 - 216$$

$$= v^3 - (6)^3 = (v - 6) \left(v^2 + (v)(6) + (6)^2 \right) \\ = \underline{\underline{(v - 6)(v^2 + 6v + 36)}}$$

$$(200) \quad 125 - 27w^3$$

$$= (5)^3 - (3w)^3 = (5 - 3w) \left((5)^2 + (5)(3w) + (3w)^2 \right) \\ = \underline{\underline{(5 - 3w)(25 + 15w + 9w^2)}}$$

$$(202) \quad 27x^3 - 64y^3$$

$$= (3x)^3 - (4y)^3 = ((3x) - (4y)) \left((3x)^2 + (3x)(4y) + (4y)^2 \right) \\ = \underline{\underline{(3x - 4y)(9x^2 + 12xy + 16y^2)}}$$

$$204) \quad 27y^3 + 8z^3$$

$$= (3y)^3 + (2z)^3 = ((3y) + (2z))((3y)^2 - (3y)(2z) + (2z)^2) \\ = \underline{\underline{(3y + 2z)(9y^2 - 6yz + 4z^2)}}$$

$$206) \quad 6x^3 - 48y^3 = 6(x^3 - 8y^3)$$

$$= 6(x^3 - (2y)^3) = 6(x - (2y))(x^2 + (x)(2y) + (2y)^2) \\ = \underline{\underline{6(x - 2y)(x^2 + 2xy + 4y^2)}}$$

$$208) \quad -2x^3y^2 - 16y^5 = -2y^2(x^3 + 8y^3)$$

$$= -2y^2(x^3 + (2y)^3) = -2y^2(x + (2y))(x^2 - (x)(2y) + (2y)^2) \\ = \underline{\underline{-2y^2(x + 2y)(x^2 - 2xy + 4y^2)}}$$

$$210) \quad (x+4)^3 - 27x^3$$

$$= (x+4)^3 - (3x)^3 = ((x+4) - (3x))((x+4)^2 + (x+4)(3x) + (3x)^2)$$

$$= (x+4 - 3x)((x^2 + 8x + 16) + 3x^2 + 12x + 9x^2)$$

$$= \underline{\underline{(4 - 2x)(13x^2 + 20x + 16)}}$$

$$212) (y-5)^3 + 125y^3$$

$$\begin{aligned}
 &= (y-5)^3 + (5y)^3 = ((y-5) + (5y))((y-5)^2 - (y-5)(5y) + (5y)^2) \\
 &= (y-5+5y)(y^2-10y+25-5y^2+25y+25y^2) \\
 &= \underline{\underline{(6y-5)(21y^2+15y+25)}}
 \end{aligned}$$

$$214) 121x^2 - 144 = (11x)^2 - (12)^2 = \underline{\underline{(11x+12)(11x-12)}}$$

$$216) 4p^2 - 100 = 4(p^2 - 25) = 4(p^2 - (5)^2) = \underline{\underline{4(p+5)(p-5)}}$$

$$\begin{aligned}
 218) & 36y^2 + 12y + 1 \\
 &= (6y)^2 + 12y + (+1)^2 \\
 &= ((6y) + (+1))^2 \\
 &= \underline{\underline{(6y+1)^2}}
 \end{aligned}$$

"middle term +"

$$a = 6y, \quad b = +1$$

$$ab = (6y)(+1) = +6y$$

$$2ab = 2(+6y) = \underline{\underline{+12y}}$$

$$220) 81x^2 + 169 = (9x)^2 + (13)^2$$

same sign
not factorable

$$222) 27u^3 + 1000$$

$$\begin{aligned}
 &= (3u)^3 + (10)^3 = ((3u) + (10))((3u)^2 - (3u)(10) + (10)^2) \\
 &= \underline{\underline{(3u+10)(9u^2-30u+100)}}
 \end{aligned}$$

$$224) 48q^3 - 24q^2 + 3q$$

$$= 3q(16q^2 - 8q + 1)$$

$$= 3q((4q)^2 - 8q + (-1)^2)$$

$$= 3q((4q) + (-1))^2$$

$$= \underline{\underline{3q(4q-1)^2}}$$

"middle term -"

$$a = 4q, b = -1$$

$$ab = (4q)(-1) = -4q$$

$$2ab = 2(-4q) = \underline{\underline{-8q}}$$

$$226) x^2 + 12x + 36 - y^2$$

$$= \{x^2 + 12x + (+6)\} - y^2$$

$$= \{(x + (+6))\}^2 - y^2$$

$$= (x + 6)^2 - y^2$$

$$= ((x+6) + y)((x+6) - y) = \underline{\underline{(x+6+y)(x+6-y)}}$$

$$\left\{ \begin{array}{l} \text{"middle term +"} \\ a = x, b = +6 \\ ab = (x)(+6) = +6x \\ 2ab = 2(+6x) = \underline{\underline{+12x}} \end{array} \right\}$$

$$228) (y-3)^3 - 64y^3$$

$$= (y-3)^3 - (4y)^3 = ((y-3) - (4y))((y-3)^2 + (y-3)(4y) + (4y)^2)$$

$$= (y-3-4y)(y^2-6y+9) + 4y^2-12y+16y^2$$

$$= \underline{\underline{(-5y-3)(21y^2-18y+9)}}$$